A. La Rosa Physics 464/564

Homework #1 Due 04-09-09

1. Light as a wave

It was mentioned during the lectures in class that the "Newton's rings" experiment supported the hypothesis that light was a wave. The following question aims to quantify and provide ma more solid argument of that hypothesis.

1A If we place a slightly convex glass surface on a flat glass and view the reflected light from the glass-air-glass interfaces, we observe a dark spot at the point of contact. Justify this observation

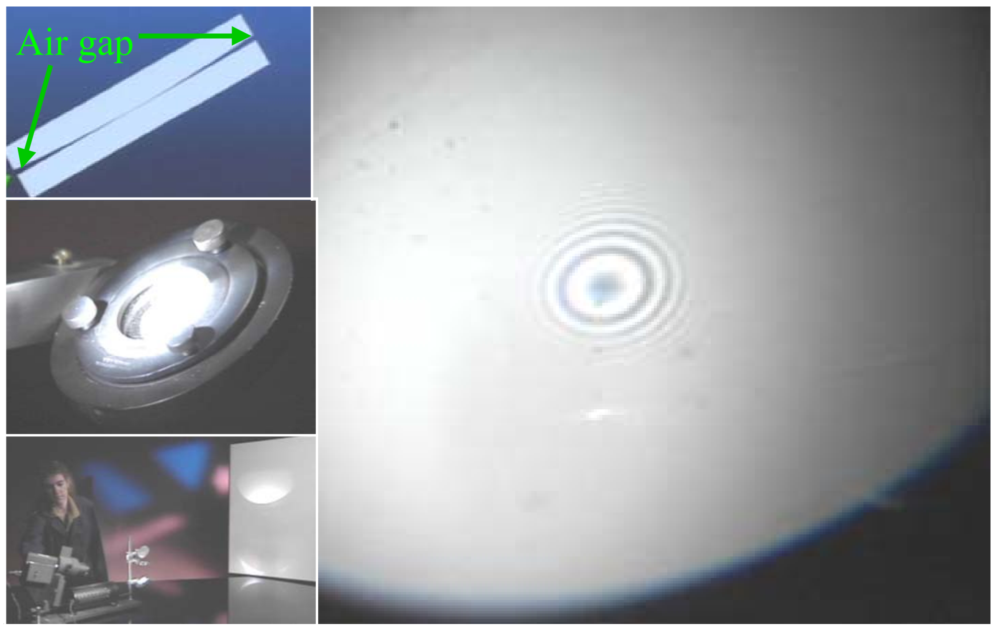

Fig.1 Experimental demonstration of the Newton's rings. Figures downloaded from http://www.physics.montana.edu/demonstrations/video/6_optics/demos/newtonsrings.html

1B Away from the center concentric bright and dark rings are observed. This is attributed to the variation in thickness of the air gap between the convex glass and the flat glass surfaces. Evaluate the different thicknesses of the air-gap at which the **dark rings** will be observed. Assume the incident light has a wavelength of 532 nm.

1C If we rather view the transmitted light the central spot is bright. Provide arguments to justify this observation.

2. Atom in thermal equilibrium with electromagnetic radiation inside a box

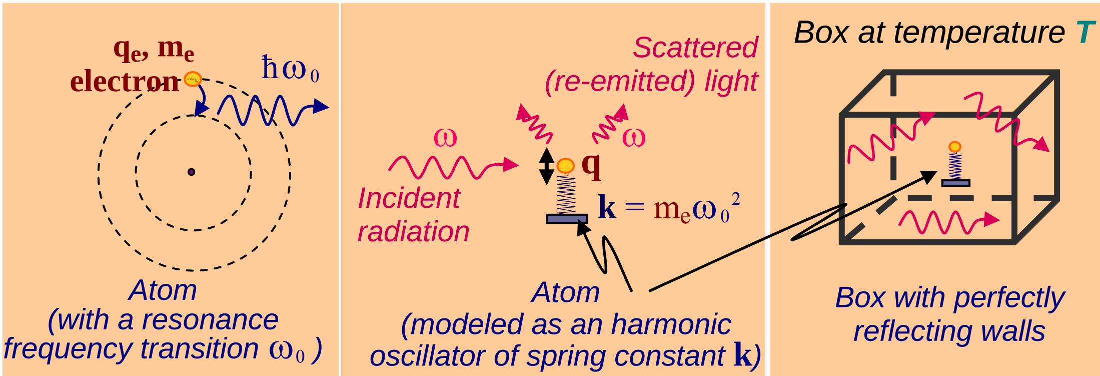

Fig. 2 Left: Atom with a pair of transition energy levels separated by $\Delta E = \hbar\omega_o$. **Center:** Such an atom modeled as a harmonic oscillator, whose associated spring constant $\mathbf{k}$ is chosen to be equal to $\mathbf{k} = m_e\omega_o^2$. When excited with radiation of frequency ω, the oscillator re-emits the radiation (a phenomenon known as scattering.) **Right:** Atom in equilibrium with electromagnetic radiation inside a box. (Energy radiated by the atom equals the energy absorbed by the atom from the radiation inside the cavity)

2A. Energy dissipation

Let W be the energy content of the harmonic oscillator. The energy radiated per second can be expressed as $\frac{dW}{dt} = -\gamma W$ **,** where the electromagnetic damping constant (which results from taking into account that an accelerate charge emits radiation)) is given by,

$$\gamma = \frac{2}{3c}\frac{q_e^{\,2}}{4\pi\varepsilon_o m_e c^2}\omega^2 \qquad (1)$$

Here q_e and m_e are the mass and charge of the electron respectively.

QUESTION: At resonance condition, when $\omega = \omega_o$, an atom radiates light of λ=532 nm. Calculate the numerical value of the corresponding electromagnetic damping constant.

2.B. Absorption of energy

The batch of electromagnetic radiation inside the box (see figure above) is described by the traveling energy flux (or power) per unit area ***I***. Since this density can depend on the frequency, it is convenient to define a ***spectral intensity*** $\mathcal{I}$, which is defined as

$\mathcal{I}(\omega)\, d\omega =$ contribution to the intensity from radiation having an angular frequency between ω and $\omega + d\omega$ (2)

That is $I = \int_0^\infty \mathcal{I}(\omega)\, d\omega$ (3)

When an atom (characterized by a resonance absorption frequency equal to ω_o) is inside a bath of electromagnetic radiation $\mathcal{I}(\omega)$ it has an ability of absorbing electromagnetic energy proportional to $\mathcal{I}(\omega)$,

$\sigma(\omega)\mathcal{I}(\omega)\, d\omega$

Here the proportionality factor $\sigma(\omega)$ is called the "scattering cross section area", and it is given by,

$$\sigma(\omega) = \frac{8\pi}{3}\frac{e^4}{m_e{}^2 c^4}\frac{\omega^4}{(\omega^2 - \omega_o{}^2)^2 + \gamma^2\omega^2} \qquad (4)$$

where $e^2 = q_e{}^2/4\pi\varepsilon_o$, q_e and m_e are the electron's charge and mass respectively.

QUESTION: Verify that σ has the units on area

QUESTION: Justify that σ can also be expressed as

$$\sigma(\omega) = \frac{8\pi}{3}\frac{e^4}{m^2 c^4}\frac{\omega_o{}^2}{(\omega - \omega_o)^2 + (\gamma/2)^2} \qquad (5)$$

Hint: $\omega^2 - \omega_o{}^2 = (\omega + \omega_o)(\omega - \omega_o)$, $\quad \omega + \omega_o \sim 2\omega_o$, $\quad etc$

QUESTION: Using ω_o corresponding to λ=532 nm make a plot of σ *vs* ω .

Hint: You may want to make the plot in units of γ.
(Pasted graphs that you construct from an excel file would be fine.)

3. Light Scattering

An atom whose emission/absorption spectrum displays a peak at the frequency ω_0, can be modeled as a harmonic oscillator of spring constant $k= m_e\ \omega_0{}^2$, where m_e is the electron's mass. Accordingly, a description of its corresponding dynamics, when excited by an external harmonic electric field $E = E_o e^{j\omega t}$ can be obtained by applying Newton's second law,

$m_e\, a =$ sum (all forces)

$$m_e \frac{d^2x}{dt^2} = q_e E_o e^{j\omega t} - m\gamma\frac{dx}{dt} - kx \qquad (6)$$

with γ given in expression (1) above.

We need to find $x = x(t)$

3.1 Show that Eq. (6) admits solutions of the form

$$x = [x_o e^{j\varphi}]e^{j\omega t} \qquad (7)$$

Notice: Real $x = x_0 \, \text{Cos}(\omega t + \varphi)$

More explicitly, show that

$$x_o = x_o(\omega) = \frac{(q_e / m_e) E_o}{\left[(\omega_o{}^2 - \omega^2)^2 \; + \; \gamma^2 \omega^2 \right]^{1/2}} \qquad (8)$$

and

$$\varphi = \tan^{-1}\left(- \, \frac{\gamma\omega}{\omega_o{}^2 - \omega^2} \right) \qquad (9)$$

3.2 Give a physical interpretation of φ.