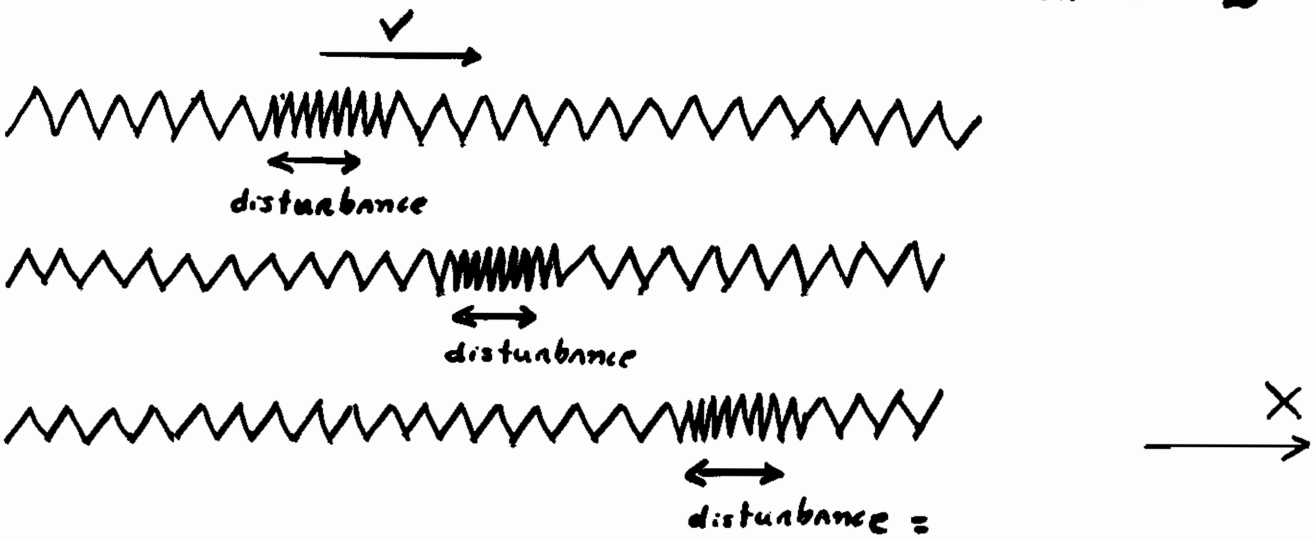
Longitudinal waves



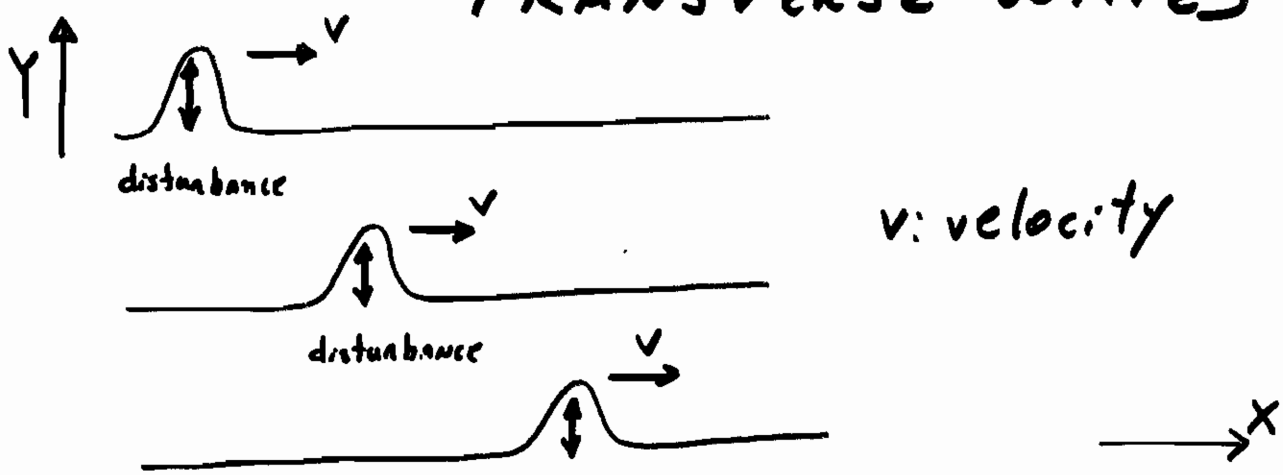
: segments of the spring move back and forth in the x direction

Longitudinal wave: disturbance is parallel to the direction of propagation

$$\boxed{\text{Disturbance} \parallel v}$$

Example: SOUND WAVE

TRANSVERSE WAVES



v: velocity

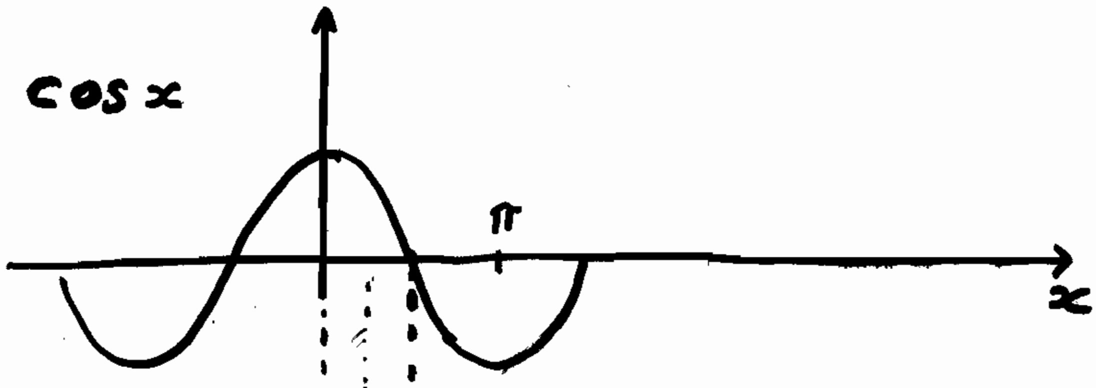
disturbance = segments of the string move back and forth along Y direction

disturbance travels in the X direction

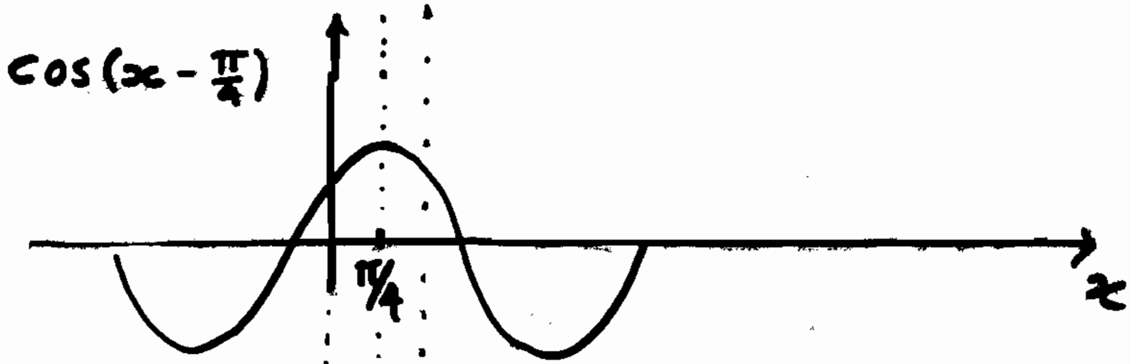
TRANSVERSE WAVE

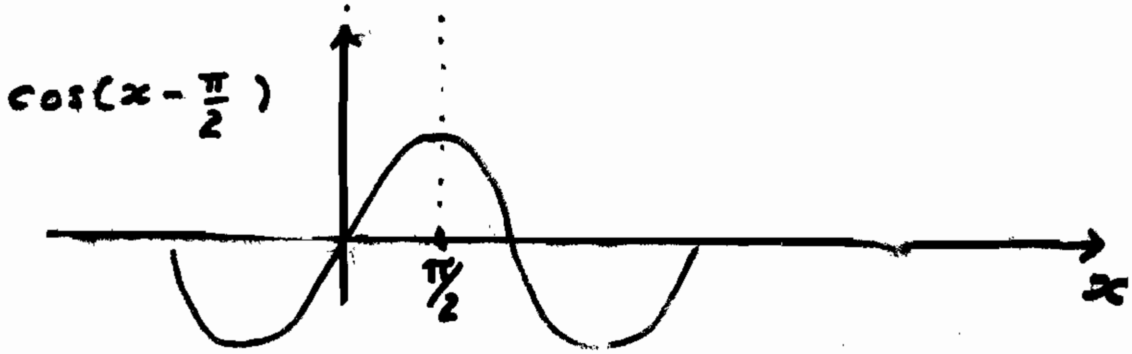
$\boxed{\text{DISTURBANCE} \perp V}$

TRAVELING WAVES



QUESTION: How does $\cos(x - \frac{\pi}{4})$ looks like?





- How does the wave

$$\cos(x - vt) \qquad v = \frac{\pi}{4}\ m/s$$

look like at different times t ?

At $t=0$

At $t = 1\,sec$

At $t=2\,sec$

So, $\cos(x - vt)$ represents a wave traveling to the right

Exercise

Make a plot of the function

$\cos(x + vt)$ $\quad v = \frac{\pi}{4}\ m/s$

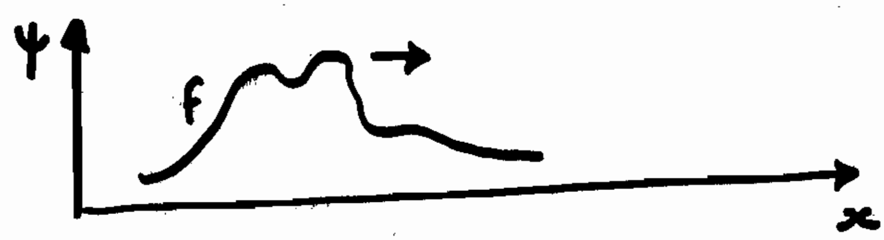
for different values of t ($t = 0, 1, 2, \ldots$), and conclude that it describes a wave traveling to the left

In general:

- Functions of the form

$$\psi = f(x - vt)$$

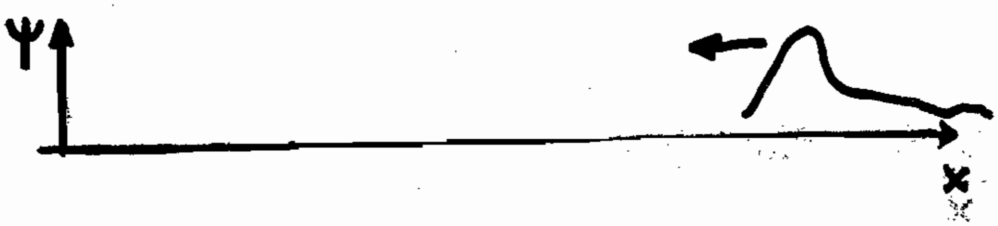
describe waves traveling to the right

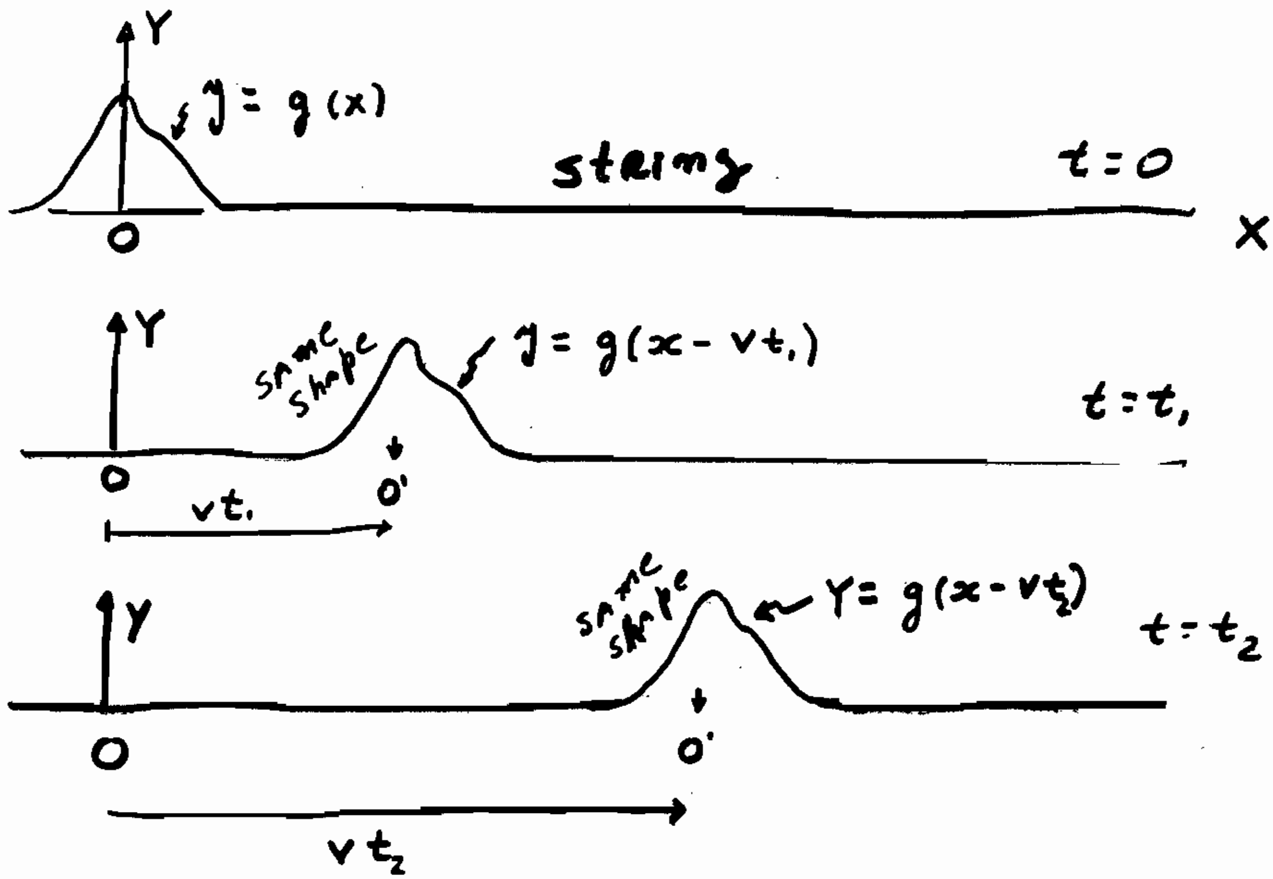


- Function of the form

$$\psi = g(x + vt)$$

describe waves traveling to the left





The wave doesn't get distorted, it keeps its shape

So, we can describe the wave as

$$Y = g(x - vt)$$

g determines the shape of the wave

velocity at which the wave propagates

example sin
cos
exp
etc

FUNCTIONS of 2 variables

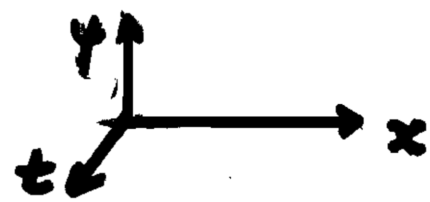
- $$\psi(x,t) = g(x - \underset{\sim}{v}t)$$

ψ is a function of 2 variables

g is a function of 1 variable

velocity

the variables are x and t



- Other times, the wave is described in the following way

$$\psi(x,t) = g(kx - wt)$$

QUESTION: what is the velocity of propagation?

$$= g\left(k\left[x - \underset{\sim}{\frac{w}{k}}t\right]\right)$$

velocity $= \dfrac{w}{k}$

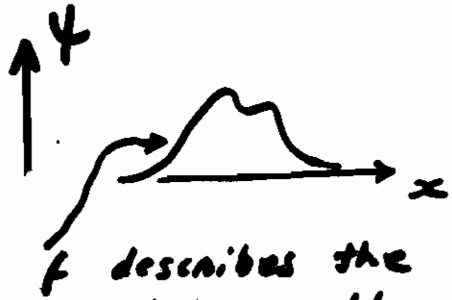
WAVE EQUATION (Section 2.1.1)

If $\Psi = f(x - vt)$

describe a wave,

what type of equation

does it satisfy ?



f describes the shape of the wave

$$\Psi = f(\alpha) = f(\underbrace{x - vt}_{\alpha})$$

NOTICE

$$\hookrightarrow \frac{\partial \Psi}{\partial x} = \frac{df}{d\alpha} \cdot \underbrace{\frac{\partial \alpha}{\partial x}}_{=1} = \frac{df}{d\alpha}$$

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{d^2 f}{d\alpha^2} \quad (1)$$

f could be any function: SIN, EXP, ..., etc

Ψ can represent a physical property: pressure, electric field, ..., etc.

$$\hookrightarrow \frac{\partial \psi}{\partial t} = \frac{df}{d\alpha}\underbrace{\frac{\partial \alpha}{\partial t}}_{-v} = -v\frac{df}{d\alpha}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -v\frac{\partial}{\partial t}\left(\frac{df}{d\alpha}\right) = -v\frac{d^2 f}{d\alpha^2}\underbrace{\frac{\partial \alpha}{\partial t}}_{-v} = v^2\frac{d^2 f}{d\alpha^2}$$

thus

$$\frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = \frac{d^2 f}{d\alpha^2} \qquad (2)$$

Substracting (2) from (1) we obtain.

$$\boxed{\frac{\partial^2 \psi}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0} \qquad (3)$$

called the WAVE EQUATION

It is amazing!

No matter what is the shape of the function f,

the wave $\psi = f(x - vt)$ satisfies Eq 3

f can be: sin, cos, , , exp, etc.

* Exercise: Show that the function $\psi = f(x + vt)$ is also a solution of equation (3)

Harmonic Waves

Let's consider the harmonic wave

$$\Psi(x,t) = A \sin(kx - \omega t)$$

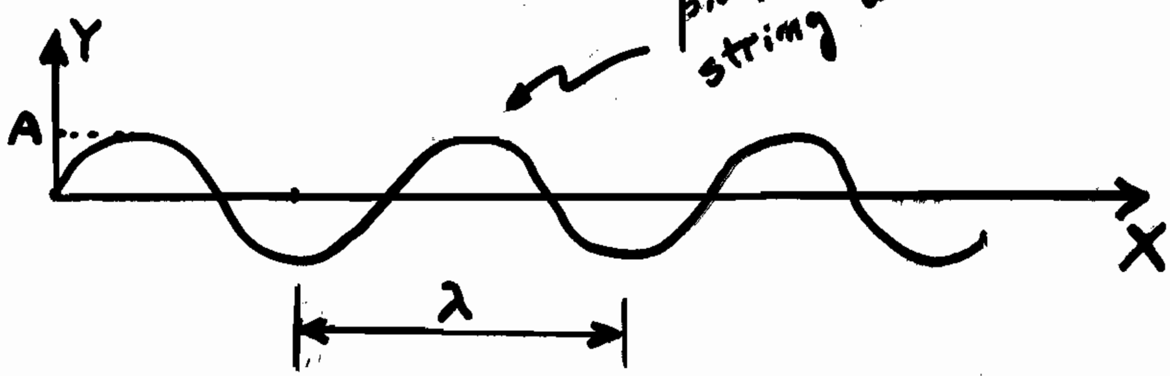
$k = 10\ \text{cm}^{-1}$

$\omega = 2\ \text{rad/sec}$

a) What is the spatial peridicity of the wave?

Let's take a snapshot of the wave at, for example, $t=0$. The resulting profile will be described by,

$$\Psi(x,0) = A \sin(kx)$$



$$\text{spatial periodicity} = \frac{2\pi}{k} = \lambda$$

It is also called the WAVELENGTH

(For the particular case where $k = 10\ \text{cm}^{-1}$,

$\lambda = 0.628$ cm)

At $t=0$

$$\psi(x,0) = \psi(x+\lambda, 0)$$

At any time t

$$\psi(x,t) = \psi(x+\lambda, t)$$

b) What is the temporal periodicity of the wave?

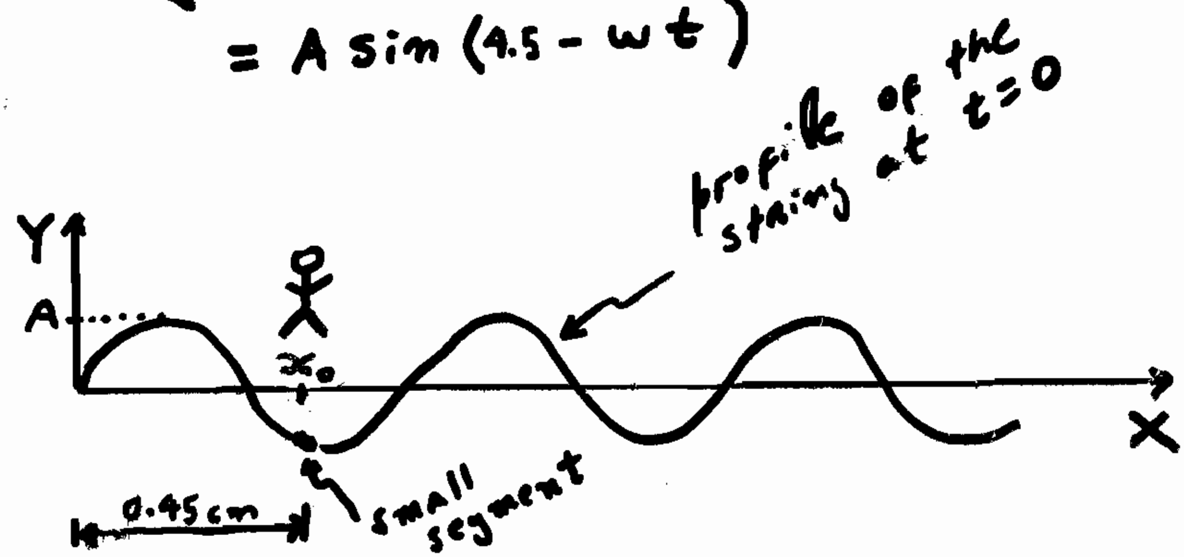
To answer this question let's track the motion followed by a particular segment of the string; the one located at $x_0 = 0.45$ cm for example.

A small segment located around $x = 0.45$ cm follows the motion described by

$$\psi(x_0, t) = A \sin(K x_0 - w t)$$

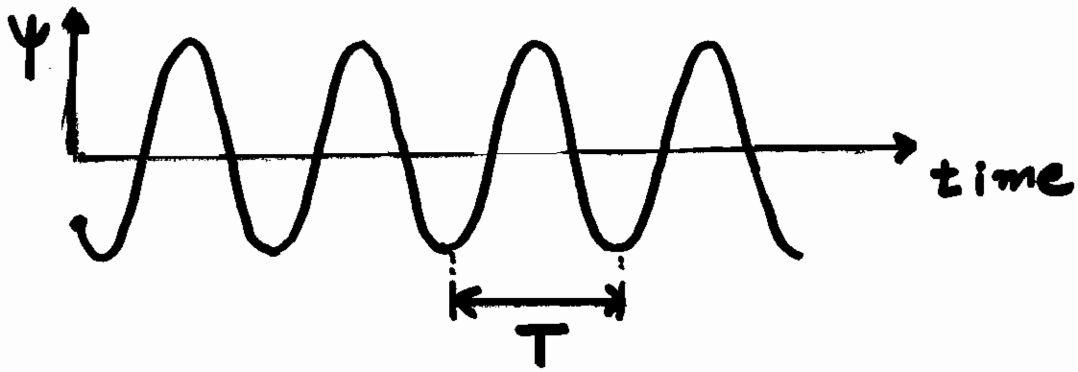
$K = 10\,cm^{-1}$, $x_0 = 0.45$ cm

$$= A \sin(4.5 - w t)$$



So, the small segment located at $x_0 = 0.45$ cm will undergo a up and down motion according to

$$\psi = A \sin(4.5\,\text{rad} - \omega t); \qquad \omega = 2\,\frac{\text{rad}}{\text{sec}}$$



temporal periodicity (of a given segment of the string) $= \dfrac{2\pi}{\omega} =$ we call it PERIOD T

Definition: Frequency $f = \dfrac{1}{T}$

$$= \frac{\omega}{2\pi}$$

Summary (harmonic waves)

$$\psi = \underbrace{A}_{\text{Amplitude}} \sin(\underbrace{kx - wt}_{\text{phase}})$$

wavelength $\lambda = \dfrac{2\pi}{k}$

frequency $f = \dfrac{w}{2\pi}$

phase velocity $v = \dfrac{w}{k}$

Notice $v = \lambda f$

$$\psi = A \sin\left(\frac{2\pi}{\lambda} x - 2\pi f t\right)$$