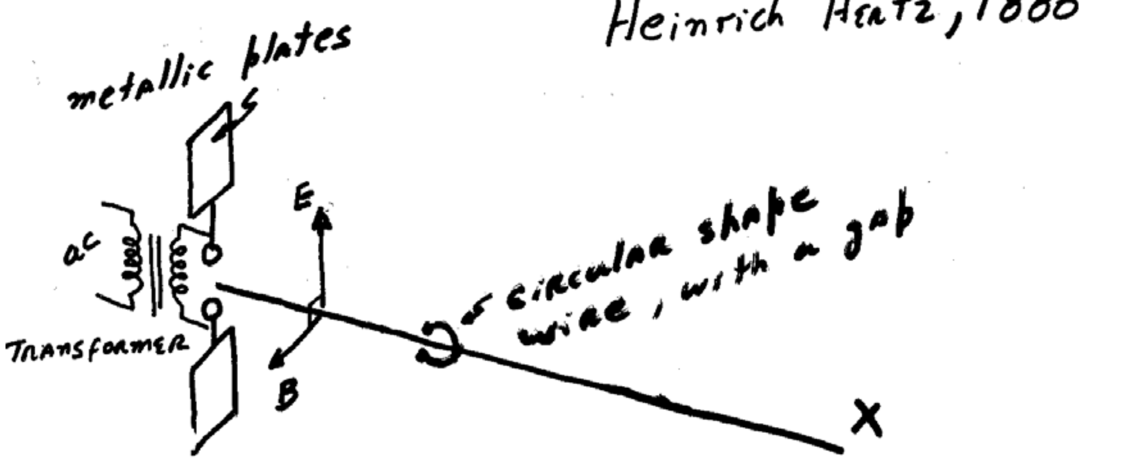
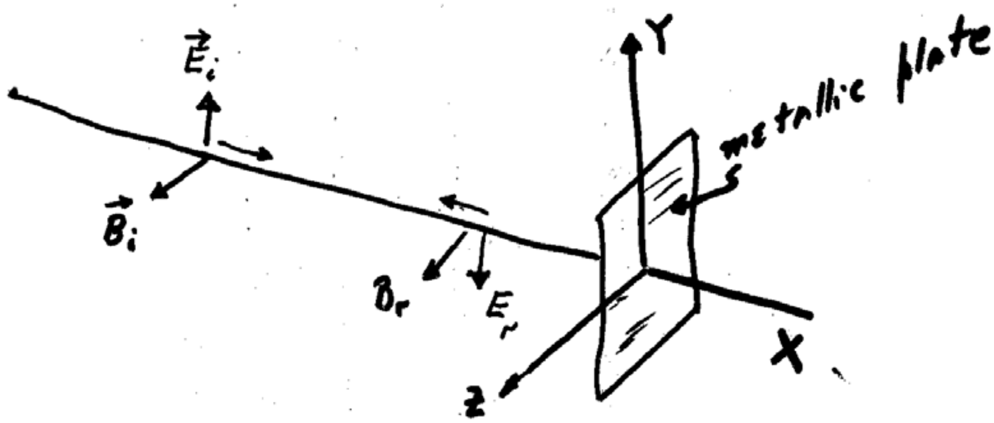
DETECTION of EM WAVES

Heinrich Hertz, 1888



$\rightarrow \quad \vec{\boldsymbol{E}}_i = E_m \, \mathrm{Cos}(\, kx - \omega t \,) \; \hat{\boldsymbol{y}}$



$\rightarrow \quad \vec{\boldsymbol{E}}_i = E_m \, \mathrm{Cos}(\, kx - \omega t \,) \; \hat{\boldsymbol{y}}$

$\leftarrow \quad \vec{\boldsymbol{E}}_r = E_m \, \mathrm{Cos}(\, kx + \omega t + \varphi_R \,) \; \hat{\boldsymbol{y}}$ $\quad \varphi_R$ is included to account for possible phase shift upon reflection

Using the identities $\quad$ Cos(a+b) = Cos(a) Cos(b) - Sin(a) Sin(b)

Cos((a-b) = Cos(a) Cos(b) + Sin(a) Sin(b)

gives $\quad$ Cos(a+b) + Cos((a-b) = 2 Cos(a) Cos(b)

Cos(A) + Cos(B) = 2 Cos[(A+B)/2] Cos[(A-B)/2]

$$\vec{\boldsymbol{E}}_{total}(x,t) = \vec{\boldsymbol{E}}_i + \vec{\boldsymbol{E}}_r = 2\,\mathrm{Cos}\,[\, kx + \varphi_R/2 \,]\,\mathrm{Cos}\,[\omega t + \varphi_R/2\,] \; \hat{\boldsymbol{y}}$$

We require the total electric field to be zero at the metal plate (i. e. at x=0) and at all time.

At x=0:

$$\vec{E}_{total}(0,t) = 2\,\mathrm{Cos}[\varphi_R/2]\,\mathrm{Cos}[\omega t + \varphi_R/2]\;\hat{y} = \mathbf{0}$$

This requires $\varphi_R = \pi$

Thus,

$$\rightarrow \quad \vec{E}_i = E_m\,\mathrm{Cos}(kx - \omega t)\;\hat{y}$$

$$\leftarrow \quad \vec{E}_r = E_m\,\mathrm{Cos}(kx + \omega t + \pi)\;\hat{y} = -E_m\,\mathrm{Cos}(kx + \omega t)\;\hat{y}$$

$$\vec{E}_{total}(x,t) = \vec{E}_i + \vec{E}_r = 2\,\mathrm{Sin}(kx)\,\mathrm{Sin}(\omega t)\;\hat{y}$$

On the other hand, before we also found that

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$$

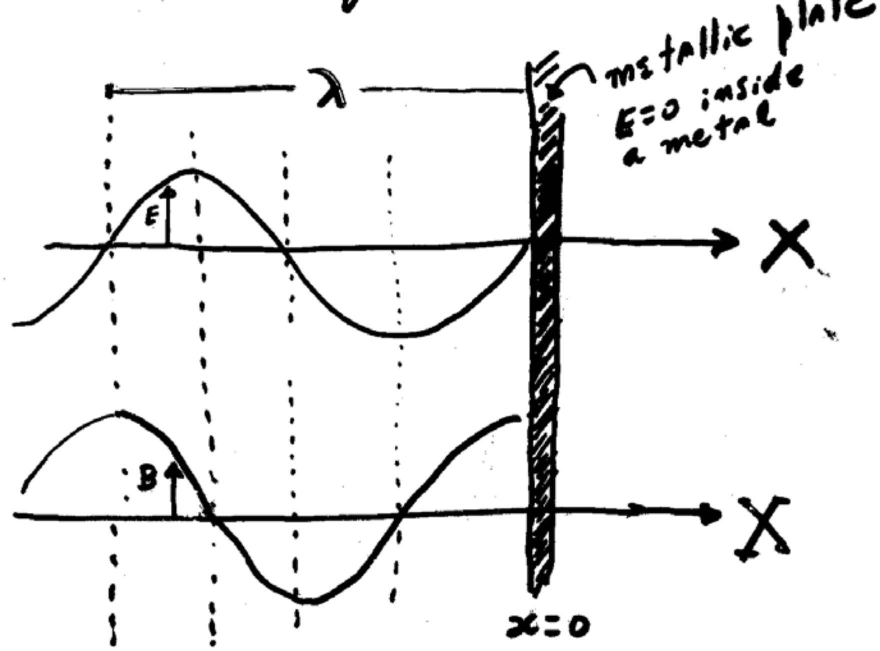
From which one obtains,

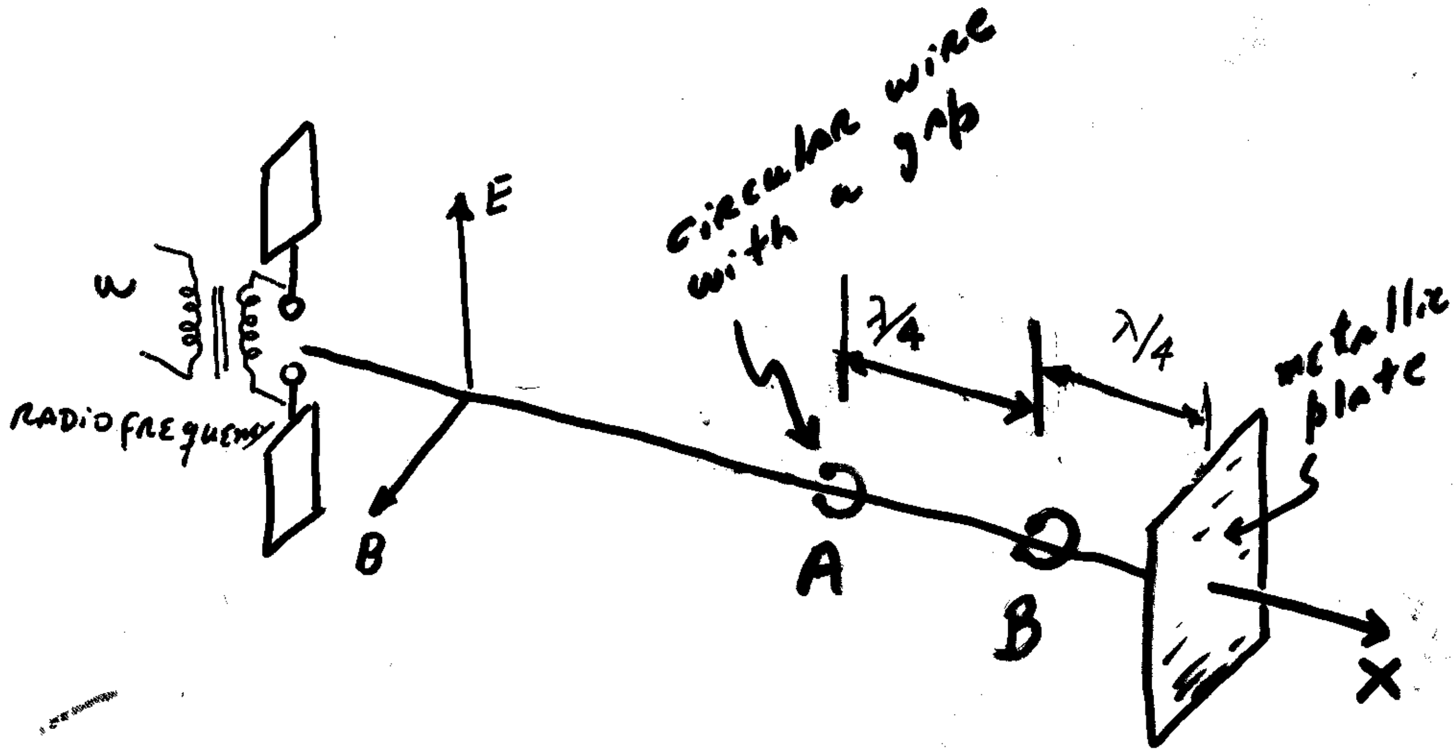
$$2k\mathrm{Cos}(kx)\,\mathrm{Sin}(\omega t) = -\frac{\partial B_z}{\partial t}$$

$$B_z = 2\frac{k}{\omega}\,\mathrm{Cos}(kx)\,\mathrm{Cos}(\omega t)$$

$$\vec{B}_{total}(x,t) = 2\,\mathrm{Cos}(kx)\,\mathrm{Cos}(\omega t)\;\hat{z}$$

Notice E_{total} and B_{total} will have nodes and antinodes along the X axis





ANTINODE A $\mathcal{E}$ electromotive force along the wire $\vec{B}(t)$ $\mathcal{E}$ will produce a spark

NODE B $\vec{B}=0$ NO $\mathcal{E}$ NO spark.

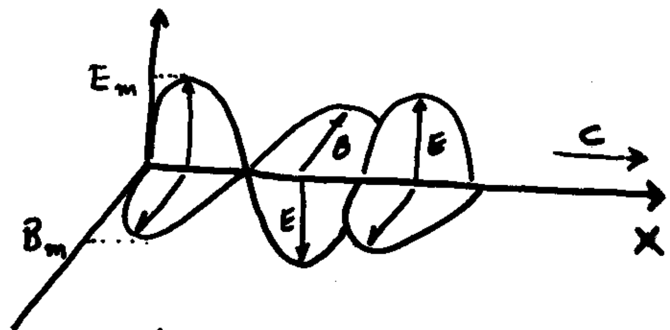
By measuring the distance between 2 antinodes, Hertz was able to figure out $\lambda/2$. Since he knew the frequency ω of the oscillator (transformer), he could calculate the velocity λf $\left(f = \frac{\omega}{2\pi}\right)$ of the electromagnetic waves. $V = 300{,}000$ Km/s

Relationship between the amplitudes E_m and B_m -

Let's consider a sinusoidal wave

$$E = E_m \cos(kx - \omega t)$$

$$B = B_m \cos(kx - \omega t)$$



$$c = \frac{\omega}{k}$$

EM propagating in the free space

and use equation ① (see page 5 in these notes)

$$\frac{\partial E}{\partial x} = -k E_m \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = +\omega B_m \sin(kx - \omega t)$$

$$\underbrace{\text{eq ① implies}}_{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}} \longrightarrow \quad k E_m = \omega B_m \quad \text{or} \quad \boxed{B_m = \frac{E_m}{c}}$$

$\vec{E}$ and $\vec{B}$ are perpendicular to one another

$\vec{E} \times \vec{B}$ points in the direction where the wave propagates toward to.

indicates vector product