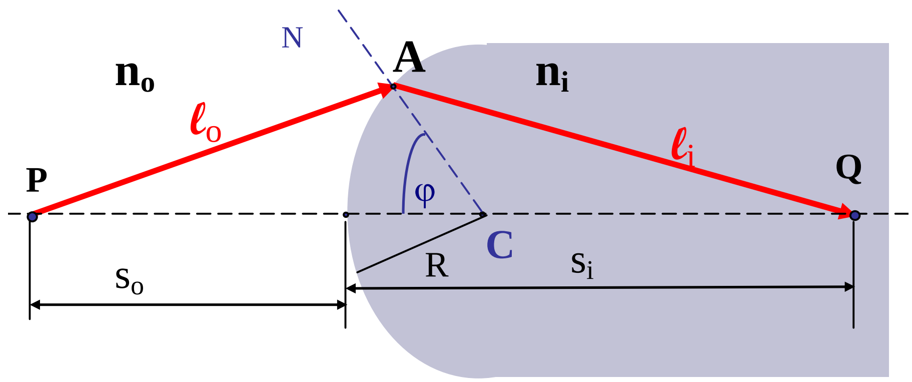
Physics 464/564

Homework #5 Due May 14th, 2009

1. The figure shows a point P being imaged at Q by a spherical surface. Assume $n_i > n_o$.



1A Demonstrate that ℓ_o and ℓ_i can be expressed, respectively, as

$$\ell_o^2 = (s_o + R)^2 + R^2 - 2\,(s_o + R)\; R\; Cos(\varphi)$$

$$\ell_i^2 = R^2 + (s_i - R)^2 + 2\; R(s_i - R)\; Cos(\varphi)$$

1B The condition for the optical path length $\ell = n_0\,\ell_o + n_i\,\ell_i$ to be stationary can be expressed as $\frac{d\ell}{d\varphi} = 0$. Show that this condition leads to the following condition to be satisfied by ℓ_o and ℓ_i :

Either

$$\varphi = 0$$

or

$$\frac{n_o}{\ell_o} + \frac{n_i}{\ell_i} = \frac{1}{R}\left[n_i \frac{s_i}{\ell_i} - n_o \frac{s_o}{\ell_o} \right]$$ *The Lens Equation* (exact version)

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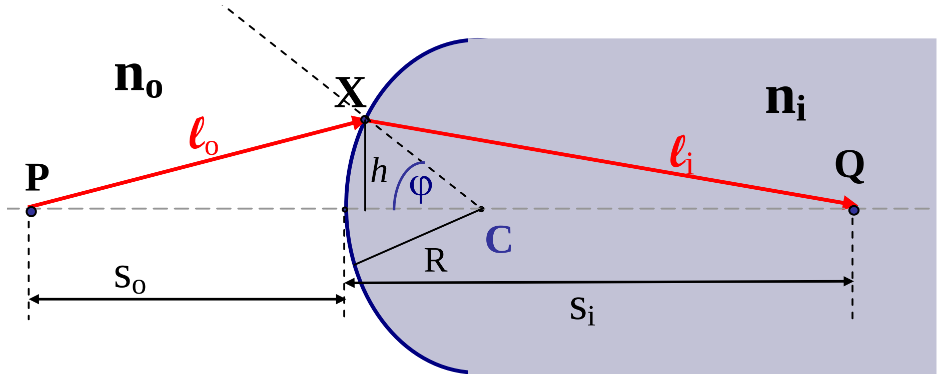
2. The figure shows a point X on the spherical surface that is at a distance h from the optical axis.

Show that for h/R <<1,

$$Cos(\varphi) = 1 - \frac{1}{2}\left(\frac{h}{R}\right)^2 - \frac{1}{8}\left(\frac{h}{R}\right)^4 + \ldots$$



Using the expression for ℓ_o and ℓ_i obtained in question 1 above, show that they can be expressed in the following approximate form,

$$\ell_o = s_o\left[1+(\frac{1}{R}+\frac{1}{s_o})\frac{R^2}{s_o}\left(\frac{h}{R}\right)^2+\frac{1}{4}(\frac{1}{R}+\frac{1}{s_o})\frac{R^2}{s_o}\left(\frac{h}{R}\right)^4+\ldots\right]^{1/2}$$

and

$$\ell_i = s_i\left[1-(\frac{1}{R}-\frac{1}{s_i})\frac{R^2}{s_i}\left(\frac{h}{R}\right)^2-\frac{1}{4}(\frac{1}{R}-\frac{1}{s_i})\frac{R^2}{s_i}\left(\frac{h}{R}\right)^4+\ldots\right]^{1/2}$$

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3. **3A.** Show that when the quadratic term in h/R are discarded, the Lens Equation adopts the form

$$\frac{n_o}{s_o}+\frac{n_i}{s_i}=\frac{n_i-n_o}{R}$$

3B. When pressed to provide a better approximation, as to include the quadratic terms in h/R, show that the lens Eq. adopts the form

$$\frac{n_o}{s_o}+\frac{n_i}{s_i}=\frac{n_i-n_o}{R}+\frac{1}{2}\left[\frac{n_o}{s_o}(\frac{1}{R}+\frac{1}{s_o})^2+\frac{n_i}{s_i}(\frac{1}{R}-\frac{1}{s_i})^2\right]h^2$$