SECOND APPROACH

"Is the field E_a given in expression (9) the kind of field we would expect from oscillating charges in the plate?

If the answer is affirmative, then we would have a way to identify the index of refraction "n" in terms of the fundamental electro magnetic quantities.

All we need to do is to look at expression (6), which gives the electric field due to charges oscillating on a plane cross section

$$E_a' = -\frac{\eta_0 q x_0}{2\epsilon_0}\left(i\frac{\omega}{c}\right)e^{i\omega(t-\frac{z}{c})} \qquad (10)$$

where x_0 is the "amplitude" of the oscillating charge that are responding to the incident field

We say "amplitude" because, as we know, the motion of the charges has a phase lag relative to the incident field.

Indeed: $$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + m\omega_0^2 x = qE = qE_0 e^{i\omega t}$$

admits a solution of the form $x = x_0 e^{i\omega t}$

provided that

$$(-m\omega^2 + ib\omega + m\omega_0^2)\, x_0 e^{i\omega t} = q E_0 e^{i\omega t}$$

$$\Rightarrow \quad x_0 = \frac{q E_0/m}{(\omega_0^2 - \omega^2) + i\left(\frac{b}{m}\right)\omega} \qquad (11)$$

Replacing (11) in (10)

$$E_a' = -\frac{n_0 q}{2\epsilon_0} \frac{q E_0/m}{(\omega_0^2-\omega^2) + i\left(\frac{b}{m}\right)\omega} \left(i\frac{\omega}{c}\right) e^{i\omega(t-\frac{z}{c})}$$

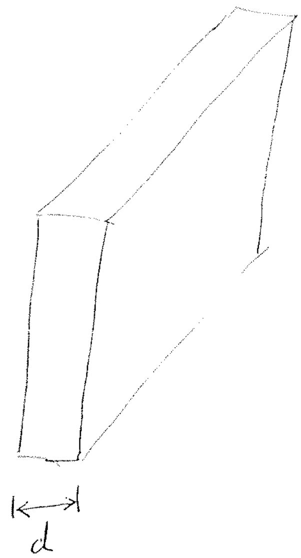
$$E_a' = \frac{n_0 q^2}{2m\epsilon_0} \frac{\omega}{c} \frac{1}{(\omega_0^2-\omega^2) + i\left(\frac{b}{m}\right)\omega} e^{i\omega(t-\frac{z}{c}) - i\frac{\pi}{2}} \qquad (12)$$

Comparing (9) and (12), we notice the fields E_a and E_a' are very similar. In fact they will be equal if

$$\cancel{a\Delta E_0}$$

$$\cancel{\omega}\Delta = \frac{n_0 q^2 \cancel{\omega}}{2m\epsilon_0 c} \frac{1}{(\omega_0^2-\omega^2) + i\left(\frac{b}{m}\right)\omega}$$

$$\frac{(n-1)d}{\cancel{c}} = \frac{n_0 q^2}{2m\epsilon_0 \cancel{c}} \frac{1}{(\omega_0^2-\omega^2) + i\left(\frac{b}{m}\right)\omega}$$



Since η_0 is the # of charges per unit area,

$\frac{\eta_0}{d}$ will be the # of charges per unit volume.

Let's call $\frac{\eta_0}{d} = N$

thus, we obtain an expression for the index of refraction n

$$\boxed{n = 1 + \frac{N q^2}{2 m \epsilon_0} \frac{1}{(\omega_0^2 - \omega^2) + i\left(\frac{b}{m}\right)\omega}} \qquad (13)$$

The lesson is the following

In expression (12) the electric field E_a' does not invoke at all that the speed of waves has changed. E_a' is simply the contribution to the electric field at P coming from all the changes oscillating in the thin glass. ALL these contributions are waves that travel at speed c.

We have found that E_a' is identical to E_a given in expression (9) where it is assumed that the wave slows down while traveling inside the glass of thickness d.