A. La Rosa **Physics 464/564**

Homework #2 Due 04-16-09

1. Electromagnetic Modes

When describing the mechanical vibration of, for example, a cylindrical rod, those particular states of motion in which all the particles in the rod oscillate at the same frequency are called MODE. Similarly, in the description of electromagnetic waves inside a cavity, a single value of ω specifies a mode.

If we had just two parallel metal plates separated by a distance "*a*" a mode could be described by the following electric field,

$$E(x,t) = E_o SIN(\frac{2\pi}{\lambda}x)\, SIN(\omega_x t)$$

$$= E_o SIN(\omega_x \frac{1}{c}x)\, SIN(\omega_x t) \qquad (1)$$

It was explained in class that, in order to satisfy the boundary conditions, (E=0 in a metal) the values for ω_x are restrained to have some discrete values,

$$\omega_x = n\,\pi\frac{c}{a} \text{ , where } n = 1, 2, 3, \ldots \text{ (One dimensional case)} \qquad (2)$$

1A. For the **one dimensional case**, calculate how many electromagnetic modes there exist when consider a frequency interval between ω_x and $\omega_x + \Delta\omega_x$.

Indicate whether or not the value you found depends on the value of ω_x.

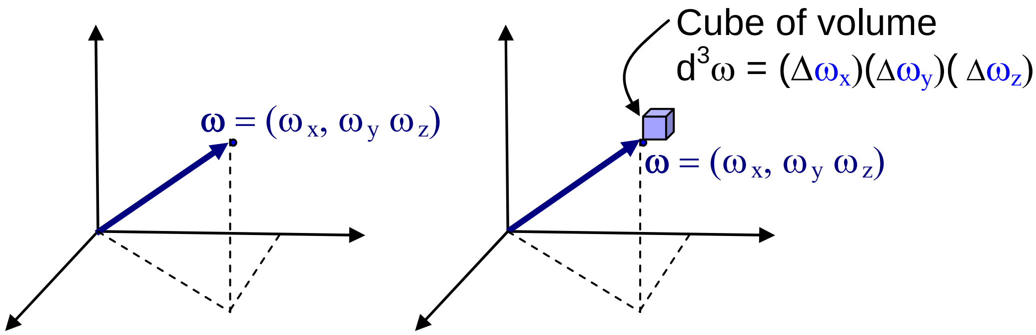
1B. For the **3D case** of a cubic metallic box of side "*a*", a single mode is specified rather by three numbers: $(\omega_x, \omega_y, \omega_z)$. See figure below.

Each of these three frequency numbers are subjected to the condition given in expression (2) above.

Question: Obtain an expression to calculate the number of modes in the cubic domain (shown in the figure below) of volume $d^3\omega = (\Delta\omega_x)\ (\Delta\omega_y)\ (\Delta\omega_z)$.

1C. For the **3D case**, consider of a cubic metallic box of side "*a*". Derive an expression to calculate $N(\omega)d\omega$, the latter defined as the **number of modes per unit volume** whose frequencies have a magnitude $|\boldsymbol{\omega}| \equiv \omega$ between ω and $\omega + d\,\omega$.

Hint: We are being asked to find the number of modes inside a spherical shell or radius ω and $\omega + d\,\omega$. Notice, however, that only the positive values of ω count as different modes, then the volume to consider is 1/8 of the volume of the shell.

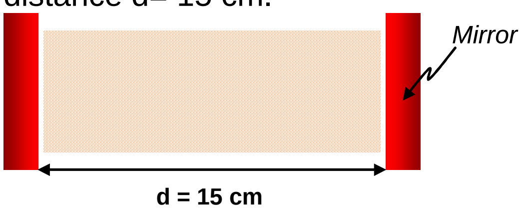


2. Spectral Electromagnetic Energy Density (U)

Let $U(\omega)d\omega$ be energy per unit volume inside the cube contributed by modes having frequency between ω and $\omega + d\omega$.

Using the expression you have obtained for $N(\omega)d\omega$ in question 1C above, derive an expression for $U(\omega)$.
(To answer the question above completely, you will need the average energy of each mode: let's call it W_{mode}. Accordingly, give your answer in terms of W_{mode}.

3. A laser cavity resonator consists of two parallel mirrors separated by a distance d= 15 cm.



3A. Derive an expression to calculate the frequencies (and corresponding wavelengths) of the different modes that are possible to be established inside the cavity.

3B. Evaluate numerically the difference in frequency between two contiguous modes. Assume $c = 3x10^8$ m/s. Express your answer in units of GHz.

4. A laser cavity resonator consists of two parallel mirrors separated by a distance d= 15 cm.

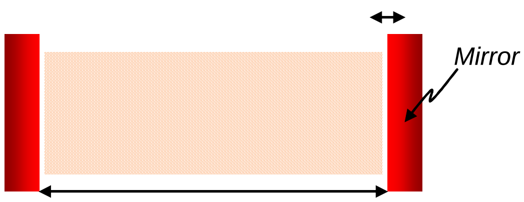
4A. List the frequencies (and their corresponding wavelengths) of a few cavity modes whose wavelengths are around λ=532 nm.

4B Demonstrate that $(\Delta\lambda)/\lambda = (\Delta f)/f$

For λ = 532 nm calculate the corresponding frequency in units of Hz as well as in units of GHz

How many modes with wavelengths in the range between 532 nm and 533 nm could be established the cavity?

5. A laser cavity resonator consists of two parallel mirrors separated initially by a distance d= 15 cm, but this time you can vary the separation distance.
Let's consider a specific mode, m = 600,000 for example.



5A. What is the wavelength and frequency corresponding to the mode m= 600,000?

5B. Derive expressions that allows calculating how the wavelength and the frequency of the mode m changes when the separation between mirrors vary from d to d +Δd.
Hint: Show and justify very explicitly all the steps leading to

$$f_m(d') = f_m(d)\ [\ 1 - (\Delta d / d)\ ]$$

$$\lambda_m(d') = [\lambda_m(d)\ ]\ [\ 1 + (\Delta d / d)\ ]$$

5C. What value of Δd is required to change the frequency of the mode m by c/2d?