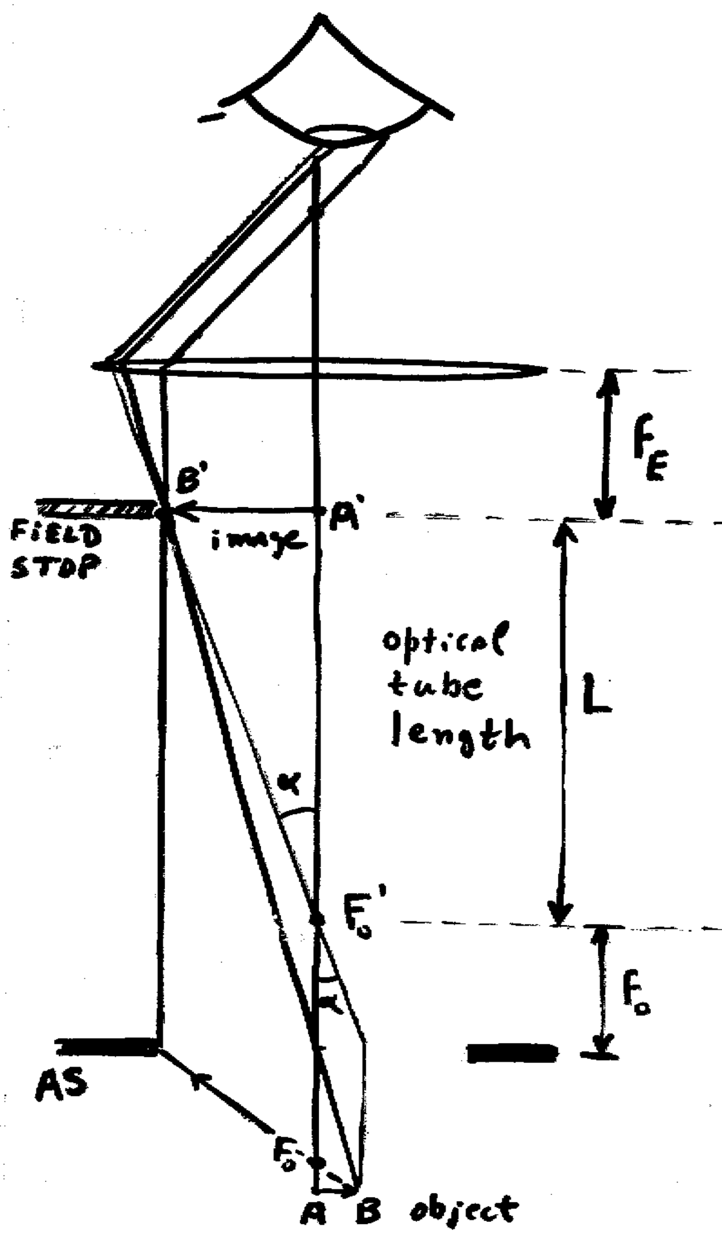
MAGNIFICATION

$$M = \text{angular magnification} \times \text{transverse magnification}$$

$$= \frac{250\text{ mm}}{f_E} \times \frac{L}{f_o}$$



Angular Magnification by the Eyepiece

$$\boxed{M_{A,E} = \frac{250\text{ mm}}{f_E}}$$

Image seen at ∞

$$\tan\alpha = \frac{A'B'}{L}$$

$$\tan\alpha = \frac{AB}{f_o}$$

$$\Rightarrow \frac{A'B'}{AB} = \frac{L}{f_o}$$

thus

Transverse Magnification by the Objective Lens

$$\boxed{M_{T,O} = \frac{L}{f_o}}$$

The total magnification of the microscope is:

$$M_{microscope} = M_{A,E} \times M_{T,O} = \frac{250\,mm}{f_E} \times \frac{L}{f_o}$$

Final image viewed at infinity

In this oversimplified version of the microscope (as presented in the previous diagram) the value of the optical tube length L is taken to be equal to $L = 160$ mm.

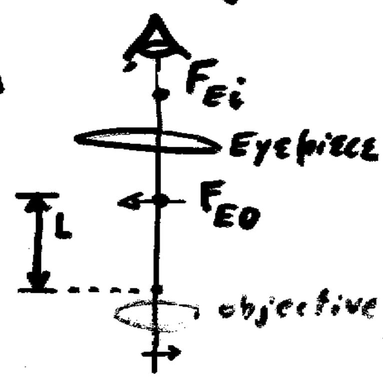
Still, small variations of L are allowed for fine focus of the specimen according to the particular application:

↳ TO VIEW THE FINAL IMAGE at INFINITY:
L is adjusted such that the first image (the one produced by the objective lens) is located at the focal point of the eyepiece.

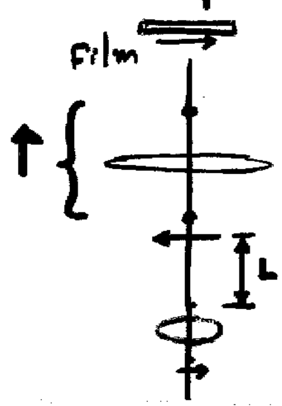
↳ TO RECORD THE FINAL IMAGE in a PHOTOGRAPHIC FILM:
L is adjusted to place the first image at a distance bigger than f_E as to create a real image into the film

Example In a microscope, the objective lens bears the designation EFL = 16 mm, and the eyepiece bears 12 X, by how much must L be raised or lowered when changing from visual observation to photography (Assume the photographic film is 60 mm away from the eyepiece.)

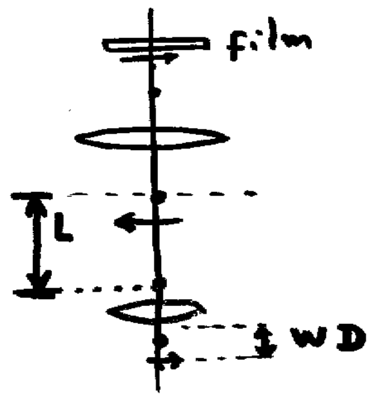
Qualitatively: • If the microscope is set for visual observation (image at ∞) the first image is located at the focal point F_{EO}



• For photography, we need a real image. What we can do is to leave the objective lens in place and lift the eyepiece until the final image is formed at the film.



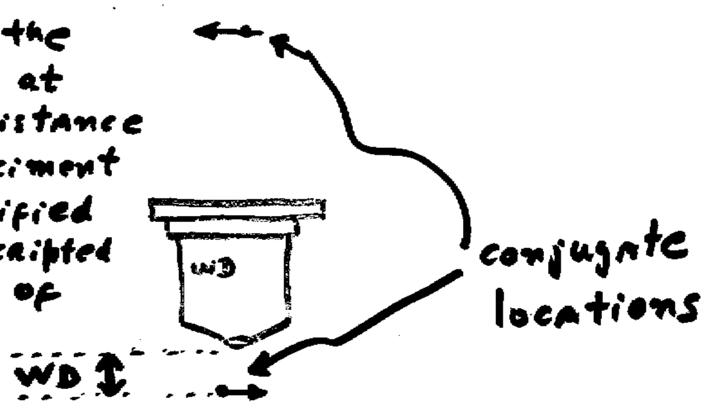
- Alternatively, for photography, we can raised the whole mechanical tube length and refocus the specimen such that the objective lens places the first image at the proper place such that the eyepiece produces a real image on the film.



The penalty in this procedure is that the objective lens will not be working at the proper working distance (WD) it was designed for.

(Objective lenses work best when use to image conjugate points distances they were designed for

Do not place the objective lens at a different distance from the speciment than the specified by the WD incripted on the barrel of the lens



)

Analytical solution:

Eyepiece 12.5X $\Rightarrow$ $12.5 = \text{Magnification} = \dfrac{250\ \text{mm}}{f_E}$

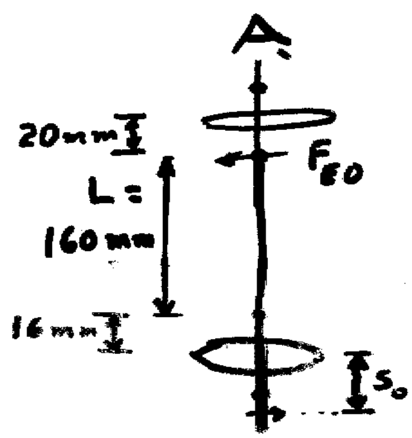
$$\Rightarrow \quad f_E = 20\ \text{mm}$$

objective lens: $f_o = 16\ \text{mm}$

MODE: VISUAL OBSERVATION at ∞

Let's find the position of the object s_o

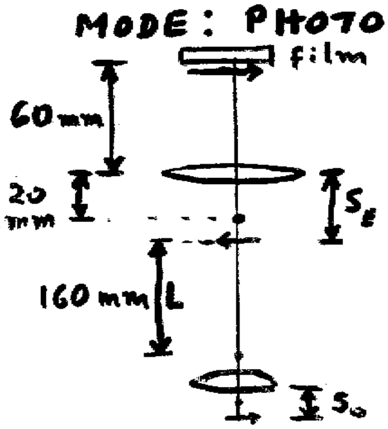
Taking the standard value for the optical tube length to be L = 160 mm, we have



$$\frac{1}{s_o} + \frac{1}{160+16} = \frac{1}{16\ \text{mm}}$$

$\Rightarrow$ $s_o = 17.6$ mm (the working distance WD)

MODE: PHOTOGRAPHY



$$\frac{1}{s_E} + \frac{1}{60\text{mm}} = \frac{1}{20\text{mm}} \Rightarrow s_E = 30\ \text{mm}$$

So, one option is just to lift the eyepiece (together with the film) up by 10 mm, as shown in the figure at the left.

MODE: PHOTOGRAPHY

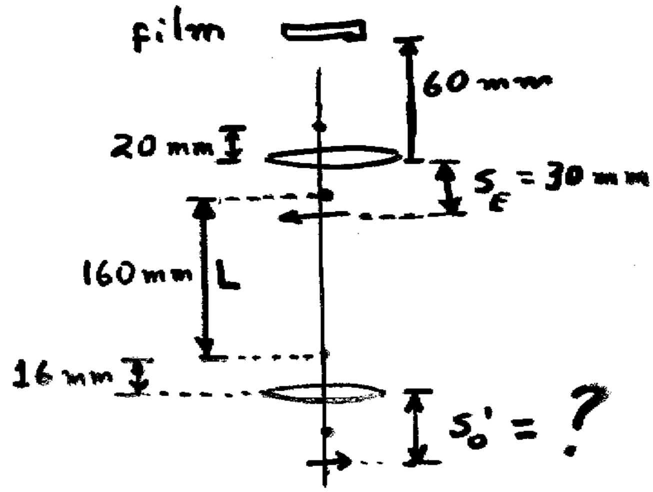
Alternatively we can move the whole microscope up (including the film)

We hope the new adjustment will not change too much the WD calculated above, WD = 17.6 mm = s_o

$$\frac{1}{160-10+16} + \frac{1}{s_o'} = \frac{1}{16\text{ mm}}$$

$$\frac{1}{166\text{ m}} + \frac{1}{s_o'} = \frac{1}{16\text{ mm}}$$

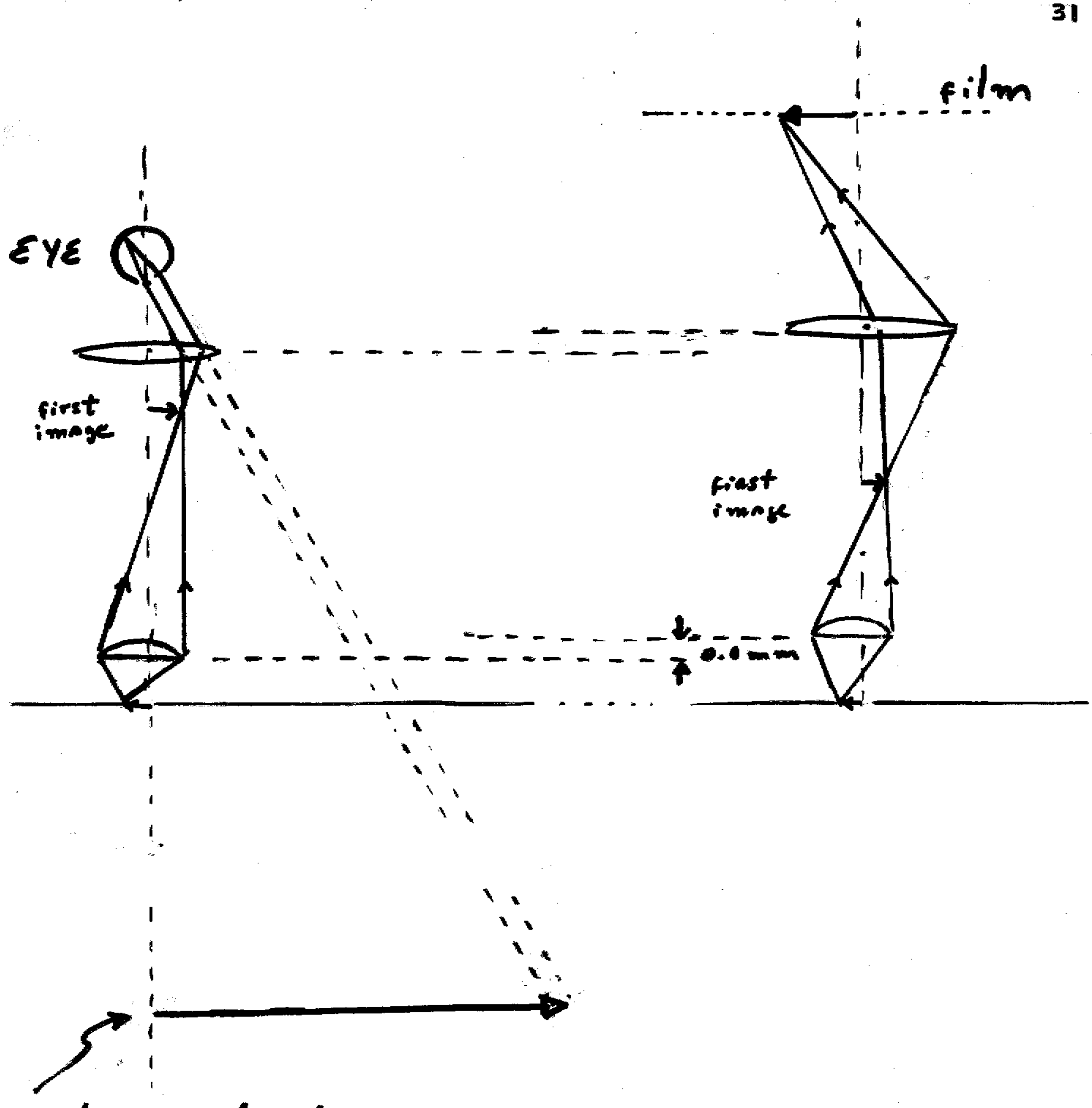
$$\Rightarrow \quad s_o' = \frac{16 \times 166}{150} = 17.7\text{ mm}$$



Notice

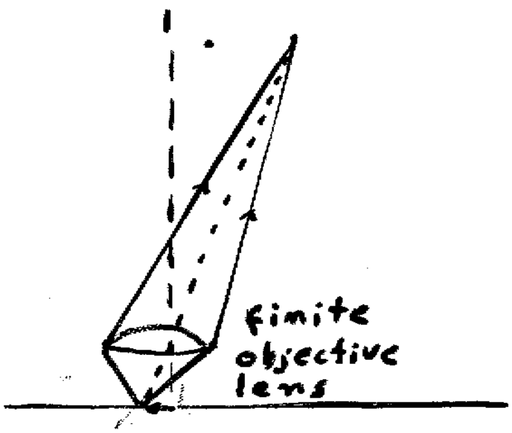
$$s_o' - s_o = 0.1\text{ mm}$$

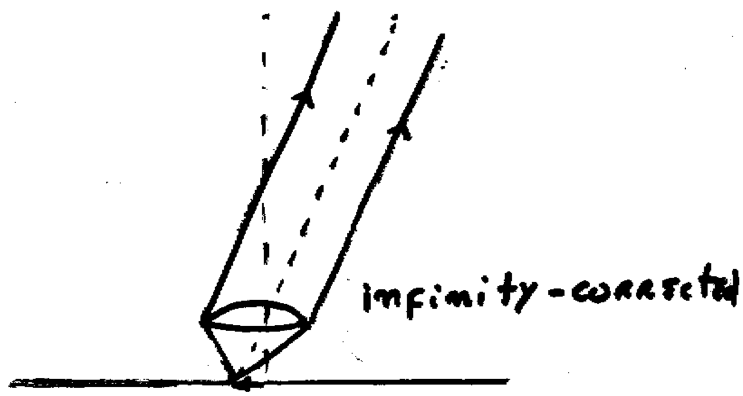
this is the distance that the whole microscope must be raised in order to form an image of the speciment at the location of the film.



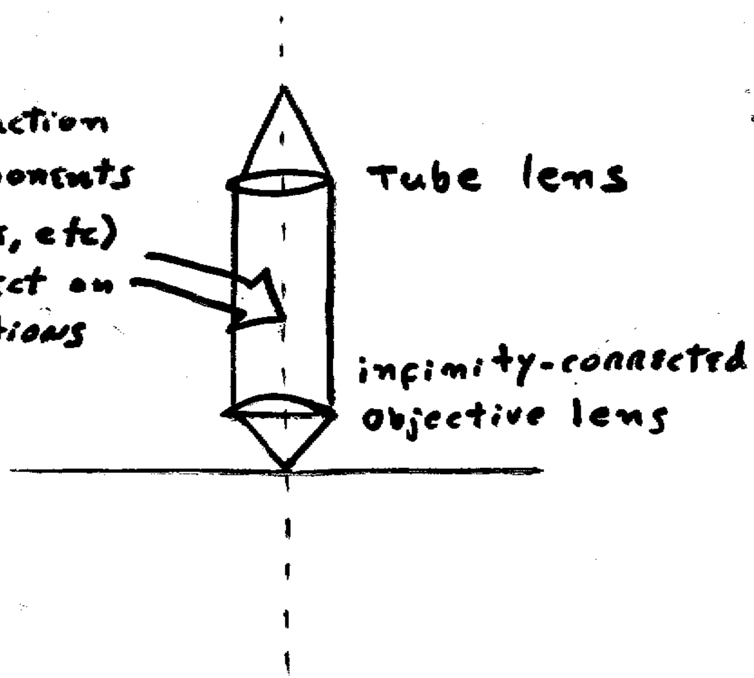
Final image located at ∞ from the eyepiece

Infinity-Corrected Objective Lens (from 1980)



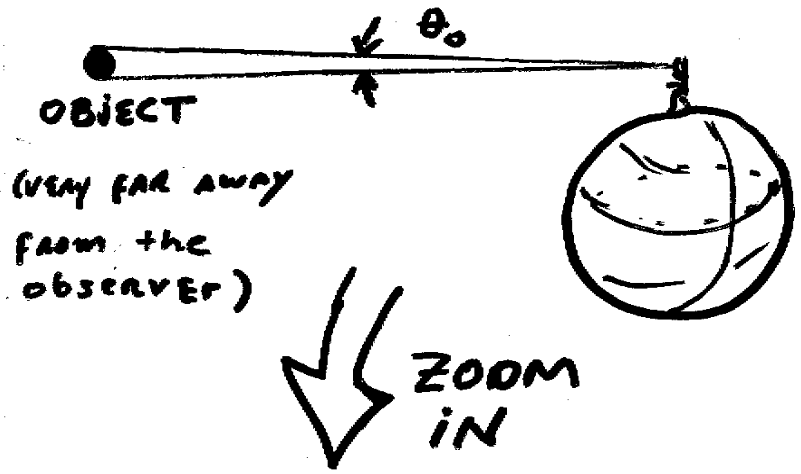


It allows introduction
of auxiliary components
(prism, polarizers, etc)
with minimal effect on
focus and aberrations

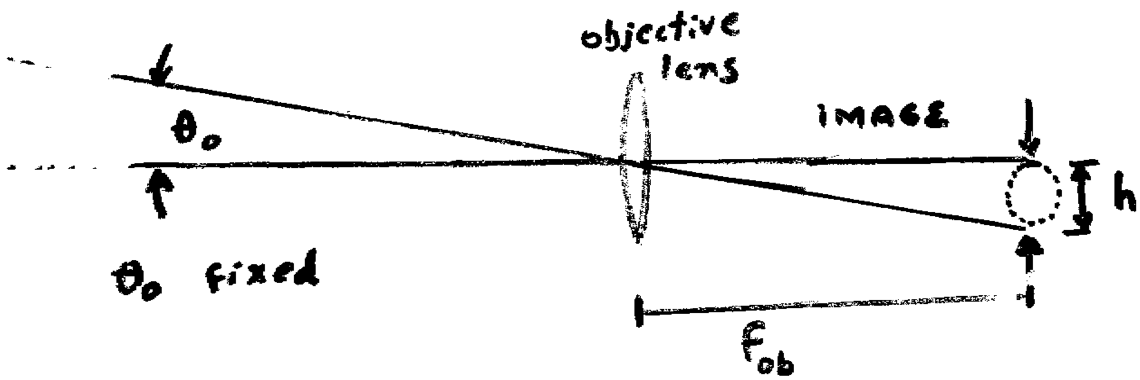


TELESCOPES

Refracting Telescope.–



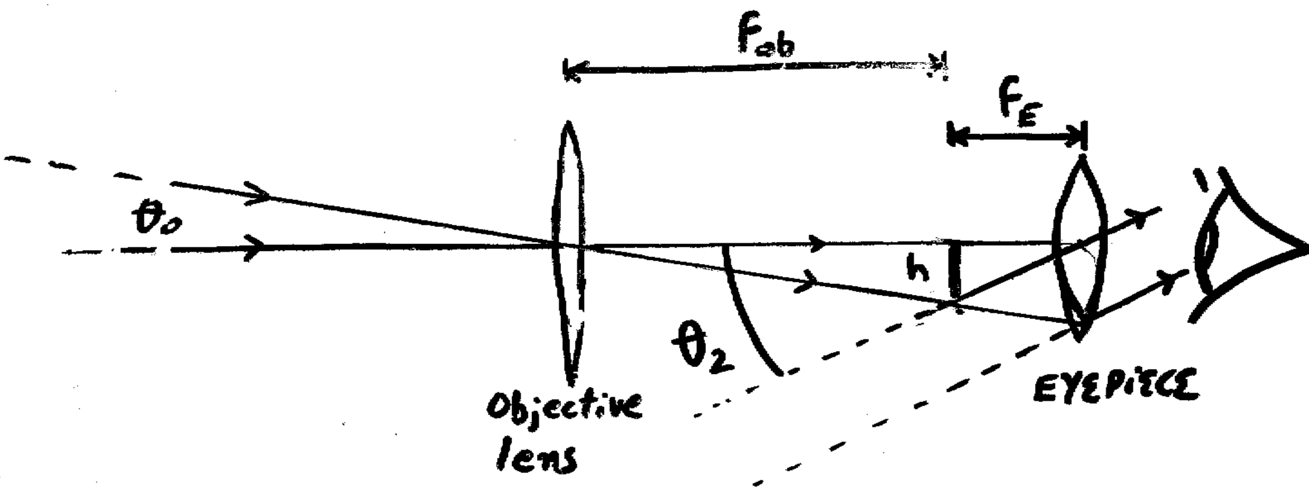
Notice, the angle θ_o is pretty much set
(we can not do anything about it to change)



A lens is used to form a image of the distant object

Notice,
the bigger f_{ob}, the larger h

$$\theta_o = \frac{h}{f_{ob}} \quad \text{or} \quad h = \theta_o \, f_{ob}$$



A second lens is used to image h at infinity.

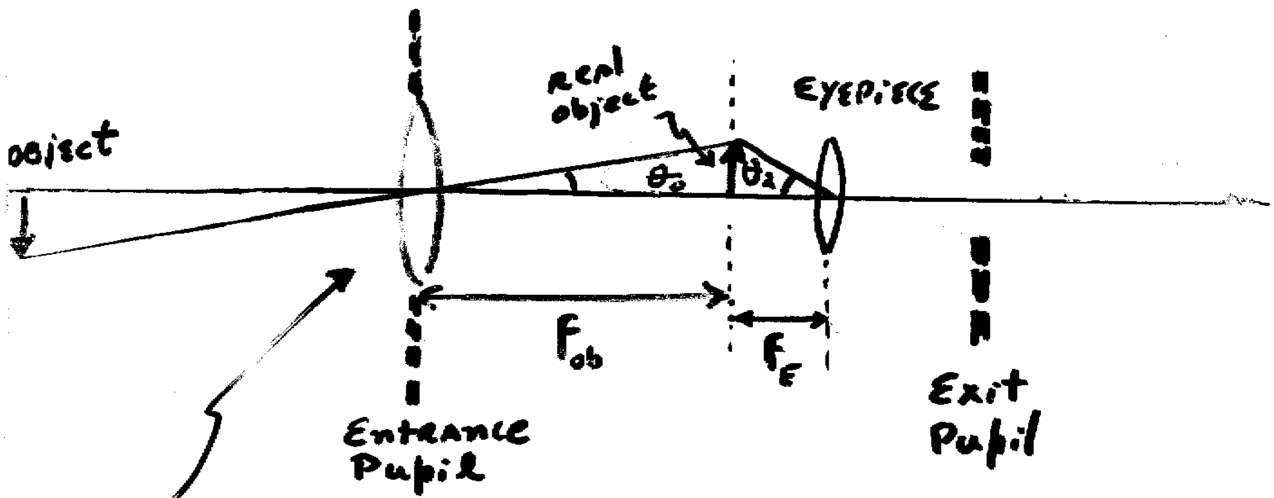
Notice: $\theta_2 = \dfrac{h}{f_E}$

Thus, we have

$$\theta_2 = \frac{h}{f_E} = \frac{\theta_0 f_{ob}}{f_E}$$

$$\frac{\text{Angular size of the object using the telescope}}{\text{angular size of the object without the telescope}} = \frac{\theta_2}{\theta_0} = \frac{f_{ob}}{f_E}$$

KEPLERIAN or Astronomical Telescope



Objective Lens
Made big in diameter to collect as much light as possible from distant stars

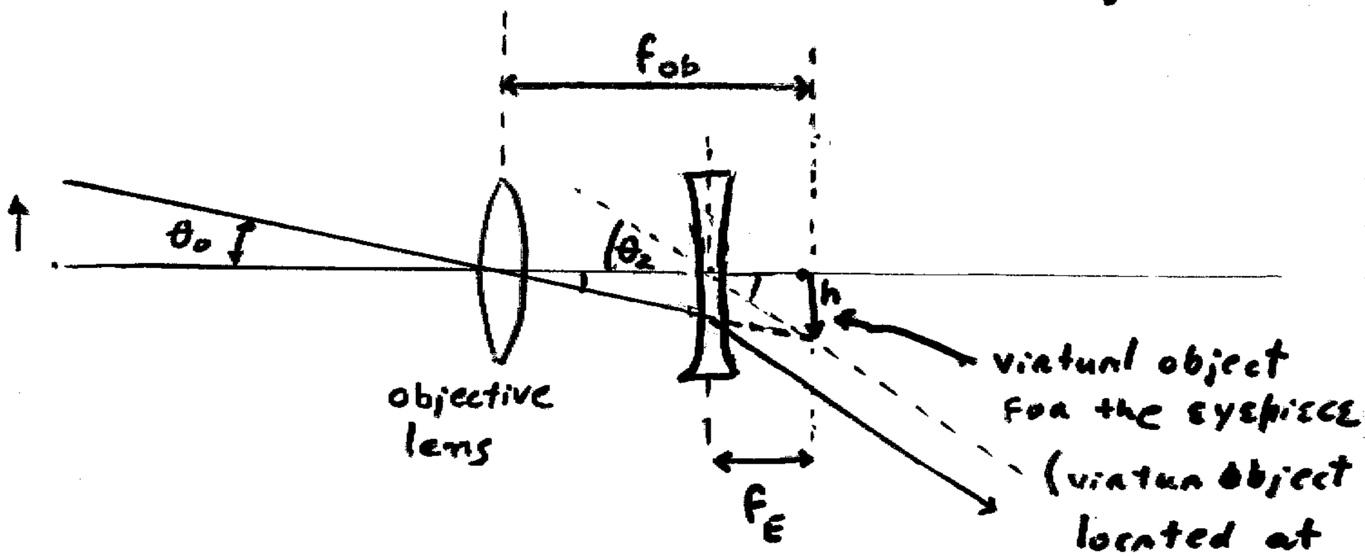
$$\frac{\theta_2}{\theta_o} = \text{Magnification} = -\frac{f_{ob}}{f_E}$$

the negative sign is introduced to indicate that the image is inverted

The telescope is designed such that the Exit Pupil matches the size of the pupil's eye

Galilean Telescope

- It produces an erect image by means of an eyepiece of negative focal length

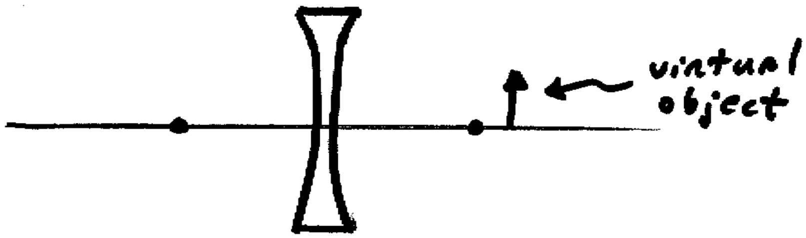


$$\theta_o = \frac{h}{f_{ob}} \quad , \quad \theta_2 = \frac{h}{f_E}$$

$$\Rightarrow \quad \text{Magnification} = \frac{\theta_2}{\theta_o} = \frac{f_{ob}}{f_E}$$

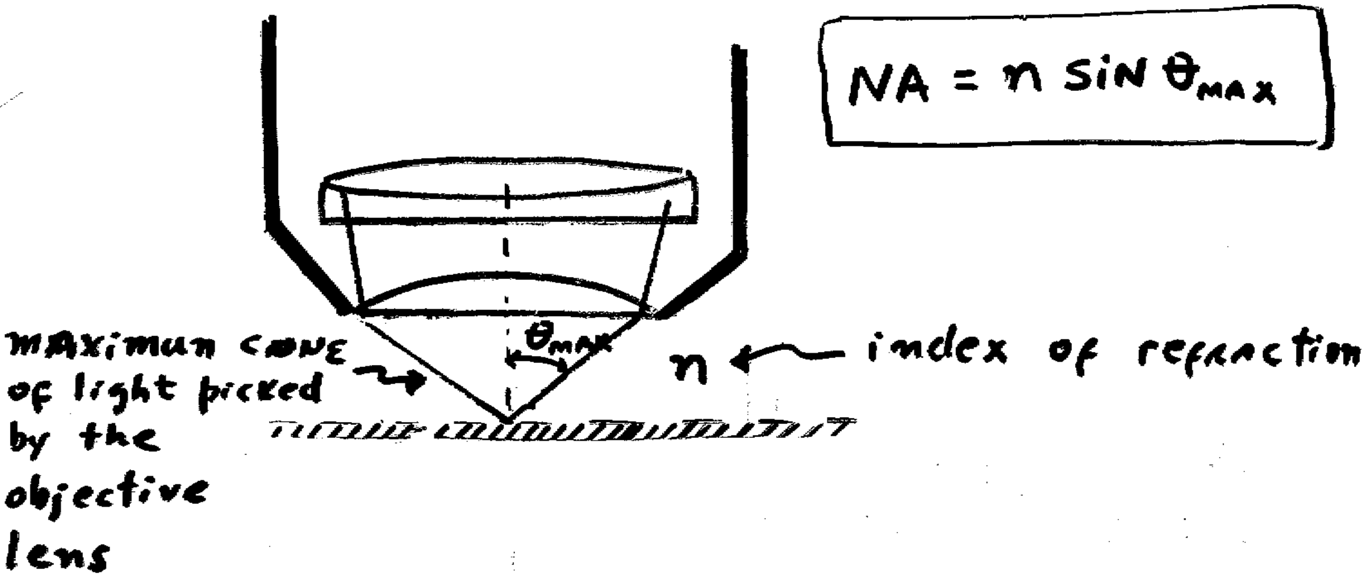
Notice, this set up shortens the length of the telescope (compared to the astronomical telescope).

Exercise Image a virtual object located close to the focal point of a negative lens.

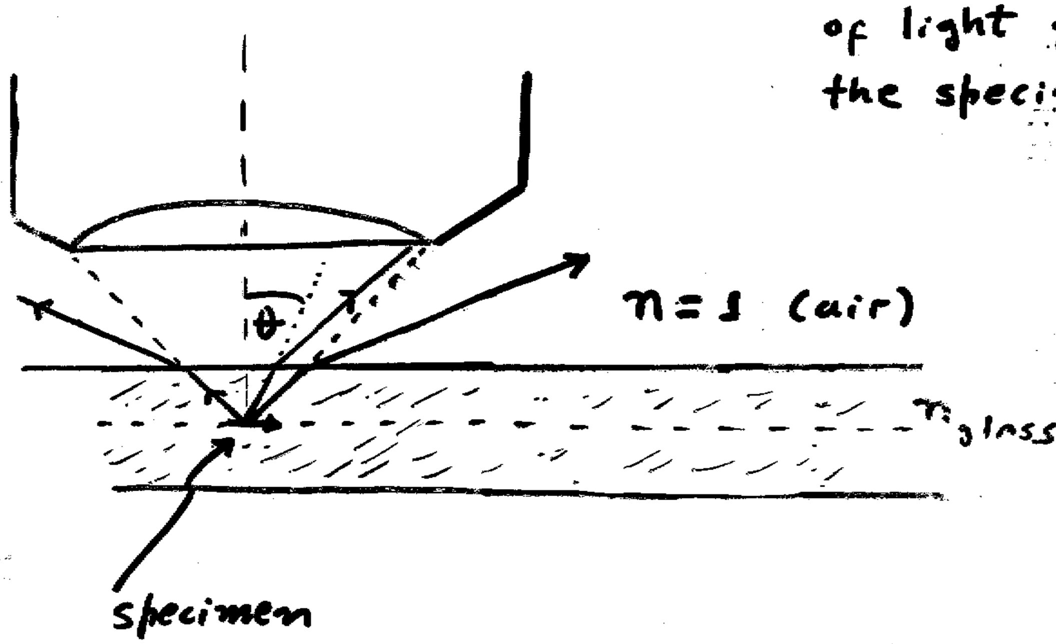


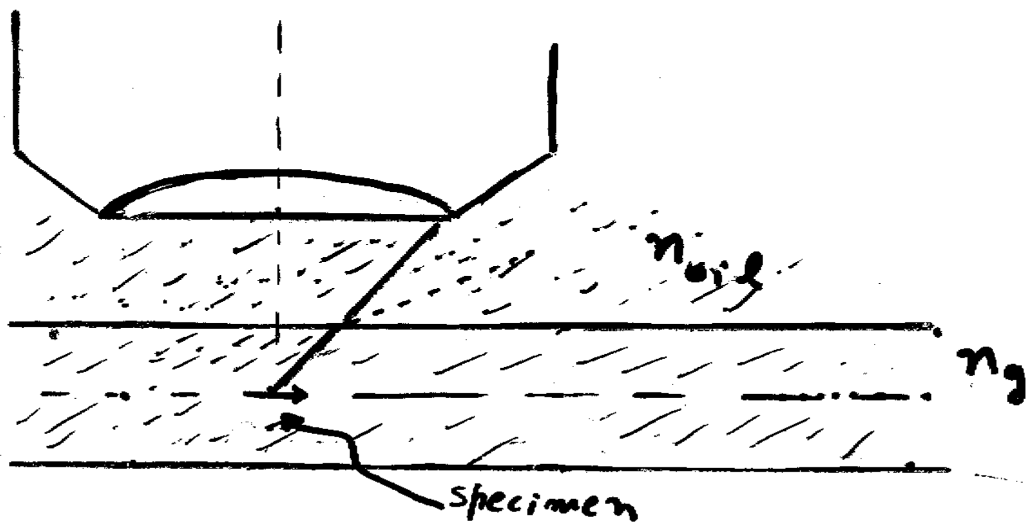
Numerical Aperture NA

$$\boxed{NA = n \sin \theta_{MAX}}$$



Looking through a cover slide decreases the cone of light gathered from the specimen





$n_{oil} = n_{glass}$

Filling the space between the Objective Lens and the cover glass with a substance (i.e. oil) that matches the index of refraction of the glass helps to increase the cone of light gathered by the objective lens from the specimen

RESOLVING POWER:

The smaller lateral dimension able to be distinguished by a lens

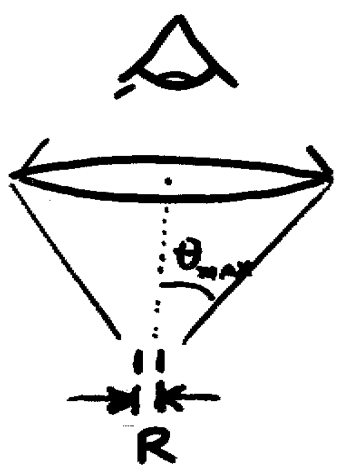
$$R \sim \frac{\lambda}{2 \times NA}$$

finest possible resolution

For green light ($\lambda = 500\,nm$) R is of the order of $250\,nm$

$\Leftrightarrow$ $4000 \frac{\text{lines}}{\text{mm}}$ at best

The smaller R the better resolution



This would be the best we could do with the most expensive conventional optical microscope available on the world.

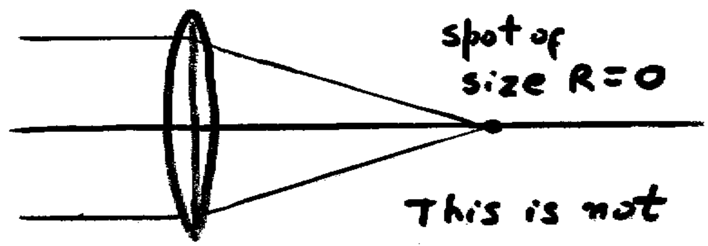
Looking at thing smaller than 100 nm are out of reach for a $\lambda = 500\,nm$ microscope.

In the Geometrical Optics approximation (i.e. $\lambda = 0$) ray tracing would have given $R = 0$ (an extraordinary resolution!) Unfortunately $R = 0$ with $\lambda \neq 0$ does not happen.

The limited resolution can be explained in WAVE OPTICS

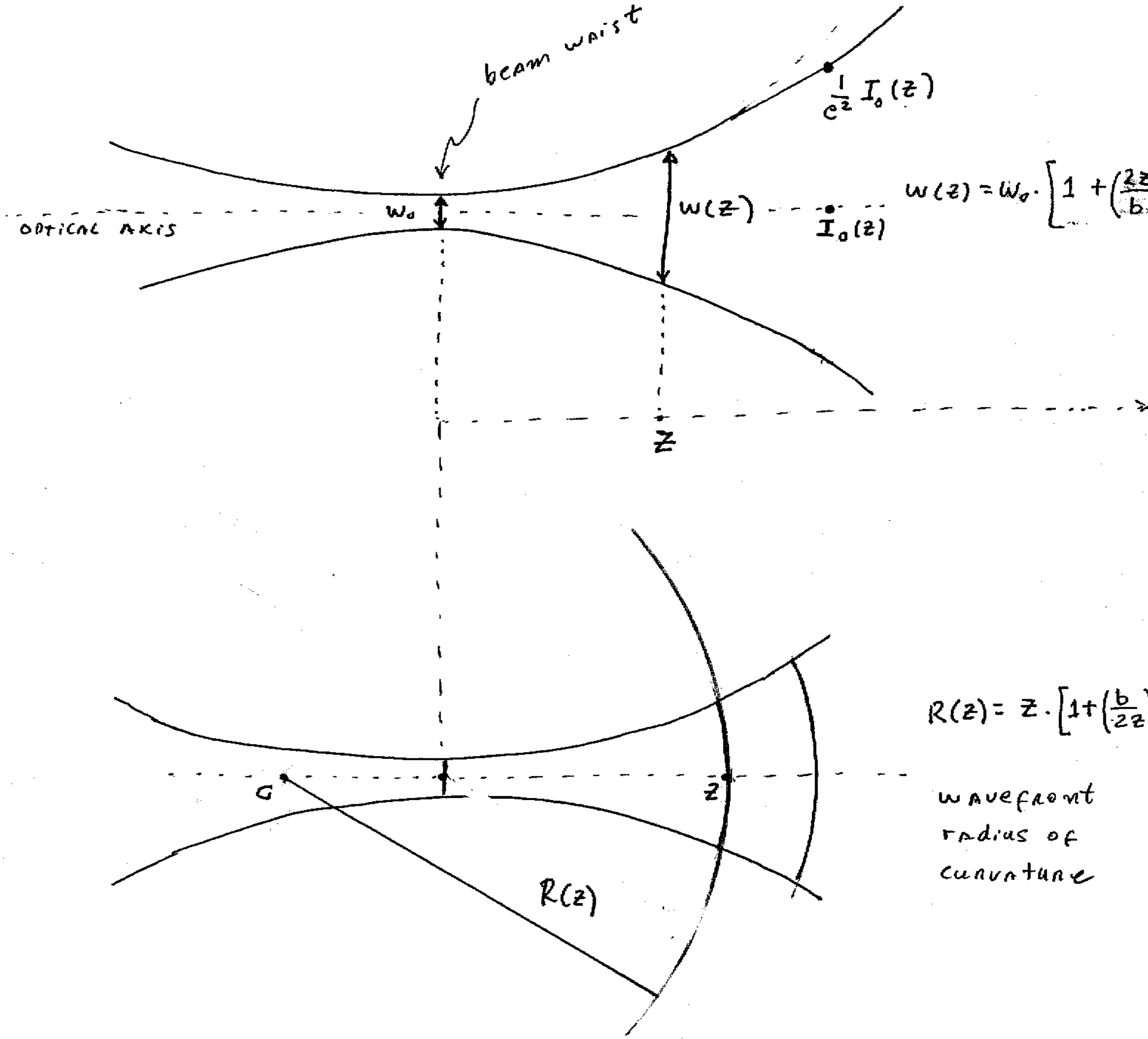
Gaussian Beams

The limited resolving power of an objective Lens (R can not be zero) is correlated also to its inability to concentrate light down to a spot of size zero, as we have frequently drawn in our previous diagrams.



This is not quite true (unless we are working in the geometrical optics domain)

GAUSSIAN BEAMS



$$w(z) = w_0 \cdot \left[1 + \left(\frac{2z}{b}\right)^2\right]$$

$$R(z) = z \cdot \left[1 + \left(\frac{b}{2z}\right)^2\right]$$

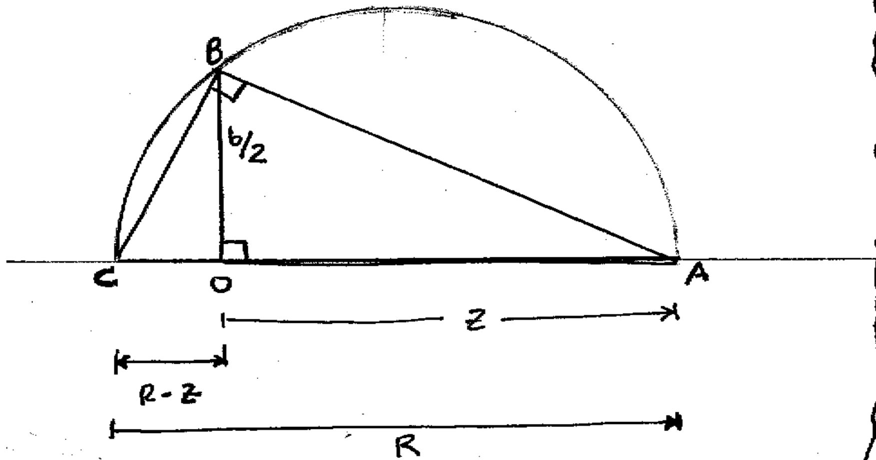
the influence of diffraction is through b

$$b = \frac{2\pi w_0^2}{\lambda} \qquad \text{"confocal parameter"}$$

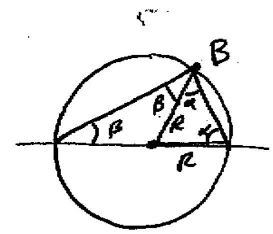
If λ were zero, the beam would be a perfectly collimated one

$$R = z\left[1 + \left(\frac{b}{2z}\right)^2\right]$$

$$\Rightarrow \quad R - z = z\left(\frac{b}{2z}\right)^2 \quad \Rightarrow \quad (R-z).\ z = \left(\frac{b}{2}\right)^2$$



Just remember



Notice

$\beta + \beta + \alpha + \alpha = 18$

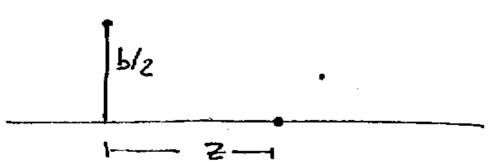
$\Rightarrow \quad \beta + \alpha = 90°$

So, $\measuredangle B = 90°$

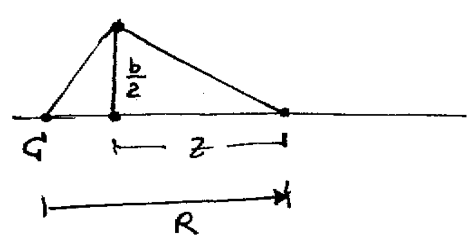
$$\frac{BC}{BA} = \tan A = \frac{b/2}{z} \quad \Rightarrow \quad \left(\frac{b}{z}\right)^2 = (R-z)\,z$$

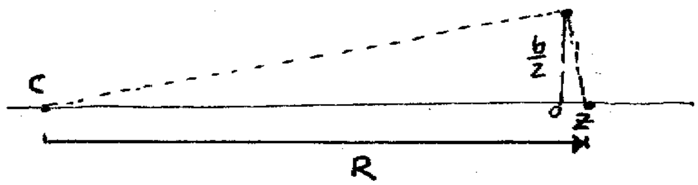
$$\frac{BA}{BC} = \tan C = \frac{b/2}{R-z}$$

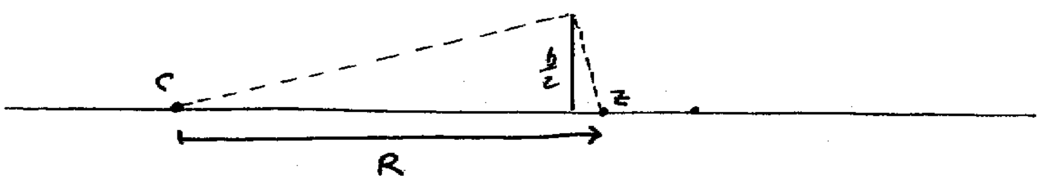
Given $\left(\frac{b}{2}\right)$, the diagram above allows to locate the wavefront center of curvature for each value of z

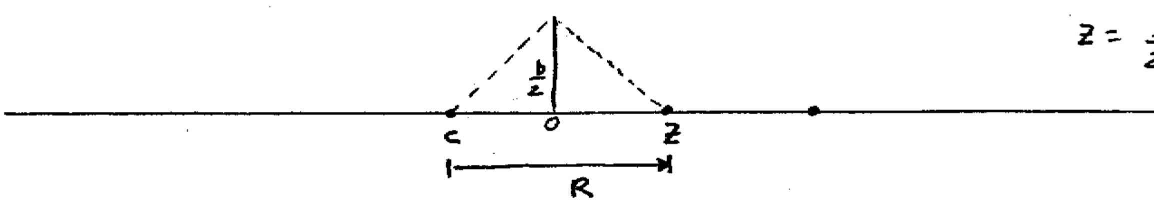


$\Rightarrow$

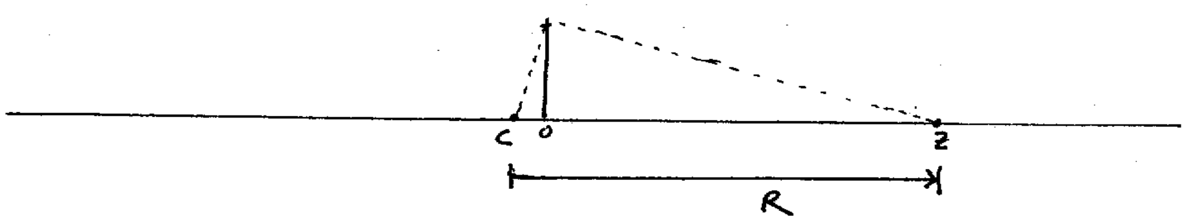


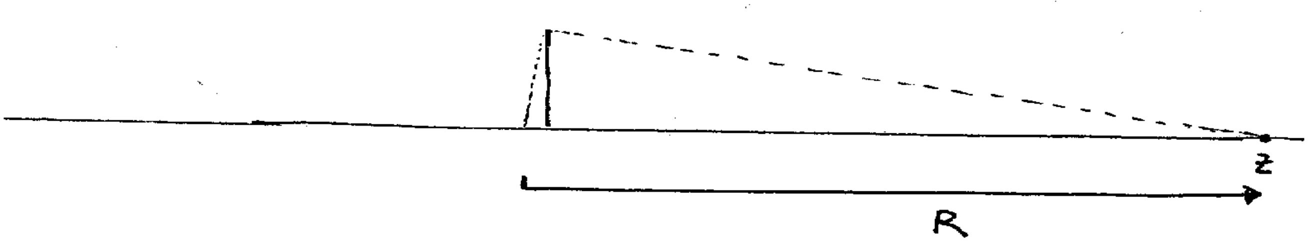






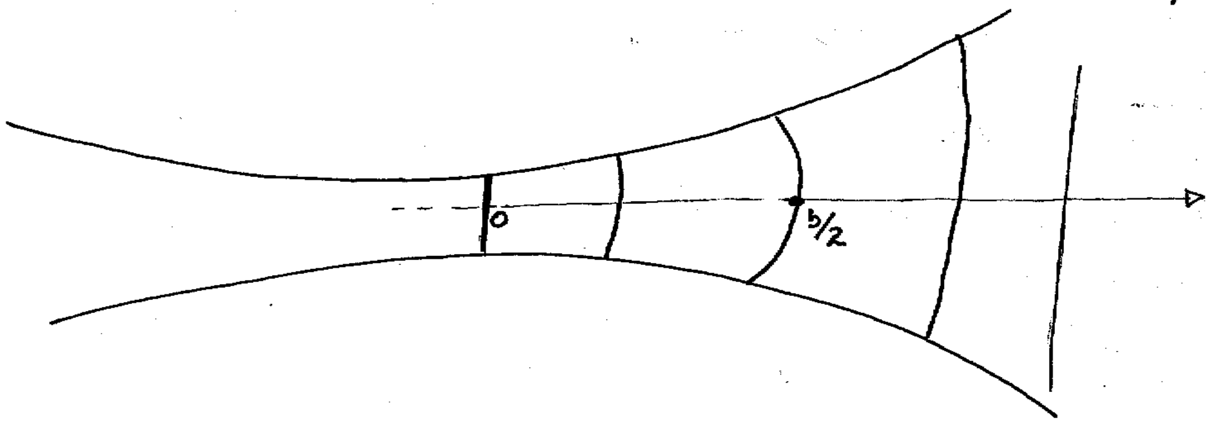
$z = \frac{b}{z}$





- So, at the beam waiste ($z=0$) the wavefront is a plane ($R=\infty$)

At $z=\infty$, again the wavefront is a plane ($R=\infty$)



- The position z at which R is minimum happens at:

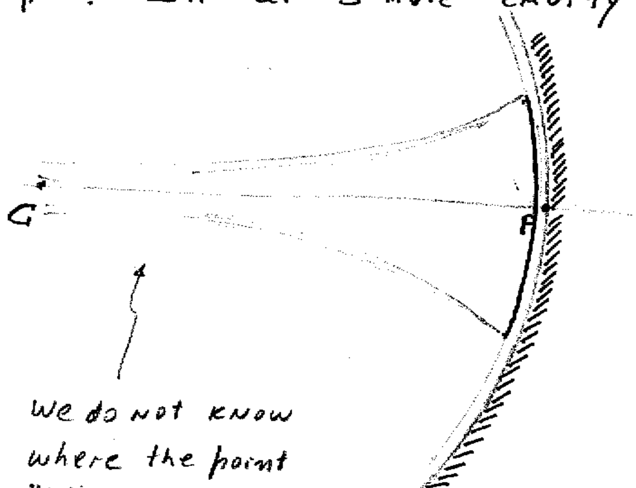
$$\frac{dR}{dz} = \frac{d}{dz}\left(z + \frac{b^2}{4z}\right) = 1 - \frac{b^2}{4z^2} = 0$$

$$\Rightarrow \boxed{z = \frac{b}{2}}$$

position at which the wavefront is minimum.

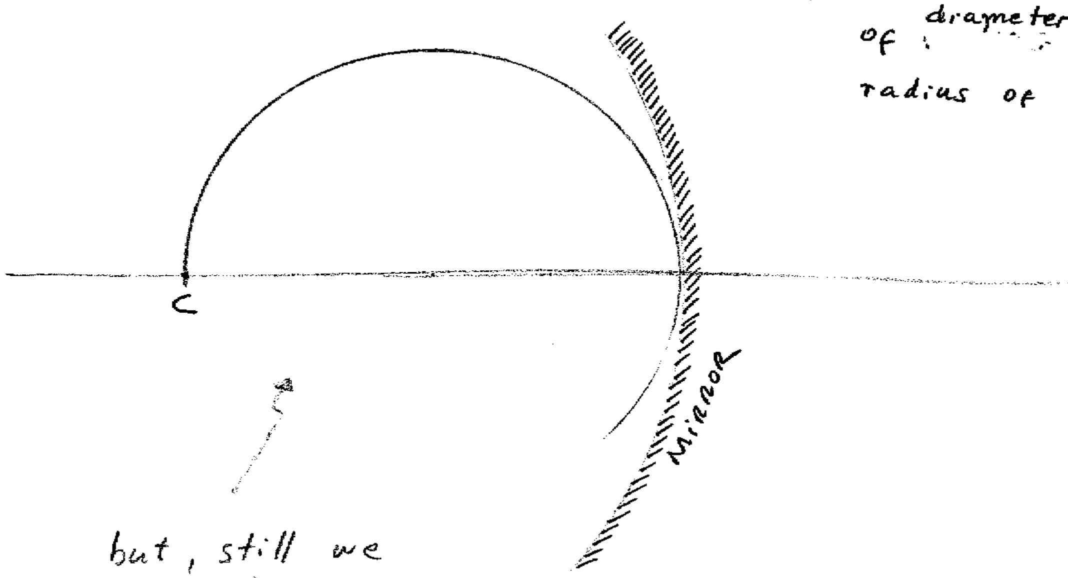
Location of the beam waist of a two-mirror laser-cavity

Principle: In an stable cavity, the radius of curvature of the wavefront must match the radius of curvature of the mirror.

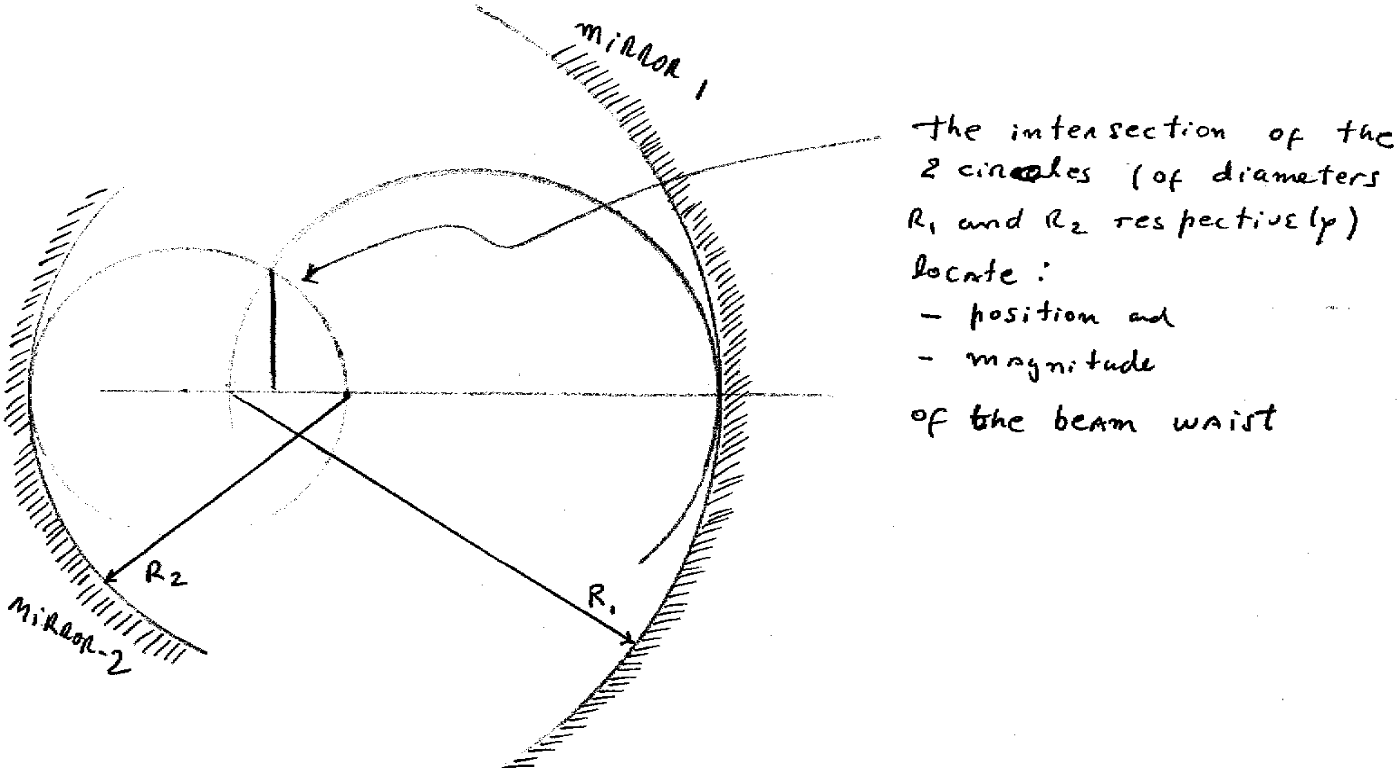


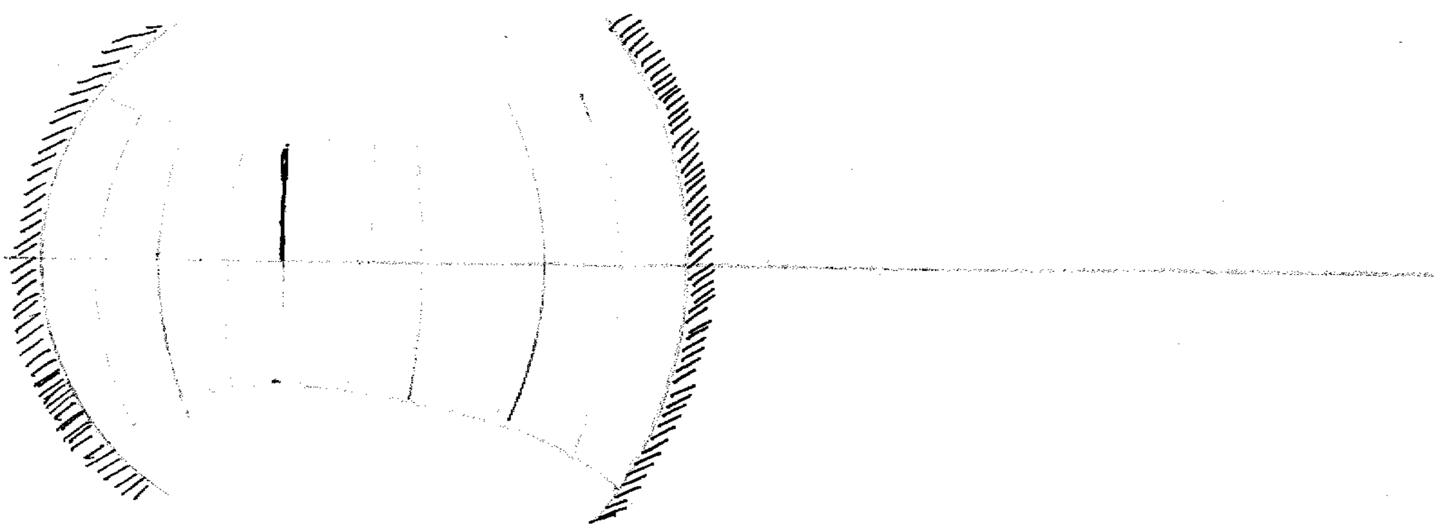
we do not know where the point "O" (beam waiste) is.

So, the geometrical method we are using, implies to draw a circumference with CP as diameter. Or, equivalent, a circumference of diameter equal to the radius of the mirror.



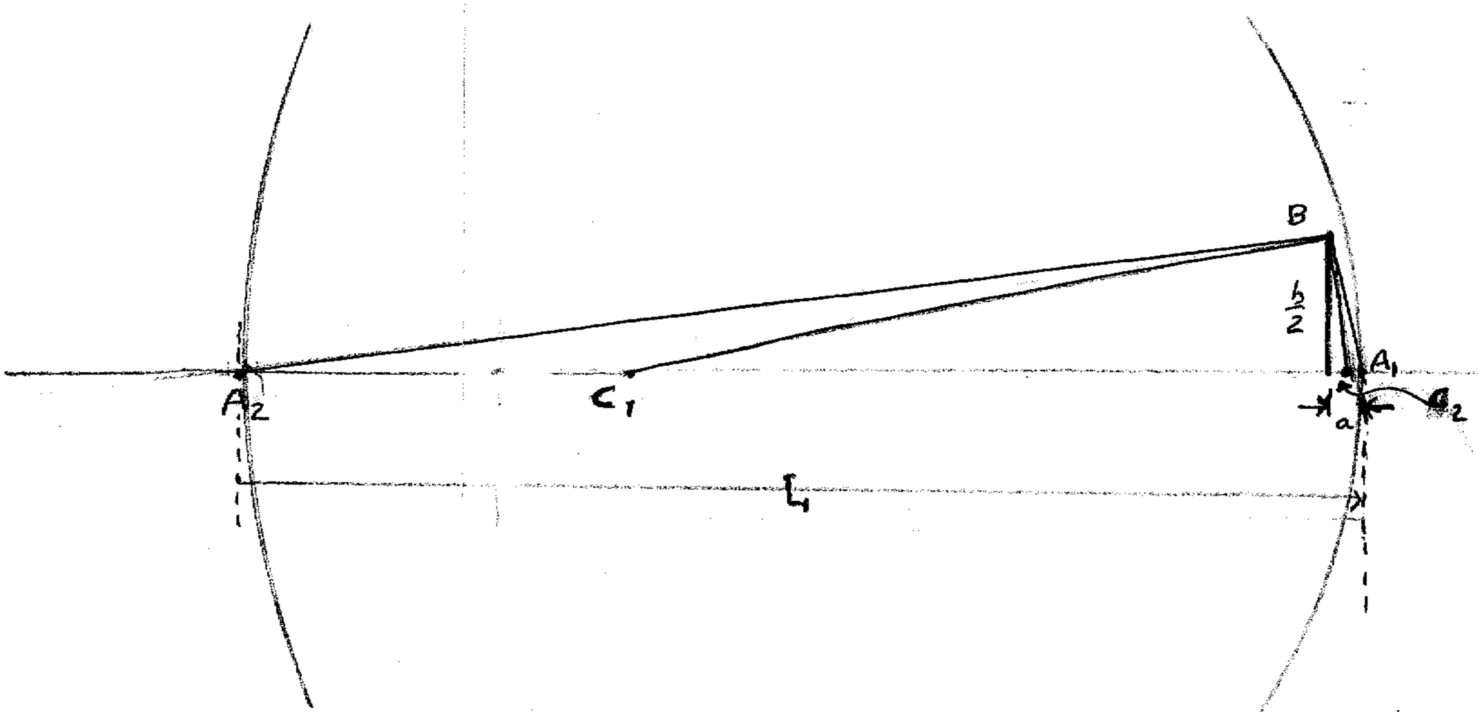
but, still we do not know where the beam waist is, neither the value of b





Conversely, if the required position and size of the beam waist is known, the circle method can be used to obtain approximate values for the mirror curvatures and cavity dimensions

Given $\frac{b}{2}$, we want the beam waist at a distance "a" from one of the mirrors



i) Given $\frac{b}{2}$ and a, draw BA_1

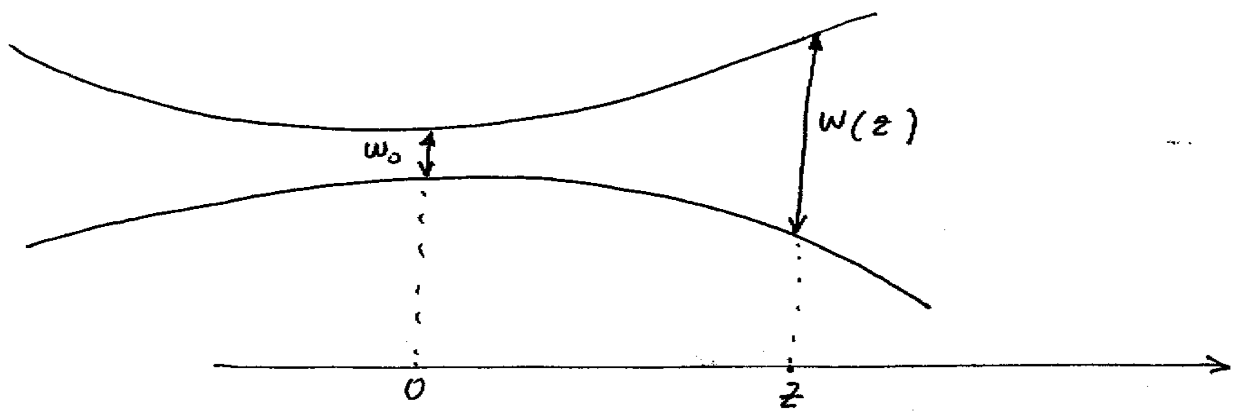
ii) Draw BC_1 such that $BC_1 \perp BA_1$

This determines the required radius of curvature of one of the mirrors. This radius is equal to $\overline{C_1A_1}$

iii) If the length of the cavity is L_1, that determines the point A_2.
Draw $\overline{A_2B}$ ~~and~~ A perpendicular determines BC_2

iv) A_2C_2 becomes the required radius of mirror-2.

Geometric method to find how the radial extent of the Gaussian beam $w(z)$ varies as a function of z



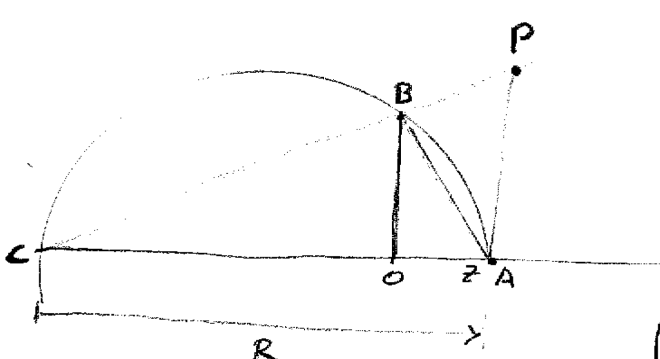
$$\left(\frac{w(z)}{w_0}\right)^2 = 1 + \left(\frac{2z}{b}\right)^2$$

Since $R = z\left[1+\left(\frac{b}{2z}\right)^2\right]$

$$\frac{R}{z} - 1 = \left(\frac{b}{2z}\right)^2 \Rightarrow 2z$$

$$\frac{R-z}{z} = \left(\frac{b}{2z}\right)^2 \Rightarrow \left(\frac{2z}{b}\right)^2 = \frac{z}{R-z}$$

$$\left[\frac{w(z)}{w_0}\right]^2 = 1 + \frac{z}{R-z} = \frac{R-z+z}{R-z} = \frac{R}{R-z} =$$



Notice in the graph that

$$\frac{CA}{CO} = \frac{PA}{OB} =$$

$$\frac{R}{R-z} = \frac{PA}{OB}$$

Therefore

$$\left[\frac{w(z)}{w_0}\right]^2 = \frac{PA}{BO}$$

GRAPHIC METH to evaluate $w(z)$