# Chromatic Aberration (CA)

6. LONGITUDINAL CA
7. LATERAL CA

$$f = f(\lambda)$$

Chromatic aberration results from the fact that

↳ The focal length of a lens depends on the index of refraction $n$

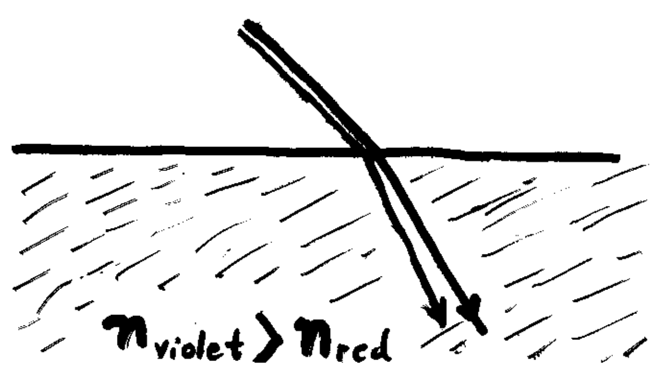
$$\frac{1}{f} = \frac{n_{lens} - n_{air}}{n_{air}} \times \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$$

and

↳ $n$ depends on the wavelength of wavelength $\lambda$ of the light

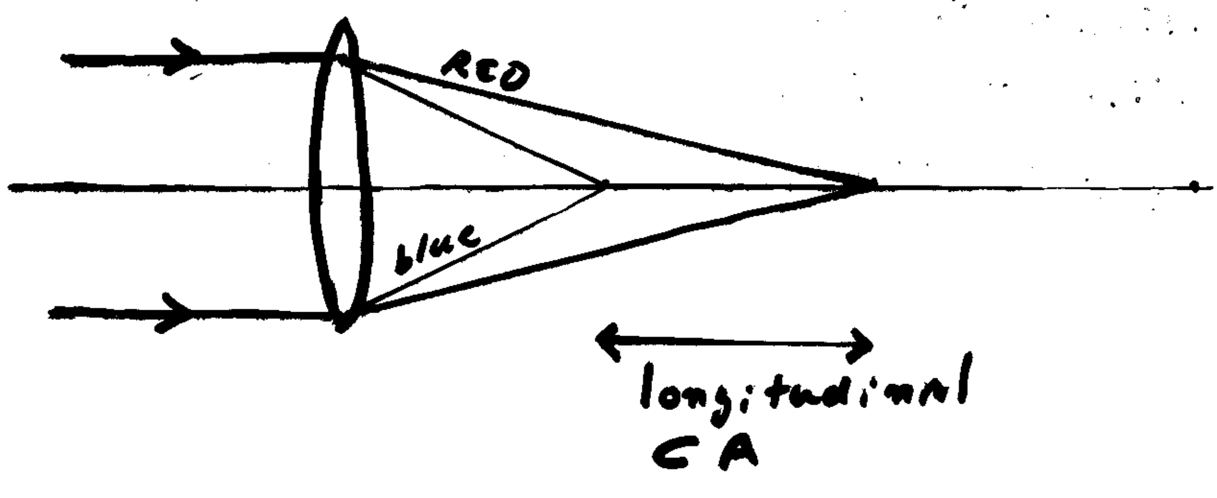
$$n = n(\lambda)$$

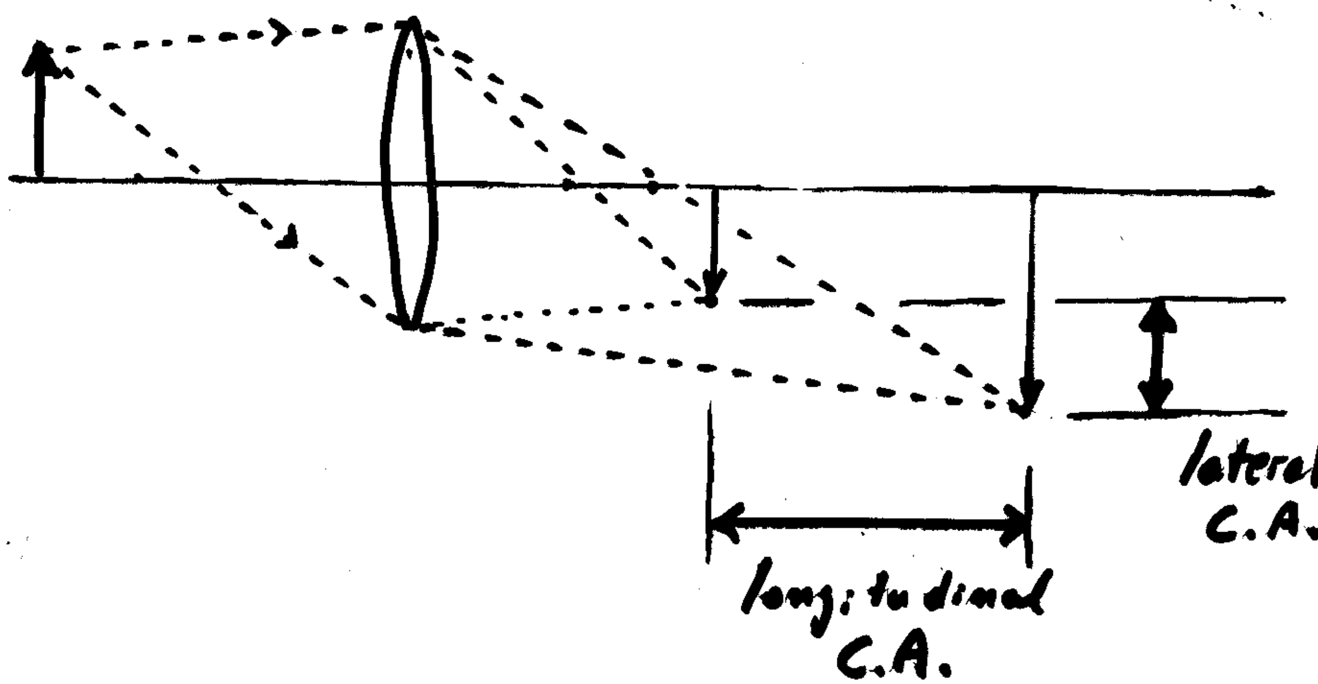
That is, different colors refract at different angles



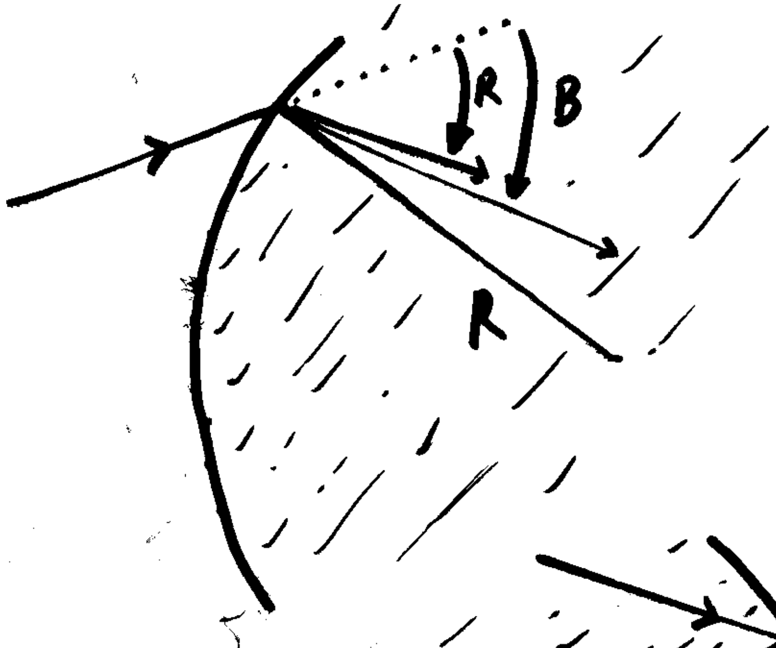

Thus,

focal length $f = f(\lambda)$

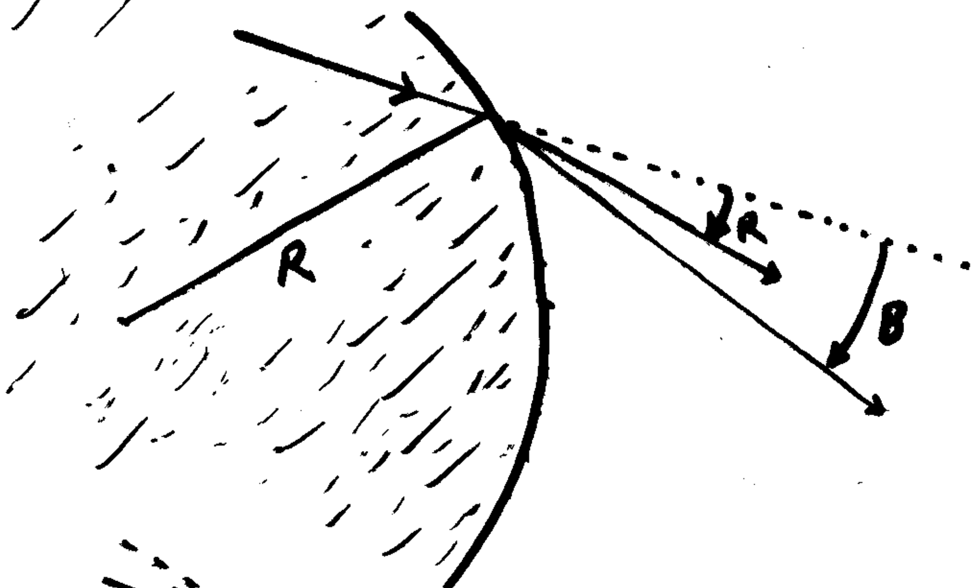

RED
blue
longitudinal
C A


lateral
C.A.
longitudinal
C.A.

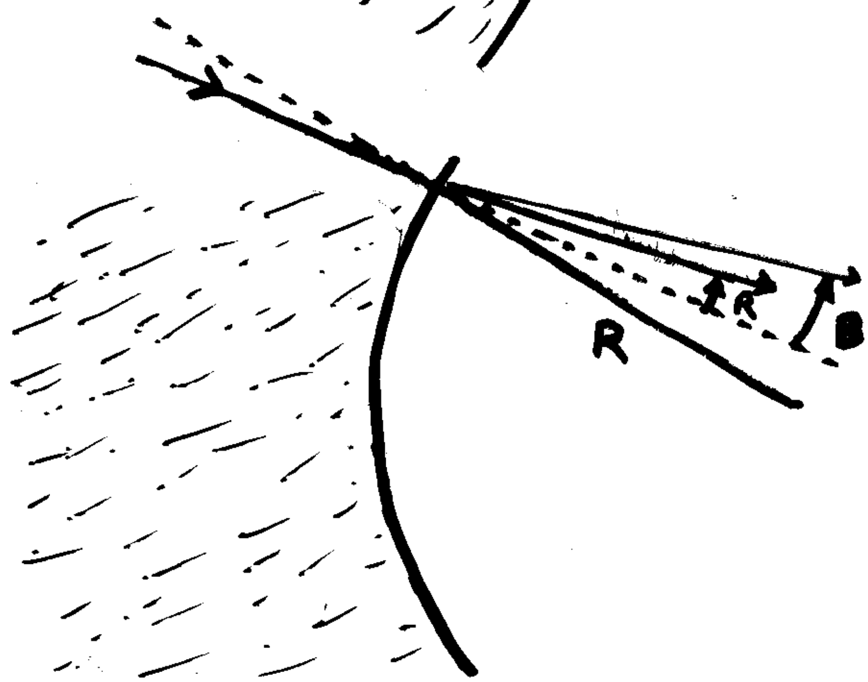
How to correct this chromatic aberration?




OPTION-1




OPTION-2

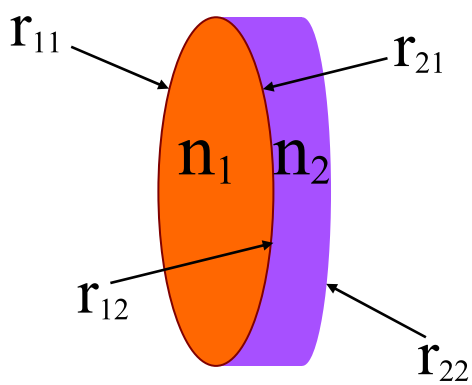


The previous ray-tracing diagrams suggest that chromatic aberration might be corrected by properly combining a converging lens with a diverging lens.

Indeed that is the case in the design of the achromatic doublet lens.

ACHROMATIC DOUBLET

Consisting of a crown equiconvex lens cemented to a negative flint glass lens




Let's assume we want an "achromat doublet" of 15 cm focal length. Lets call it $f_{desired}$= 15 cm. Since, in general, the focal length depends on the wavelength, $f_{desired}$ is conveniently specified as that associated with yellow light (the Fraunhofer wavelength $\lambda_D$= 587.6 nm). Thus,

$$f_{desired} = 15 \text{ cm}, \quad \text{at } \lambda = \lambda_D$$

Considering a doublet as two thin lenses, its effective focal length in terms of its lens components can be expressed as,

$$\frac{1}{f_{doublet}} = \frac{1}{f_1} + \frac{1}{f_2} \qquad (1)$$

The focal length of the individual lenses is given by

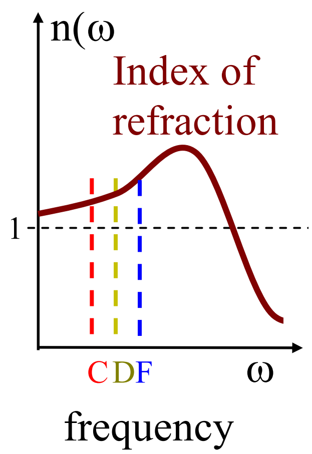
$$\frac{1}{f_1} = (n_1 - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = (n_1 - 1)\,\rho_1 \qquad (2a)$$

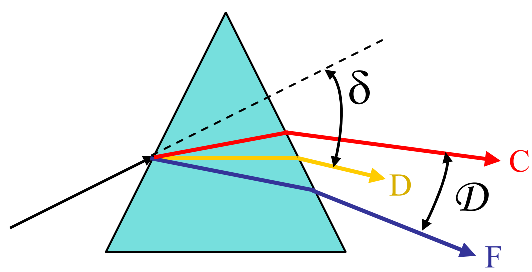
$$\frac{1}{f_2} = (n_2 - 1)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right) = (n_2 - 1)\,\rho_2 \qquad (2b)$$

The focal length of any doublet is then given by,

$$\frac{1}{f_{doublet}} = (n_1 - 1)\,\rho_1 + (n_2 - 1)\,\rho_2 \qquad (3)$$

The value of $1/f_{doublet}$ in general will depend on $\lambda$.




**Dispersion power Δ**
(measured at minimum deviation of the D line)

$$\Delta \equiv \frac{\mathcal{D}}{\delta} = \frac{n_F - n_C}{n_D - 1}$$

The condition of achromaticity, around the wavelength of the yellow light $\lambda_D$, can be expressed as,

$$\frac{d}{d\lambda}\left(\frac{1}{f_{doublet}}\right) = 0 \;, \qquad \text{at } \lambda = \lambda_D$$

Or, equivalently, using expression (3),

$$\rho_1 \left.\frac{dn_1}{d\lambda}\right|_{\lambda=\lambda_D} + \rho_2 \left.\frac{dn_2}{d\lambda}\right|_{\lambda=\lambda_D} = 0 \qquad (4)$$

The derivative of n at $\lambda=\lambda_D$ can be approximated using the red and blue Fraunhofer wavelengths, $\lambda_C$= 656.3 nm and $\lambda_F$= 486.1 nm,

$$\left.\frac{dn}{d\lambda}\right|_{\lambda=\lambda_D} \approx \frac{n_F - n_C}{\lambda_F - \lambda_C}$$

In addition, using, the reciprocal of the dispersion power $\Delta$

$$V \equiv \frac{1}{\Delta} = \frac{n_D - 1}{n_F - n_C}$$ known as "Abbe number",

we obtain,

$$\rho_1 \left.\frac{dn_1}{d\lambda}\right|_{\lambda=\lambda_D} = \rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C}$$

$$= \rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} \frac{n_{1D} - 1}{n_{1D} - 1}$$

$$= \rho_1 \frac{1}{\lambda_F - \lambda_C} \frac{1}{V_1} \frac{n_{1D} - 1}{1}$$

and using (2a)

$$\rho_1 \left.\frac{dn_1}{d\lambda}\right|_{\lambda=\lambda_D} = \frac{1}{\lambda_F - \lambda_C} \frac{1}{V_1} \frac{1}{f_{1D}}$$

So, the condition for achromaticity, expression (4), becomes

$$\frac{1}{V_1}\frac{1}{f_{1D}}+\frac{1}{V_2}\frac{1}{f_{2D}}=0 \qquad (5)$$

(For two given materials, their corresponding Abbe number values are tabulated.)

Expression (5), together with the designer's requirement that the doublet has a desired focal length of $f_{desired}$=15 cm at $\lambda = \lambda_D$, where

$$\frac{1}{f_{desired}}=\frac{1}{f_{1D}}+\frac{1}{f_{2D}}, \qquad (6)$$

constitute the couple of equations from which $f_{1D}$ and $f_{2D}$ have to be determined.

The solution to (5) and (6) is:

$$\frac{1}{f_{1D}}=-\frac{V_1}{V_2-V_1}\frac{1}{f_{desired}} \text{ and}$$

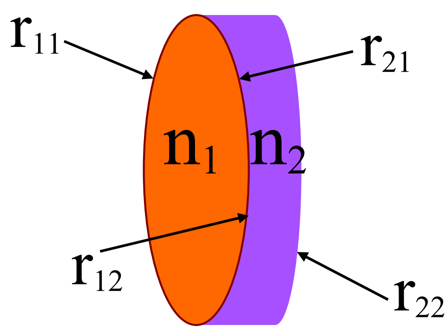
$$\frac{1}{f_{2D}}=\frac{V_2}{V_2-V_1}\frac{1}{f_{desired}} \qquad (7)$$

# DESIGN of an ACHROMAT DOUBLET of 15 cm focal length

DOUBLET

(1) Equiconvex
520/636 crown glass

(2) 617/366 flint glass




| | | Catalog code | V | $n_C$ | $n_D$ | $n_F$ |
|---|---|---|---|---|---|---|
| | | $\frac{n_D - 1}{10V}$ | $\frac{n_D - 1}{n_F - n_C}$ | 656.5 nm | 587.6 nm | 486.1 nm |
| (1) | Borosilicate crown | 520/636 | 63.59 | 1.51764 | 1.52015 | 1.52582 |
| (2) | Dense flint | 617/366 | 36.60 | 1.61218 | 1.61715 | 1.62904 |

| | | | $1/f_{1D}$ | $1/f_{2D}$ |
|---|---|---|---|---|
| $f_{desired}$ | $\frac{-V_1}{V_2 - V_1}$ | $\frac{V_2}{V_2 - V_1}$ | $\frac{-V_1}{V_2 - V_1}\frac{1}{f_{desired}}$ | $\frac{-V_1}{V_2 - V_1}\frac{1}{f_{desired}}$ |
| 15 cm | 2.356 | -1.356 | 0.157 $cm^{-1}$ | -0.0904 $cm^{-1}$ |
| | | | $f_{1D}$=6.35 cm | $f_{2D}$=-11.06 cm |

From expression (2)

$$\frac{1}{f_{1D}} = (n_{1D} - 1)\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = 0.52015\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right)$$

$$\left(\frac{1}{r_{11}} - \frac{1}{r_{12}}\right) = \frac{0.157}{0.52015}\text{cm}^{-1} = 0.3019\text{cm}^{-1};$$

by choosing a equiconvex lens $r_{12} = -r_{11} < 0$ gives $\frac{2}{r_{11}} = 0.3019\text{cm}^{-1}$. Thus, $r_{11} = 6.624$ cm

Similarly

$$\frac{1}{f_{2D}} = (n_{2D} - 1)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right) = (0.61715)\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right)$$

$$\left(\frac{1}{r_{21}} - \frac{1}{r_{22}}\right) = \frac{-0.0904}{0.61715}\text{cm}^{-1} = -0.14649\ \text{cm}^{-1};$$

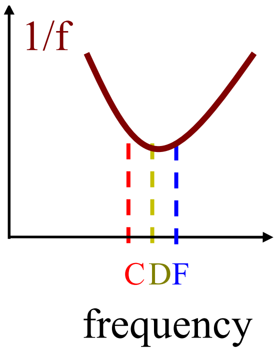
by construction $r_{21} = r_{12}$ (cemented lenses), which gives $\left(\frac{1}{r_{12}} - \frac{1}{r_{22}}\right) = -0.14649\ \text{cm}^{-1}$. Since $r_{12} = -r_{11}$, we obtain $\left(-\frac{1}{r_{11}} - \frac{1}{r_{22}}\right) = -0.14649\ \text{cm}^{-1}$. Thus,

$r_{22} = -224.21$ cm

# Observation

Although the condition $\frac{d}{d\lambda}(1/f_{doublet}) = 0$, at $\lambda = \lambda_D$, is quite appealing to use it as a requirement for obtaining a achromatic doublet, such a condition may does not necessarily ensure that 1/f is independent of the wavelength. The graph below is an example of what may happen in some cases.

That is, $1/f$ may still be different for different colors, despite the fact that the derivative of $1/f$ is zero at $\lambda = \lambda_D$.




A stronger condition is to require that, for example, that the focal length of the doublet for the red (C) and blue (F) light (Fraunhofer lines) to be equal,

$$\frac{1}{f_{doublet,C}} = \frac{1}{f_{doublet,F}} \quad \textit{requirement for achromaticity} \qquad (8)$$

Using (1) and (2) we obtain,

$$\frac{1}{f_{doublet,\,C}} = (n_{1C} - 1)\,\rho_1 + (n_{2C} - 1)\,\rho_2$$

$$\frac{1}{f_{doublet,\,F}} = (n_{1F} - 1)\,\rho_1 + (n_{2F} - 1)\,\rho_2$$

So, the condition (8) is equivalent to

$$(n_{1F} - n_{1C})\,\rho_1 + (n_{2F} - n_{2C})\,\rho_2 = 0,$$

This expression is nothing but expression (4).

Indeed, the resemblance can be more direct if we divide by the difference $\lambda_F$-$\lambda_C$.

$$\rho_1 \frac{n_{1F} - n_{1C}}{\lambda_F - \lambda_C} + (n_{2F} - n_{2C})\ \rho_2 \frac{n_{2F} - n_{2C}}{\lambda_F - \lambda_C} = 0$$

where the fractions are the approximate values of the derivative of n with respect to λ.

## SUMMARY

The procedure above ensures that the achromat doublet will have equal focal length for at least the red and blue colors.

Ref: S. Inoue aand K. Spring, Video Microscopy, Plenum Press, Second Edition

# CLASIFICATION of OBJECTIVE LENSES

## 1) Achromats. 2) Fluorites. 3) Apochromats.

Added **Plan** designation to lenses with wide flat field (low curvature of field and distortion).

| 0 | Correction for aberrations | | |
|---|---|---|---|
| Type | Spherical | Chromatic | Flatness correction |
| Achromat | * | 2λ | No |
| F-Achromat | * | 2λ | Improved |
| Neofluar | 3λ | <3λ | No |
| Plan Neofluar | 3λ | <3λ | Yes |
| Plan Apochromat | 4λ | >4λ | Yes |

* Corrected for two wavelengths

2λ Corrected for blue and red

3λ Corrected for blue, green and red

4λ Corrected for dark blue, blue, green and red

## Fluorites objectives.

- Considerably **better** corrected than **achromats**, but not quite as well corrected as the apochromats.
- Uses **fluorite crystals** (which have lower dipersion) in place of some of the glass elements.
- Correct for spherical aberrations in three wavelengths at considerably **lower cost** than the apochromats.

**Apochromatic objectives.**

- Provides the same focal length for three wavelengths, and free of spherical aberration for two wavelengths.
- Magnification still vary with wavelength (a compensating eyepiece is used to cancel the colored fringes).

## 60x Plan Apochromat Objective


http://www.microscopyu.com/articles/optics/objectivespecs.html