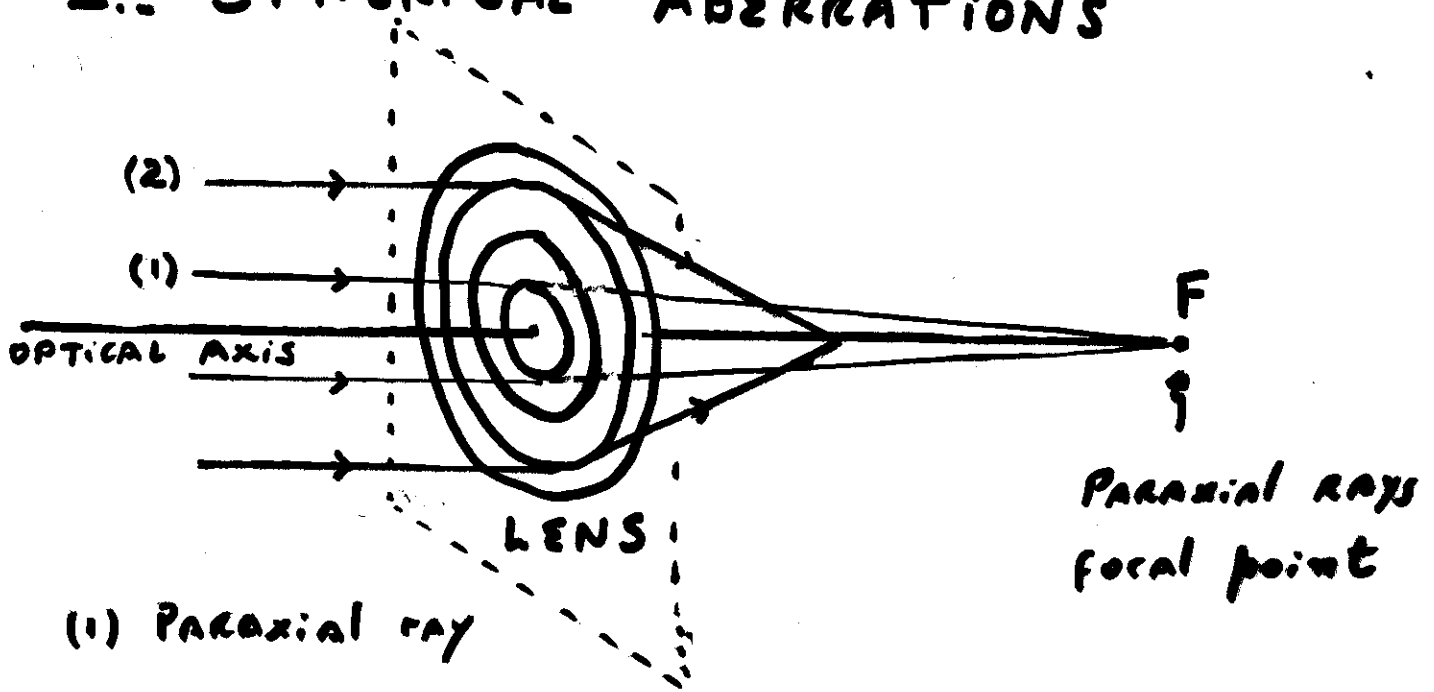


# 1. SPHERICAL ABERRATIONS



(1) PARAXIAL RAY

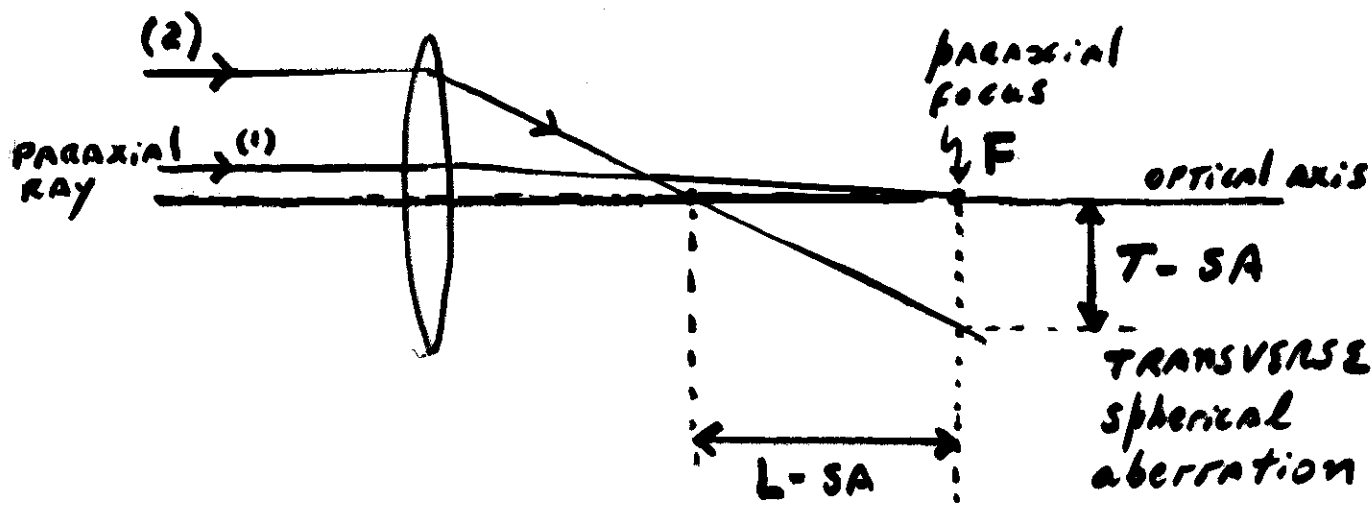
(2) NON-PARAXIAL RAY  
(marginal ray)

When using a lens with spherical aberration,

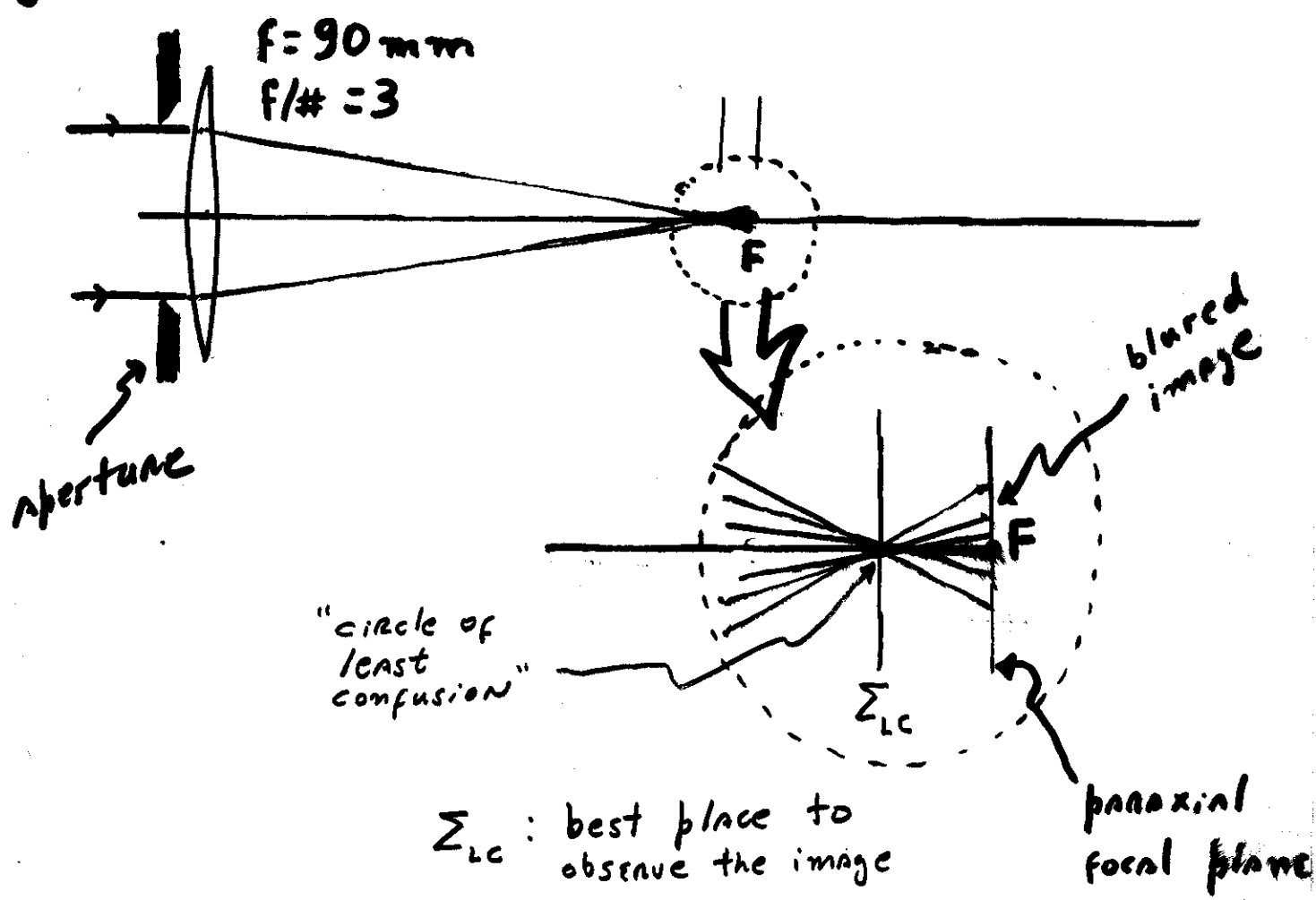
- EACH ZONE OR ANNULUS OF THE LENS has a slightly different focal length

- RAYS FROM ZONES FARTHER FROM THE axis ARE FOCUSED SHORT OF THE paraxial focal point F.

The lack of a common focal point for all zones of the lens is referred to as spherical aberration.



longitudinal spherical aberration of ray (2)



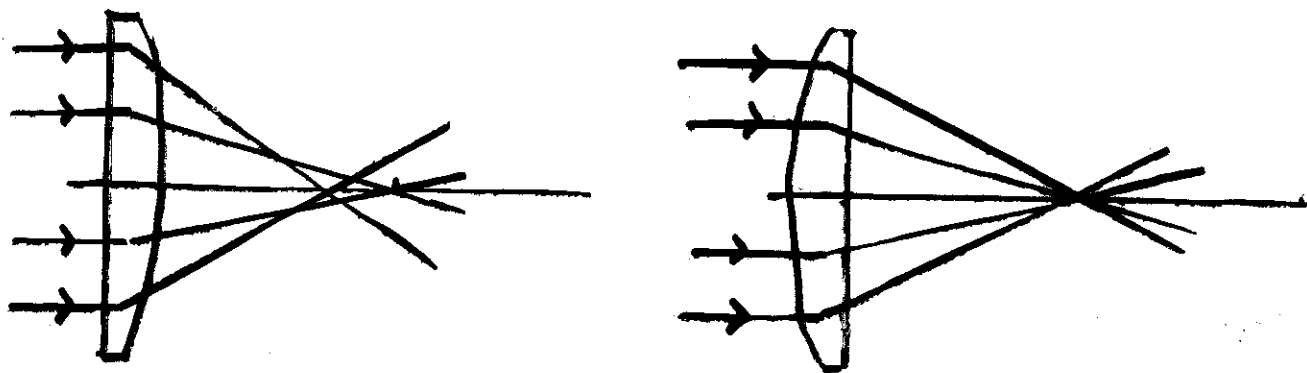
- When a screen is moved from the paraxial focal plane to the  $\Sigma_c$  plane, the image blur will have its smallest diameter. This is known as the circle of least confusion
- Notice, the previous graph suggests: spherical aberration decreases when the aperture decrease  
more general:  
the greater f/#, the less spherical aberration
- spherical aberration pertains to image analysis of an object point located at the optical axis

## NOTE-1 Minimizing Spherical Aberration

We saw above that increasing the  $f/\#$  minimizes spherical aberrations.

For the cases where the  $f/\#$  is fixed, there are other ways to minimize spherical aberrations: OPTIMIZING the shape of the lens

A striking example is presented by a PLANAR-CONVEX lens: simply turning the lens around, markedly reduces the spherical aberration

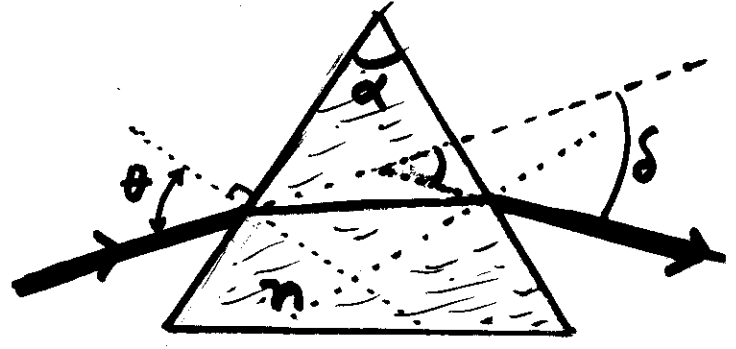


**EXPLANATION:** This can be understood by considering the lens as composed of 2 prisms joined at their bases.



When a ray strikes a prism it undergoes a net deviation  $\delta$

- $\delta$  increases with the index of refraction  $n$

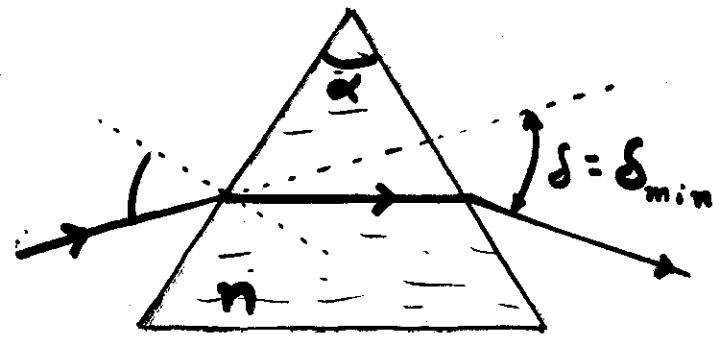
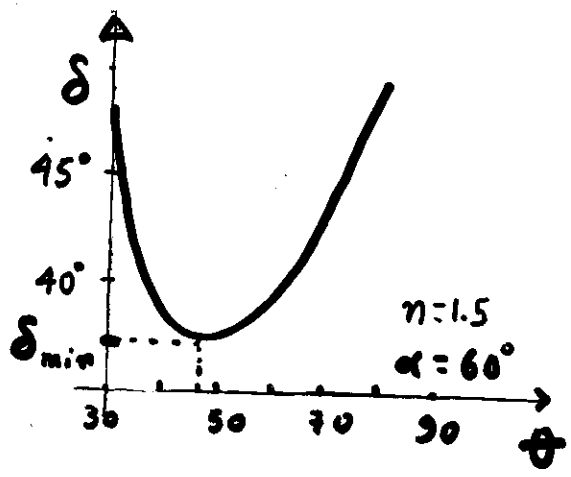


Since  $n = n(\lambda)$ , then

$\delta$  depends on the wavelength of the incident light

$n(\lambda_{blue}) > n(\lambda_{red})$  implies the deviation  $\delta$  is less for red light

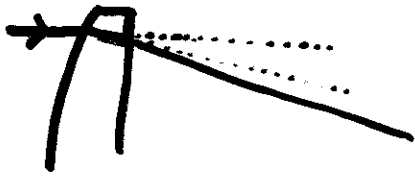
- For a given prism (i.e.  $n$  and  $\alpha$  fixed)  $\delta$  is a function of the incident angle  $\theta$



7

{ Rays for which the deviation  $\delta$  is minimum,  
traverses the prism parallel to the base.

A reinterpretation of this result,  
(in terms of our purpose of interpreting  
a lens as two prisms joined at their  
bases) is as follows:

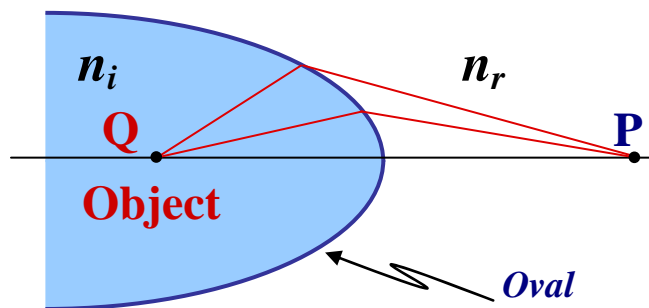


The incident ray will undergo a minimum deviation when it makes, more or less, the same angle as does the emerging ray

## Minimizing spherical aberrations

### Exploiting the existence of conjugate points in spherical lenses

Let's recall that when using aspherical lenses, aberration free imaging occurs only for a couple of, so called, conjugate points (Q and P in the figure below)



**Fig. 1** P is a perfect image of Q (and vice versa) when a proper refractive oval surface is used

We have said before that, in general, spherical refractive surface (not being an oval) will suffer from aberrations. However, it turns out, Huygens may have been the first to discover that a given object point Q could be perfectly image also by a spherical surface!! That is, all the imaging rays refracting at the spherical surface “converge” at the same (virtual in this case) image point P. (similar to the case when using an oval.)

We will consider two cases. In the first (second) case the object and virtual image points are inside the medium of higher (lower) index of refraction.

#### **Case: Imaging from glass to air**

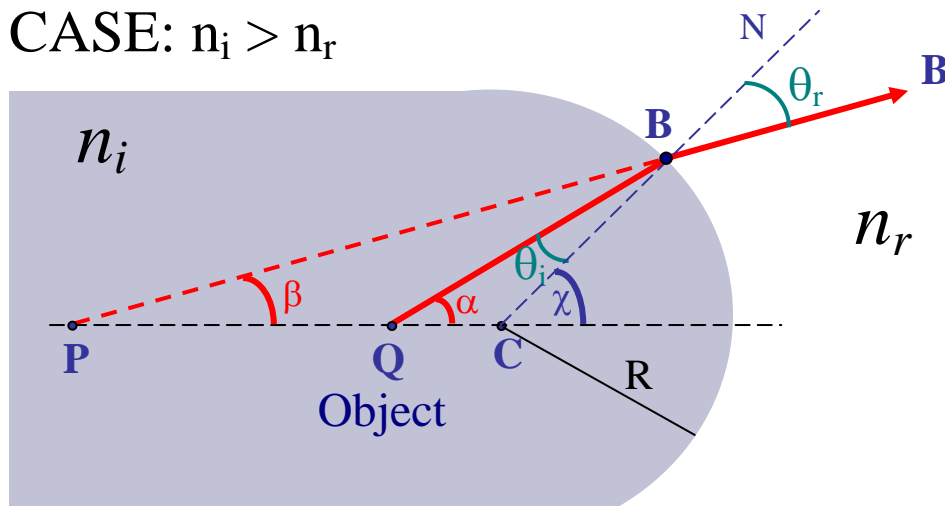
**(Object and image immersed inside the glass.)**

Let's consider a refracting spherical surface of radius “R”.

Pick up an arbitrary object point Q.

The ray QBB' images point “Q” at point “P” (the image is virtual.)

CASE:  $n_i > n_r$

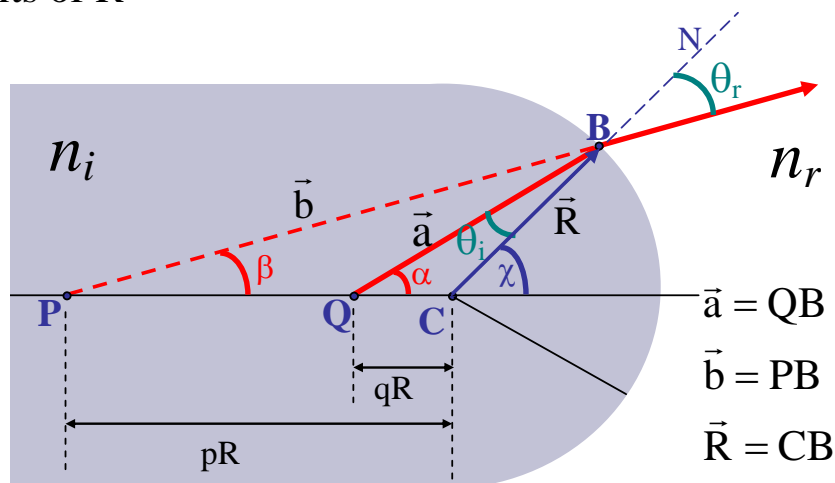


**Fig. 2** Object Q imaged through a refracting spherical surface of radius  $R$ .

But notice that, in general, not necessarily all the rays leaving from Q will be imaged at the same point “P”.

Our objective is to show that, when the location of the point Q is properly chosen, indeed all the rays leaving from Q and refracted at any point of the spherical surface will produce a (virtual) image at the common image point.

For convenience, let's express the position of Q and P, with respect to C, in units of  $R$



**Fig. 3** Scaling the positions of P and Q in terms of the radius  $R$ .

$$\vec{a} = \vec{QC} + \vec{R}$$

$$a^2 = \overline{QC}^2 + R^2 + 2\vec{R} \cdot \vec{QC}$$

$$= \overline{QC}^2 + R^2 + 2R \overline{QC} \cos X$$

Let's measure  $\overline{QC}$  in units of  $R$ .  
 thus let's consider  $\overline{QC} \equiv qR$  ( $q$  a real number) (1)

$$a^2 = q^2 R^2 + R^2 + 2R^2 q \cos X$$

$$a^2 = R^2 (q^2 + 1 + 2q \cos X) \quad (2)$$

Notice also in the figure that

$$a \sin \alpha = R \sin X \quad (3)$$

From (3) and (2)

$$\sin \alpha = \frac{\sin X}{(q^2 + 1 + 2q \cos X)^{1/2}} \quad (4)$$

Notice, giving the angle  $X$  and the position of  $Q$  relative to  $C$  (that is given the value of  $q$ ), the angle  $\alpha$  is determined.

Now, given  $X$  and  $q$ , the position of the image  $P$  must be determined. Let's find it.

$$\text{In } \Delta PCB: \overline{PC} \sin \beta = R \sin \theta_r \Rightarrow n_r \overline{PC} \sin \beta = n_r R \sin \theta_r$$

$$\text{In } \Delta QCB: \overline{QC} \sin \alpha = R \sin \theta_i \Rightarrow n_i \overline{QC} \sin \alpha = n_i R \sin \theta_i$$

$$\text{Snell's law implies: } \overline{PC} = \frac{n_i \sin \alpha}{n_r \sin \beta} \overline{QC} \quad (5)$$

If we express  $\overline{PC}$  also in units of  $R$ ,

$$\overline{PC} \equiv pR \quad (p \text{ a real number}) \quad (6)$$

result (5) becomes

$$\boxed{p = \frac{n_i \sin \alpha}{n_r \sin \beta} q} \quad (7)$$

In addition, we should obtain for  $n \sin \beta$  a expression similar to (4). That is,

$$\sin \beta = \frac{\sin \chi}{(p^2 + 1 + 2p \cos \chi)^{1/2}} \quad (8)$$

Expressions (7) and (8) give implicitly the values of  $p$  and  $\beta$ .

Let's obtain a more explicit expression for  $p$ , which will allow locating the image point  $P$  (since  $PC = pR$ )

From (4) and (8)

$$\frac{\sin^2 \alpha}{\sin^2 \beta} = \frac{p^2 + 1 + 2p \cos(\chi)}{q^2 + 1 + 2q \cos(\chi)}$$

Using this result in (7) one obtains,

$$\frac{p^2}{q^2} = \left( \frac{n_i}{n_r} \right)^2 \frac{p^2 + 1 + 2p \cos(\chi)}{q^2 + 1 + 2q \cos(\chi)} \quad (9)$$

Given  $q$  and  $\chi$  (that is, given the position of Q and B), the value of  $p$  (the position of point P) is implicit in this formula.

### Summary:

For a given point “Q”, whose distance from C (in terms of  $R$ , is  $qR$ ; i.e.  $q$  is given), we want to find the location of its image P. For that purpose, we send a generic ray QB. The angular position of B, with respect to the center C, is determined by the angle  $\chi$ .

Given  $q$  and  $\chi$ ,

- we calculate  $\alpha$  using expression (4)
- expression (9) gives implicitly the value of  $p$ , calculate  $p$  using (9); then  $\beta$  is obtained using expression (8)

Notice, for the same Q:

different values of  $\chi$  (i.e. a different point B) will give different values for  $p$  (i.e a different point P).

### Looking for aberration-free imaging

We wonder if the position of Q can be wisely chosen, such that for any point B, the corresponding position of P is the same?

That is, can we choose the value of “ $q$ ” wisely enough, such that “ $p$ ” (in expression (9)) becomes independent of

$\chi$ . ?

If that were the case, then P and Q would become conjugate points free of spherical aberrations. (In the figure, the position of P would be independent of the position of B)

For that matter,  
let's keep working out expression (5)

$$p^2 q^2 + p^2 + 2p^2 q \cos X = \left(\frac{n_i}{n_r}\right)^2 p^2 q^2 + \left(\frac{n_i}{n_r}\right)^2 q^2 + \left(\frac{n_i}{n_r}\right)^2 2p q^2 \cos X$$

$$\Rightarrow \left[ \left(\frac{n_i}{n_r}\right)^2 - 1 \right] p^2 q^2 + \left[ \left(\frac{n_i}{n_r}\right)^2 q^2 - p^2 \right] + 2p q \cos X \left[ q \left(\frac{n_i}{n_r}\right)^2 - p \right] = 0 \quad (10)$$

or

$$(q^2 + 1 + 2q \cos X - r^2 q^2) p^2 - (2r^2 q^2 \cos X) p - r^2 q^2 = 0 \quad \begin{array}{l} \text{given } q, \\ \text{solve for } p \end{array} \quad (11)$$

In general,  $p$  will depend on  $X$

However, notice in (10) that

If  $q = \left(\frac{n_i}{n_r}\right)^{-1}$  we obtain

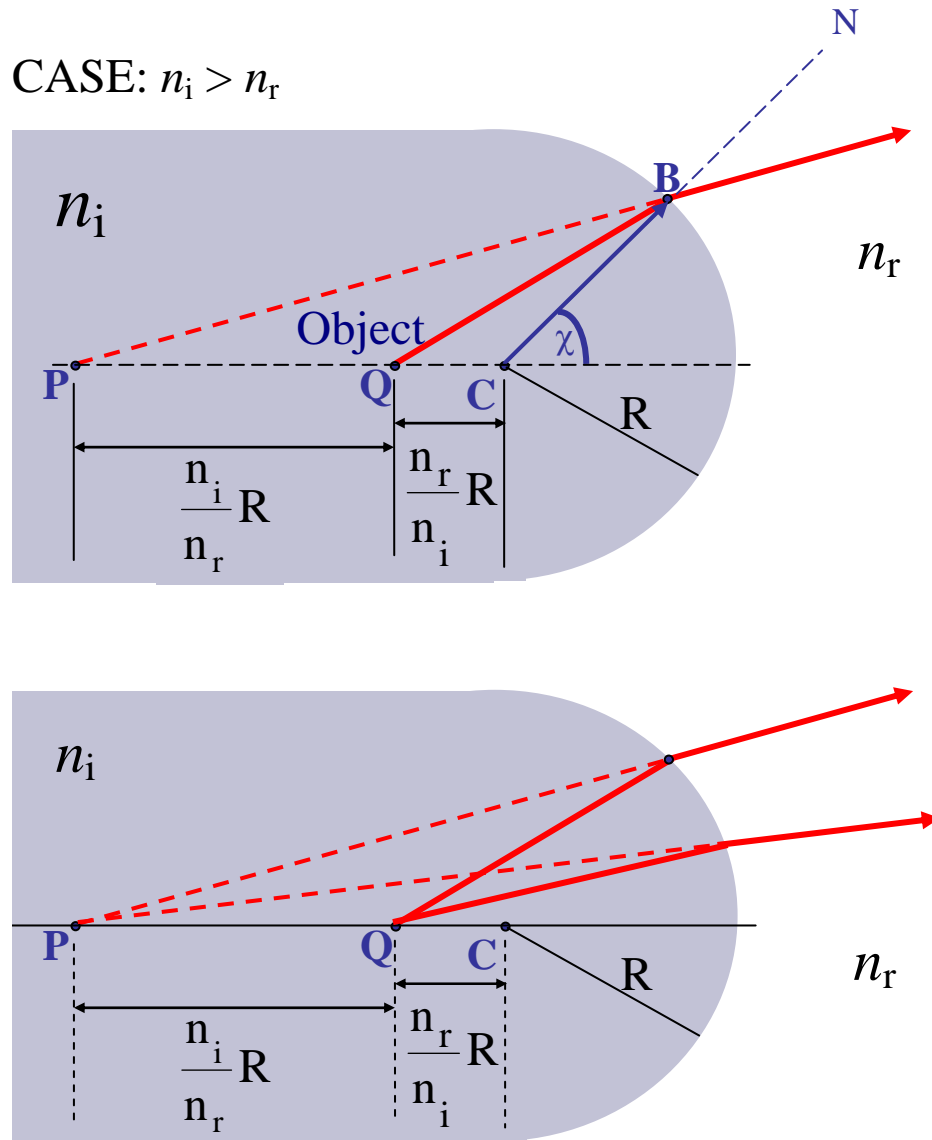
$$\left[ 1 - \left(\frac{n_i}{n_r}\right)^{-2} \right] p^2 + [1 - p^2] + 2p \left(\frac{n_i}{n_r}\right)^{-1} \cos X \left[ \left(\frac{n_i}{n_r}\right) - p \right] = 0$$

which is satisfied by the following solution for  $p$ :  $p = \frac{n_i}{n_r}$

In effect

$$\left[ \left(\frac{n_i}{n_r}\right)^2 - 1 \right] + \left[ 1 - \left(\frac{n_i}{n_r}\right)^2 \right] + 2 \cos X [0] = 0 \quad !$$

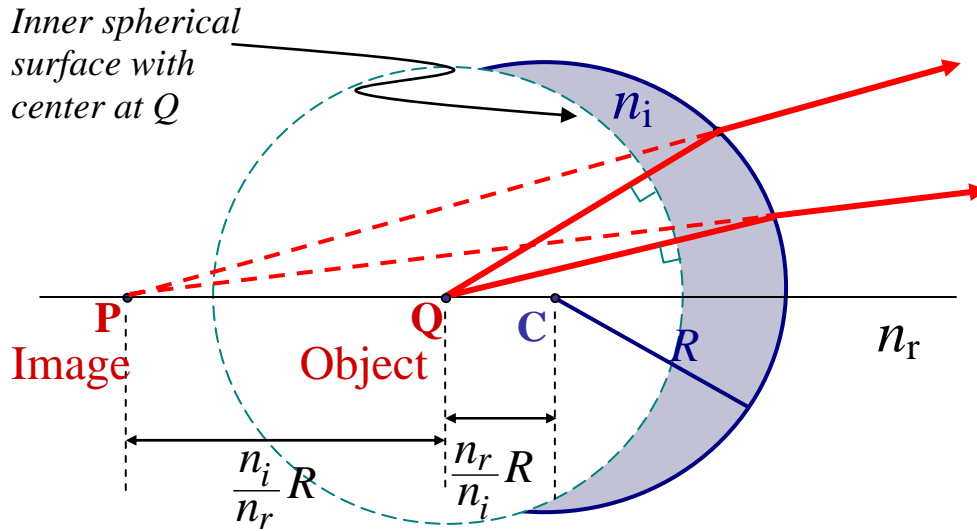
Thus, for a given refracting spherical surface of radius  $R$ , we have found a particular position for an object point  $Q$  that will be perfectly imaged at point  $P$ .



### Implementation using a converging meniscus

Instead of using a bulky (and costly) sphere, we can use a meniscus instead. The strategy is to grind another spherical surface

with center at Q; the radius of this new inner spherical interface is not critical.

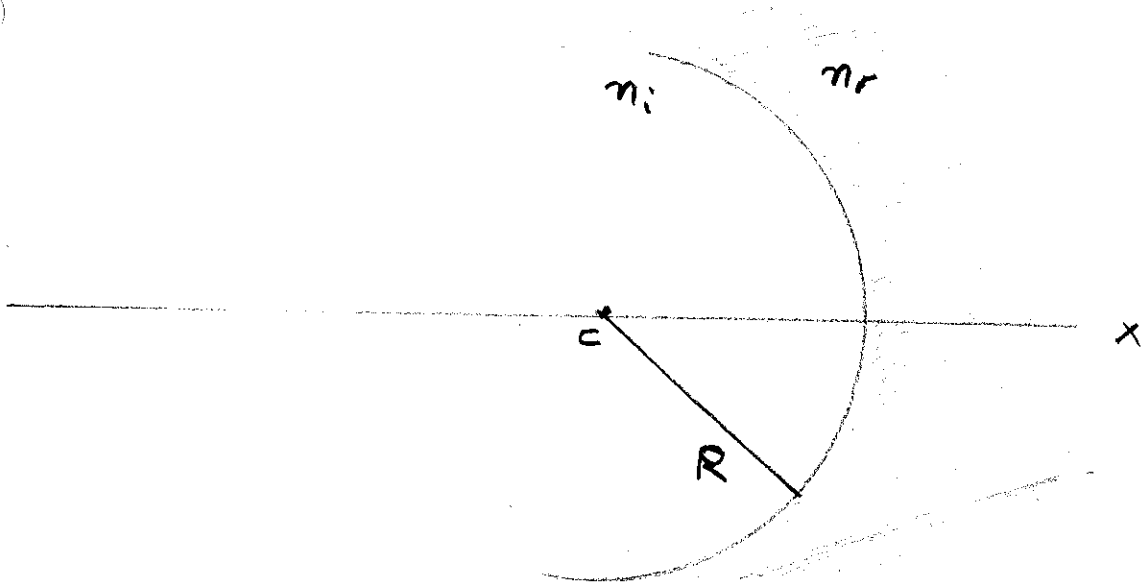


*The inner spherical surface does not affect the trajectories of the rays leaving the object Q*

**CASE:  $n_i > n_r$**

# HOMEWORK

16



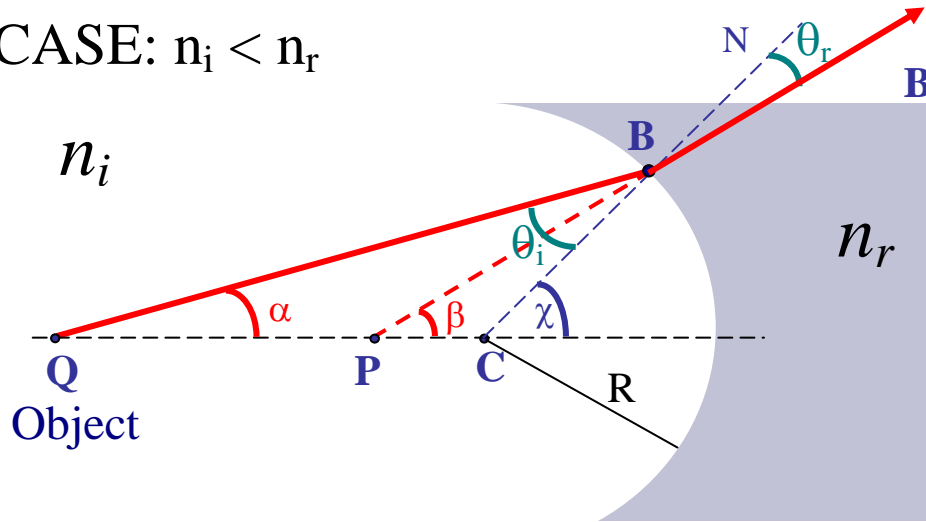
For the case  $n_i < n_r$ ,

Find the position of <sup>the pair of</sup> conjugated points (along the x-axis) that are free of aberrations.

Show explicitly all the derivation.

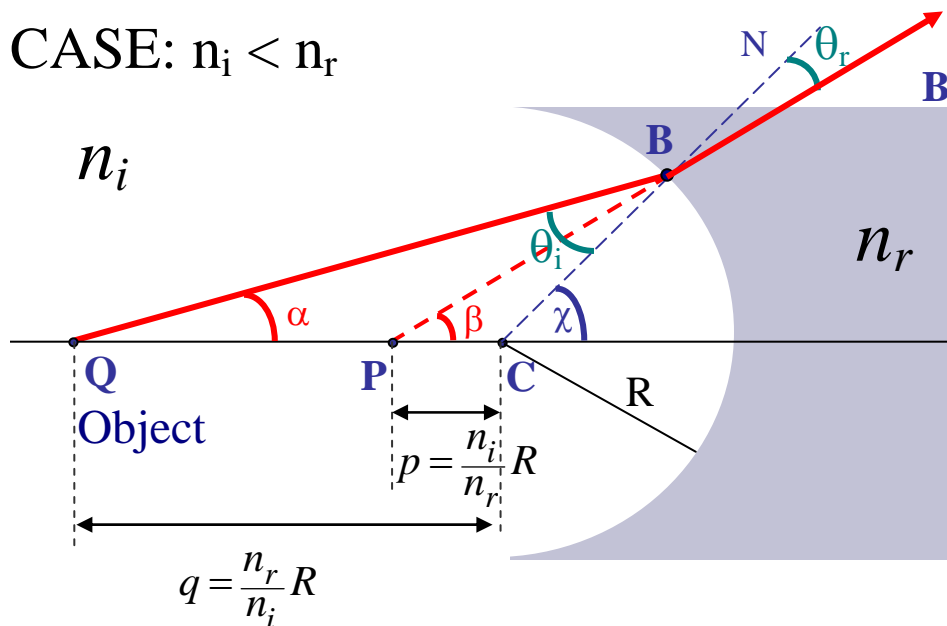
**Case 2: Imaging from air to glass  
(Object and image immersed inside the air medium.)**

CASE:  $n_i < n_r$



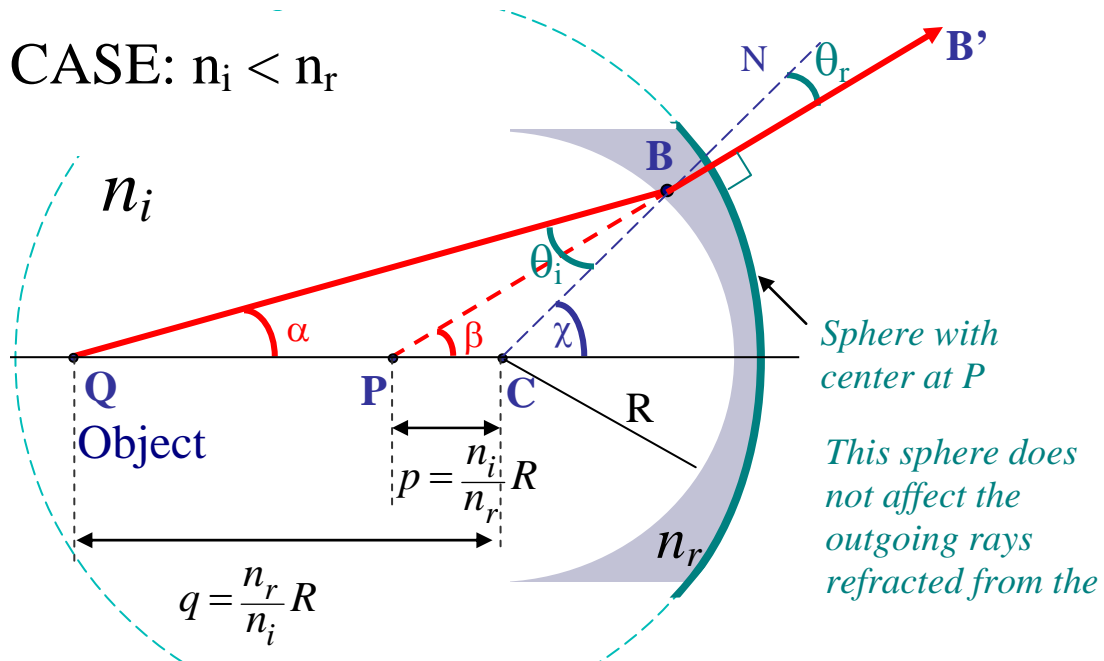
We realize the procedure is going to be identical to the previous case-1. The choice of  $q = (n_i/n_r)^{-1}$  (which give the solution  $p=(n_i/n_r)$ ) will remain the same, but this time it will imply that  $Q$  is farther away form the center than  $P$ .

CASE:  $n_i < n_r$



## Implementation using a diverging meniscus

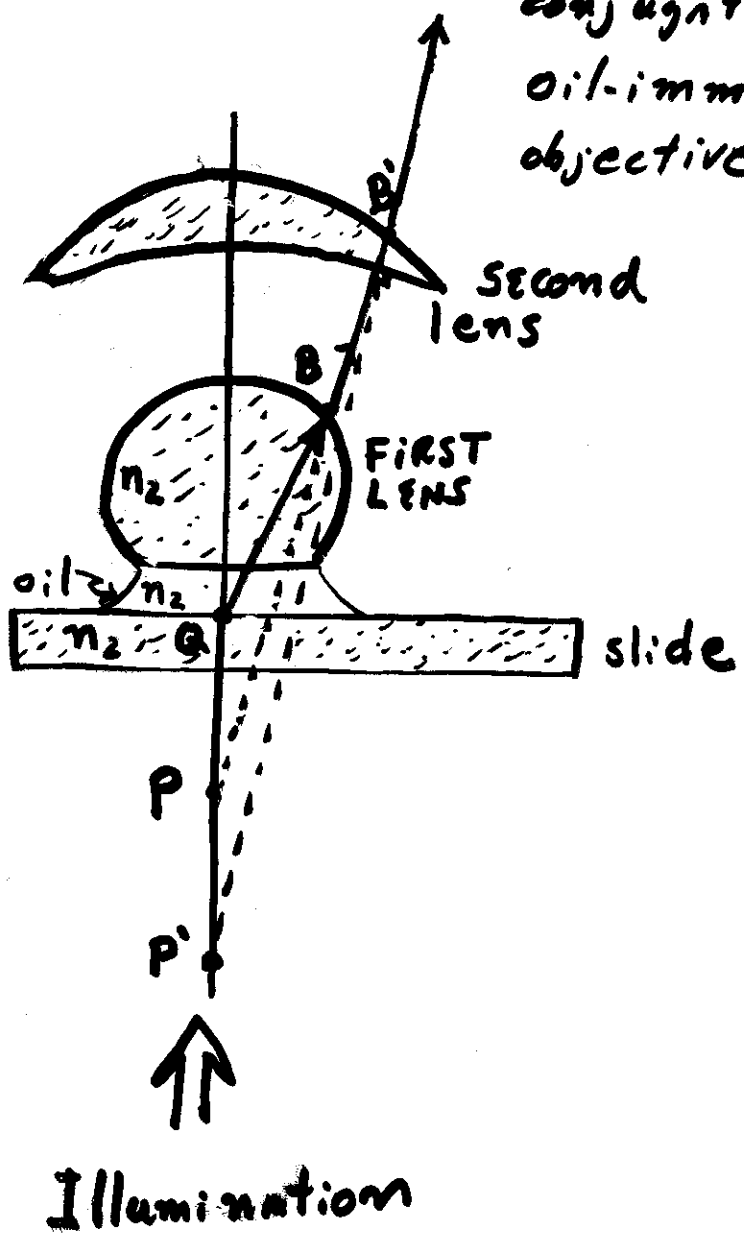
Instead of using a bulky (and costly) spherical lens, we can use a meniscus instead. The strategy is to grind another spherical surface with center at P; the radius of this new **outer spherical interface** is not critical.



Thus, similar to aspherical lenses, spherical lenses can be constructed to have zero spherical aberration for a pair of conjugated points P and Q.

APPLICATION

Spherical meniscus lenses free of spherical aberration (for two conjugated points) are used as oil-immersion microscope objective lenses.



Object under study is placed at Q

Q and P are the conjugate points (for zero SA) for the first lens

P and P' are conjugate points for the second lens