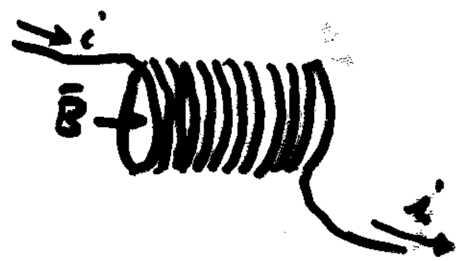
ELECTROMAGNETIC WAVES TRANSPORT ENERGY POYNTING VECTOR

Wherever there is an electric field $\vec{E}$, there is an energy density u_E (energy per unit volume, Joule/m^3) associated to it:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (\text{Joule}/m^3)$$

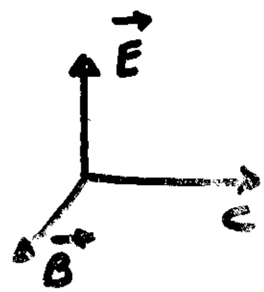
- Wherever there is a magnetic field $\vec{B}$, there is an energy density u_B (Joules/m^3) associated to it:

$$u_B = \frac{1}{2\mu_0} B^2$$



- In an electromagnetic wave, the electromagnetic energy density will be given by

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$



we can use $B = \frac{E}{c}$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$$

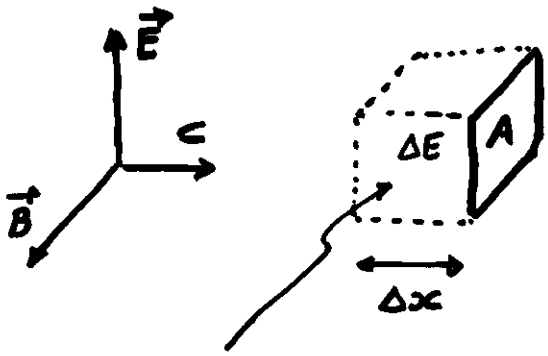
where we have used $c^2 = \frac{1}{\epsilon_0 \mu_0}$

Electromagnetic energy density (Joule/m^3)

$$\boxed{u = \epsilon_0 E^2} \quad \text{or} \quad \boxed{u = \frac{B^2}{\mu_0}} \quad \text{or} \quad \boxed{u = \frac{EB}{\mu_0 c}}$$

But this energy is propagating with velocity c

Let's consider a section of area A



the amount of energy in this cubic volume is $\Delta E = \mu A \Delta x$

In an interval of time $\Delta t = \frac{\Delta x}{c}$ all the energy in that cubic volume will have passed through the section of area A

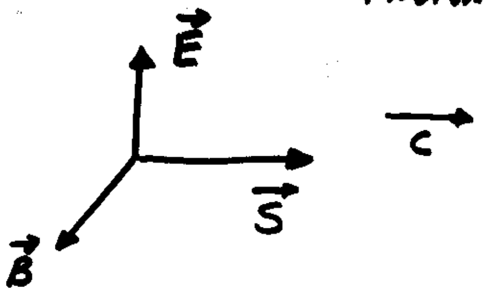
Energy per unit time crossing the area A $= \frac{\Delta E}{\Delta t} = \frac{\mu A \Delta x}{\Delta x / c} = \mu A c$

ENERGY per unit time and per unit AREA $\quad S = \mu c = \frac{E B}{\mu_0}$

$$\boxed{\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}}$$

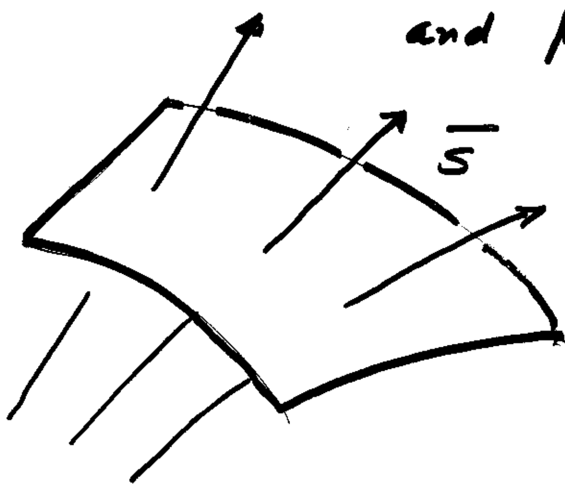
Poynting vector

indicates vector product



S is the flow of electromagnetic energy traveling through the space, per unit area and per unit time.

$$\left[\frac{J}{m^2\, sec}\right]$$



$$S = u c$$

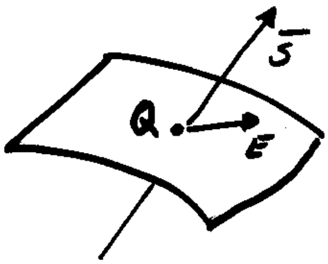
energy density

speed of light

$$\boxed{S = \frac{1}{\mu_0} E B} \qquad \boxed{S = \epsilon_0 c E^2} \qquad \boxed{S = \frac{c}{\mu_0} B^2}$$

S: power per unit area (Watt/m^2)

AVERAGE ENERGY $\langle S \rangle$: INTENSITY



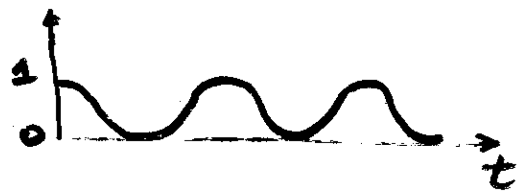
$$S = \epsilon_0 c E^2$$

Let's assume the electric field at the position "Q" is of the form

$$E = E_m \cos(\omega t)$$

The Poynting vector takes the form:

$$S = \epsilon_0 c E_m^2 \cos^2(\omega t)$$



If we take average over time:

$$\langle S \rangle = \epsilon_0 c E_m^2 \underbrace{\langle \cos^2(\omega t) \rangle}_{= \frac{1}{2}}$$

$$\boxed{I = \langle S \rangle = \frac{1}{2} \epsilon_0 c E_m^2}$$ Intensity — power per unit area (Watt/m^2)

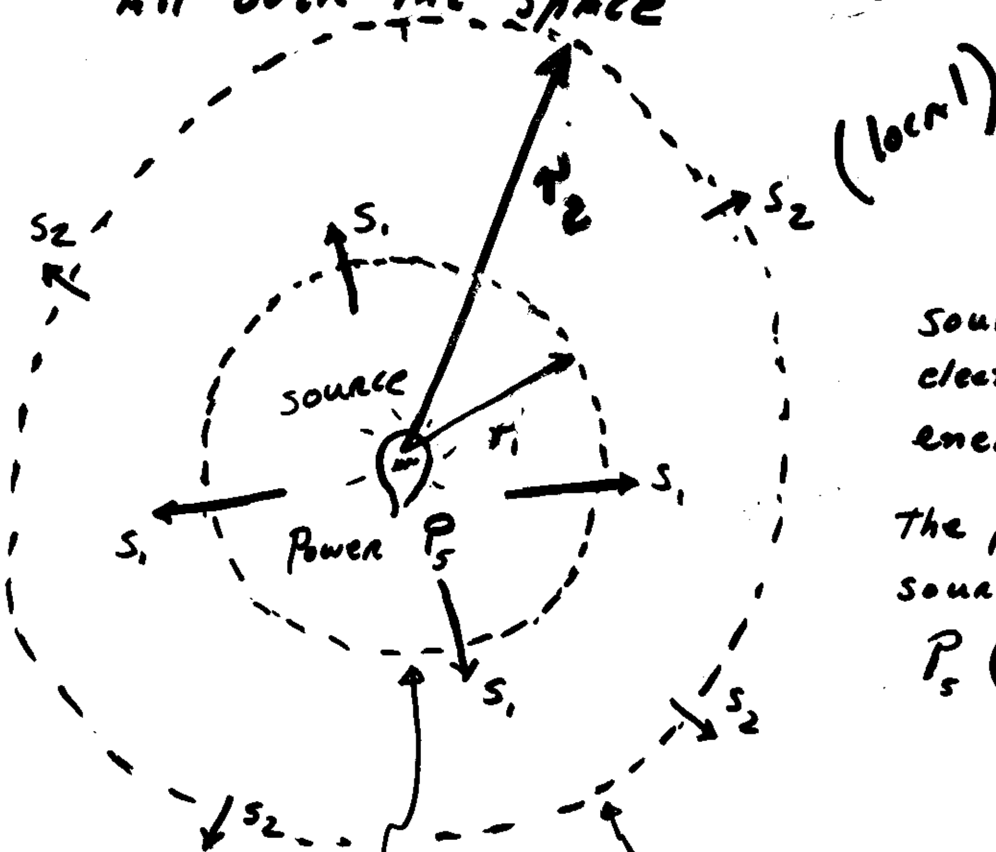
Sometimes, this result is given in terms of rms value of the electric field amplitude

$$E_{rms} \equiv \frac{E_m}{\sqrt{2}} \quad \text{(definition)}$$

$$\boxed{I = \epsilon_0 c E_{rms}}$$ Intensity (Watt/m^2)

Intensity decreases with distance from the source

CASE: PUNCTUAL SOURCE emiting energy all over the space



Source emits electromagnetic energy

The power of the source is:

P_s (Joules/sec)

Notice, conservation of energy implies:

$$P_s = \langle S_1 \rangle 4\pi r_1^2 = \langle S_2 \rangle 4\pi r_2^2$$

$$= I_1 4\pi r_1^2 = I_2 4\pi r_2^2$$

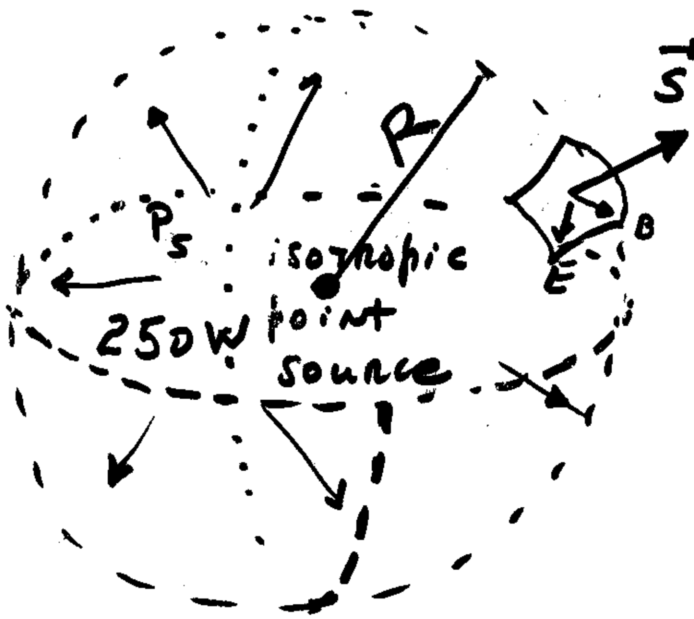
That is, for a constant value of P_s, the intensity varies with the distance r:

$$I(r) = \frac{P_s}{4\pi r^2}$$

Remember:

$$I = \langle S \rangle$$

Example: sample problem 34-1 (pg 811)



$\vec{S}$ $\left[\frac{\text{Joule}}{m^2\ \text{sec}}\right]$

$\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

$E = E_m \cos(\omega t)$

$\langle s \rangle = \frac{1}{2} \epsilon_0 c E_m^2$

$$\frac{250\,W}{4\pi R^2} = \langle s \rangle = \frac{1}{2} \epsilon_0 c\ E_m^2$$ *

$R = 1.8\,m$

$\epsilon_0 =$

$c =$

$$\Rightarrow \quad E_m^2 = \frac{2 \times 250 \frac{J}{s}}{4\pi (1.8\,m)^2\ \epsilon_0 C}$$

Electric field $E = \frac{\text{volts}}{m}$ $\left(\begin{matrix} \text{Remember} \\ E \cdot d = V \end{matrix} \right)$

$$\text{volt} = \frac{\text{work}}{\text{unit charge}} = \frac{J}{C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$$

$$= 8.85 \times 10^{-12} \frac{C^2}{J\,m}$$

Remember

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

so,

$$\epsilon_0 = \frac{C^2}{N\,m^2} = \frac{C^2}{J\,m}$$

Therefore

$$E_m^2 = \frac{2 \times 250 \frac{J}{s}}{4\pi (1.8\,m)^2 \; 8.85 \times 10^{-12} \frac{C^2}{J\,m} \times 3 \times 10^8 \frac{m}{s}}$$

$$= 0.46 \times 10^4 \frac{J^2}{m^2 C^2} = 0.46 \times 10^4 \frac{V^2}{m^2}$$

$$E_m = 0.68 \times 10^2 \frac{V}{m}$$

In team of rms: $E_{rms} = \frac{E_m}{\sqrt{2}} = 0.48 \frac{V}{m} \times 10^2$