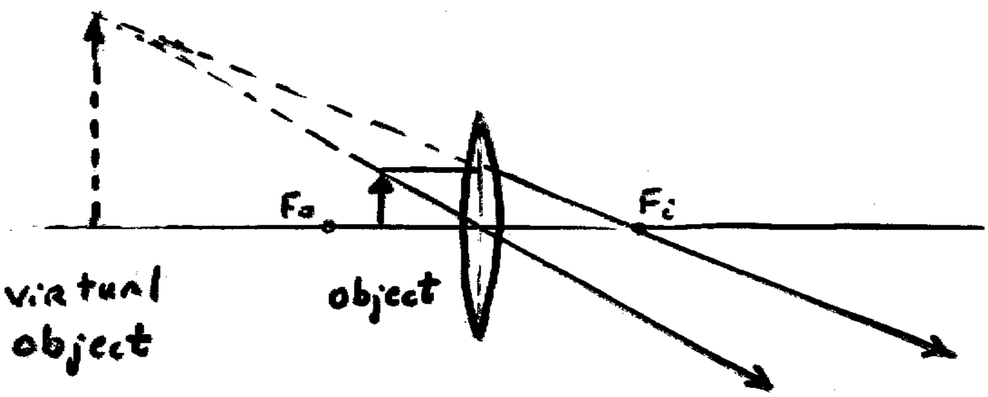
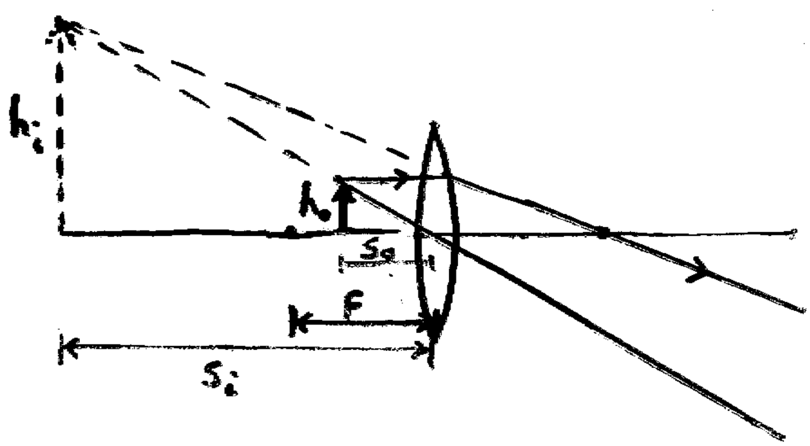
OPTICAL INSTRUMENTS

SIMPLE MAGNIFIER (5.7.3)





$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f}$$

$$s_i = -\frac{f}{f - s_o} \cdot s_o$$

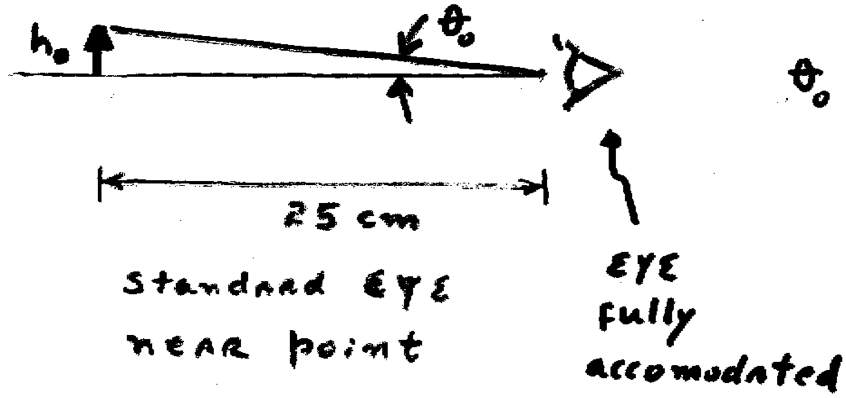
For $s_o < f$

The transverse magnification is given by

$$m = \frac{h_i}{h_o} = \frac{f}{f - s_o}$$ ← Exercise

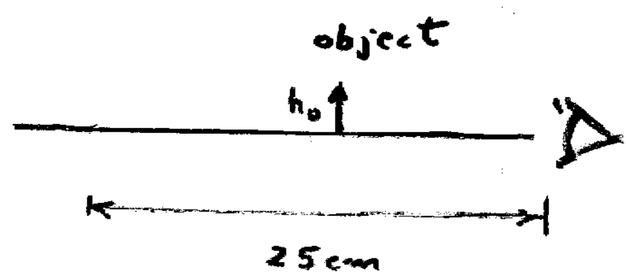
ANGULAR MAGNIFICATION

When viewing virtual images with optical instruments, the images may end up located at great distances ("at infinity"). In such cases the transverse magnification also approaches infinite which turns out to be not very useful. A more convenient term is the defined: the angular magnification

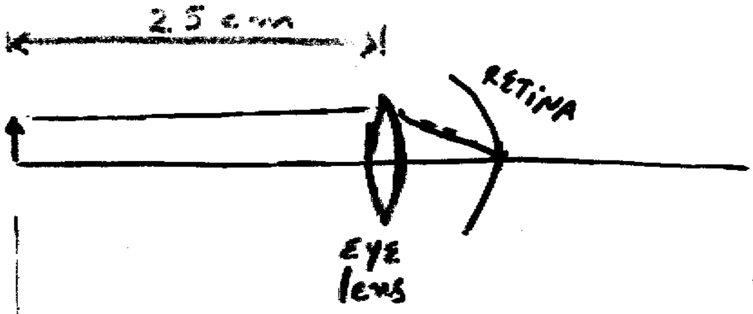


$$\theta_o = \frac{h_o}{25\text{ cm}}$$

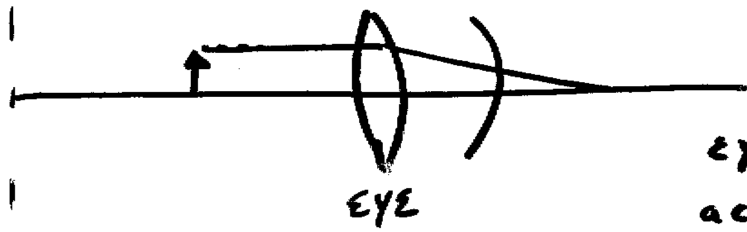
angular size of the object



eye will not be able to image the object on the retina (object is too close, Eye can not accomodate more)

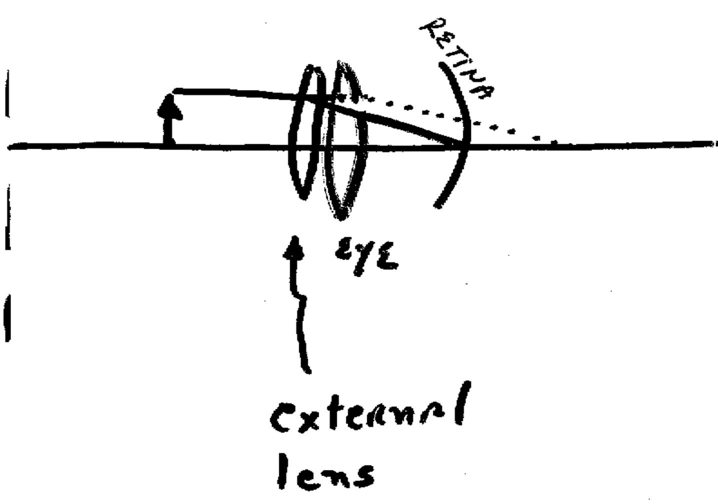


eye fully accommodated

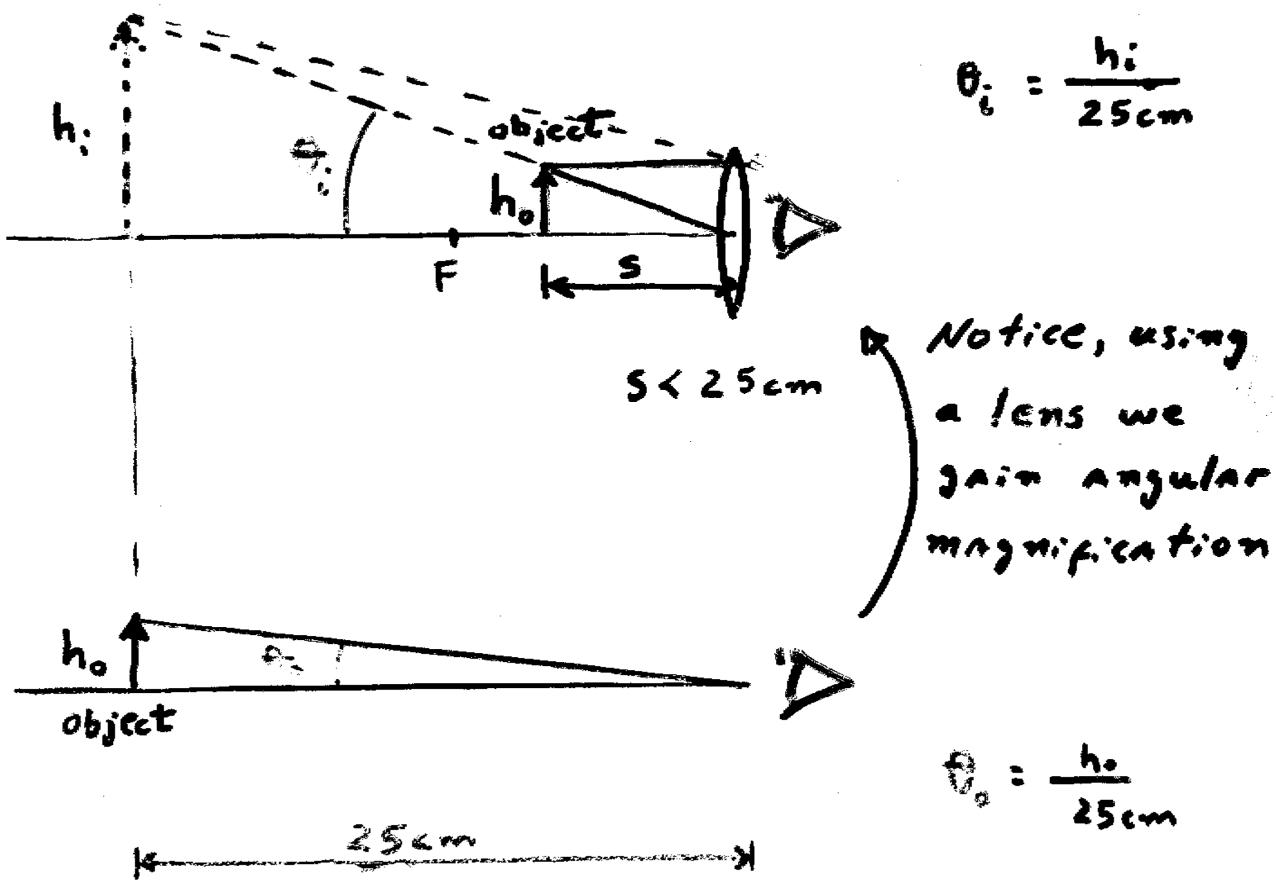


eye fully accommodated

what a lens does is to increase the focal length power (i.e. decreases the overall focal length)



In order to focus on the retina objects that are closer than 25 cm from the eye, we can use a lens*



$$\theta_i = \frac{h_i}{25\text{cm}}$$

$$\theta_o = \frac{h_o}{25\text{cm}}$$

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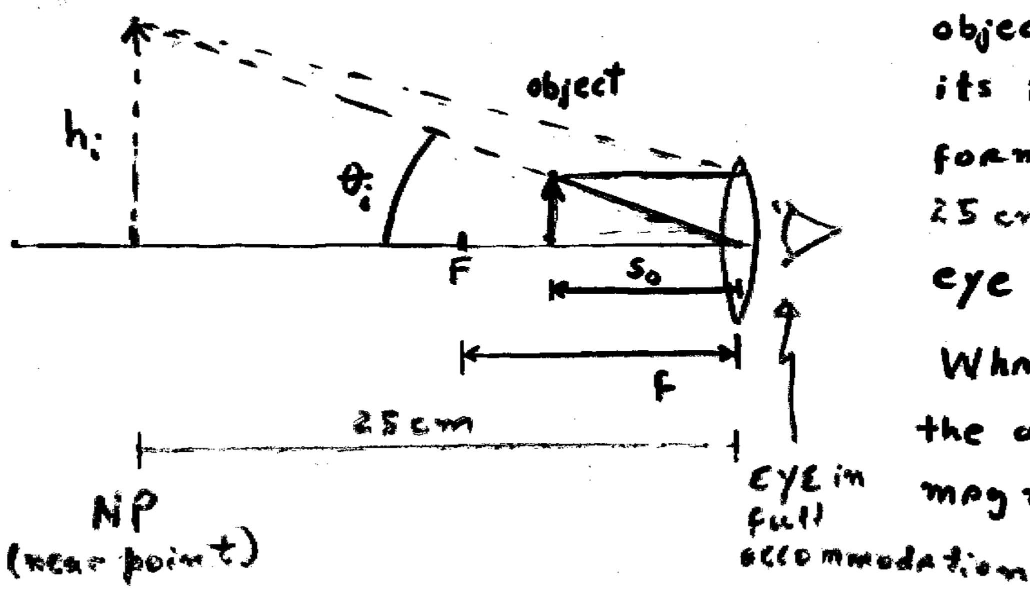
* Didn't we say that a concave lens was needed to correct a myopic eye? Why are we using a convex lens now?

Answer: A myopic eye has, relatively speaking in comparison to a normal eye, too much power. The objective of wearing glasses of negative focal length is to make it function as a normal eye. The objective is not for seeing things close to the eye

Example Given f, where should we place the object such that its image is formed at 25 cm from the eye? What will be the angular magnification?



$$s_i = -25\text{cm} \Rightarrow \frac{1}{s_o} + \frac{1}{-25\text{cm}} = \frac{1}{f}$$

$$\Rightarrow \quad s_o = \frac{f \cdot 25\text{cm}}{f + 25\text{cm}}$$

Angular size $\quad \theta_i = \dfrac{h_i}{25\text{cm}}$

From the figure

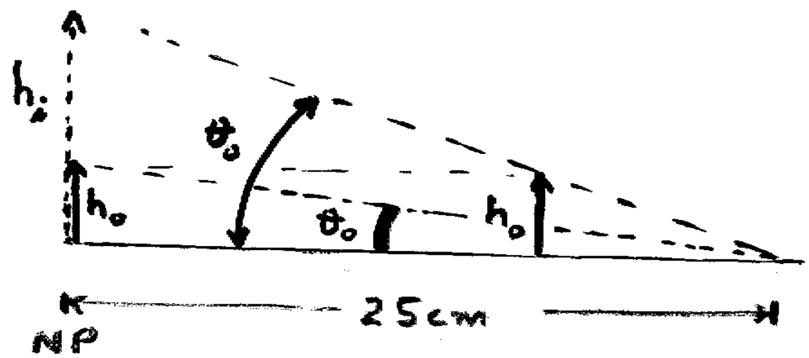
$$\theta_i = \frac{h_i}{25\text{cm}} = \frac{h_o}{s_o} = \frac{f + 25\text{cm}}{f \cdot 25\text{cm}} \cdot h_o$$

Angular magnification

Since $\theta_o = \dfrac{h_o}{25\text{cm}}$, $\quad \dfrac{\theta_i}{\theta_o} = \dfrac{f+25\text{cm}}{f}$

$$\boxed{\frac{\theta_i}{\theta_o} = \frac{25\,cm}{f} + 1}$$

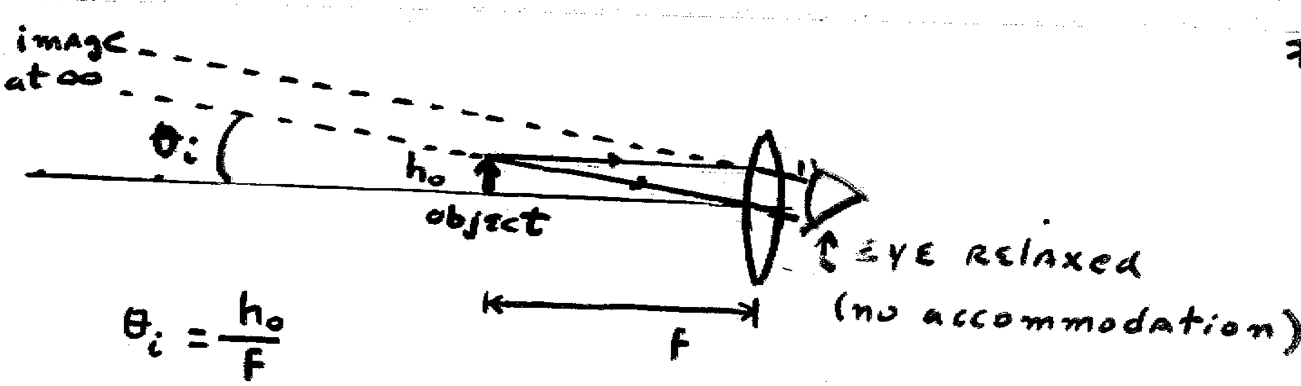
Angular magnification. Image seen at 25 cm from the eye



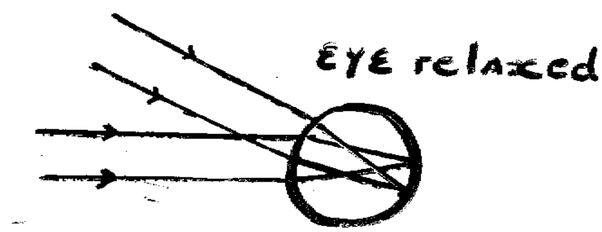
$$\text{Angular Magnification} = \frac{\theta_i}{\theta_o} = \frac{\text{Angle subtended by the IMAGE seen by the EYE}}{\text{Angle subtended by the OBJECT without the lens}}$$

<u>Example</u> Given f, where should we place the object such that its image is formed very far away from the eye (at infinity). What will be the corresponding angular magnification (when compared to the angular size without using the lens)?

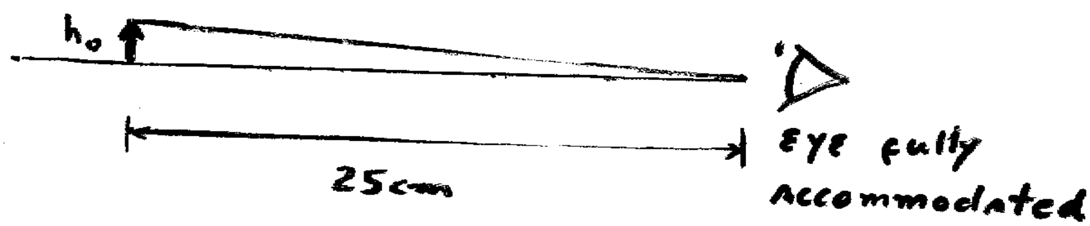
$$\frac{1}{s_o} + \frac{1}{s_i} = \frac{1}{f} \qquad s_i = \infty \Rightarrow s_o = f$$



$$\theta_i = \frac{h_o}{f}$$



compared to



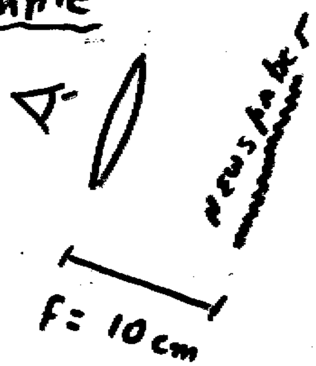
$$\theta_o = \frac{h_o}{25\,\text{cm}}$$

$$\Rightarrow \qquad \frac{\theta_i}{\theta_o} = \frac{h_o/f}{h_o/25\,\text{cm}} \qquad \boxed{\frac{\theta_i}{\theta_o} = \frac{25\,\text{cm}}{f}}$$

Angular magnification
IMAGE seen at ∞

This latest expression is most often used in citing magnifier's magnification.

Example



Dioptric power

$$\mathcal{D} = \frac{1}{f} = \frac{1}{0.1\,m} = 10\,D$$

Angular magnification

$$M_A = \frac{25\,cm}{f} = 2.5$$

which is denoted as

2.5X

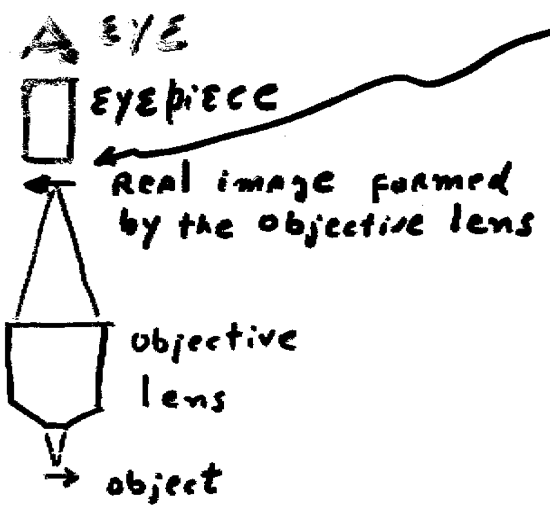
Letters from the newspaper will look 2.5 times larger on the retina when compared to viewing the newspaper at 25cm from the eye without the lens.

Single-lens magnifiers are limited by aberrations to roughly 2X or 3X

Few more complicated magnifiers are designed to operate to 10X or 20X

EYEPIECES OR OCULARS (5.7.4)

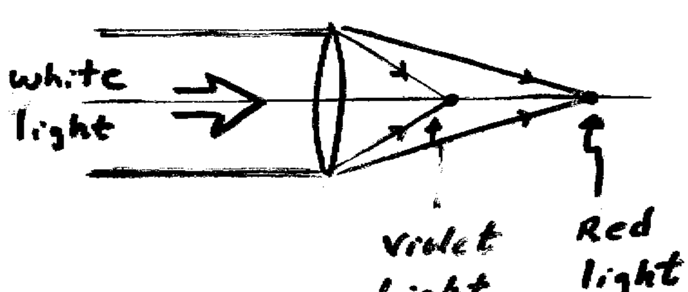
- Fundamentally they are magnifiers
- The are used to aid the EYE in viewing images formed by prior components of an optical system.



This real image serves as the object that is viewed by the eyepiece, whose angular magnification contributes to the overall magnification of the instrument

- To provide high quality images the eyepiece is corrected for aberrations and, in particular, to reduce chromatic aberrations.

WHAT is CHROMATIC ABERRATION?



light of different color focuses at different positions

longitudinal chromatic aberration

Chromatic aberration results from the fact that

↳ The focal length of a lens depends on the index of refraction

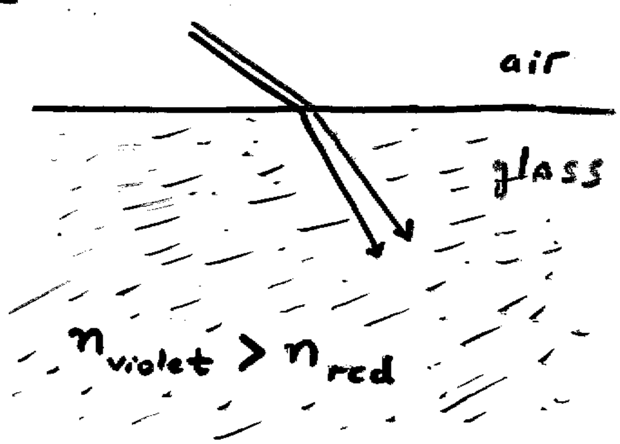
$$\frac{1}{f} = \frac{n_{lens} - n_{air}}{n_{air}} \cdot \left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

and

↳ the indices of refraction depend on the wavelength of the light

$$n = n(\lambda)$$

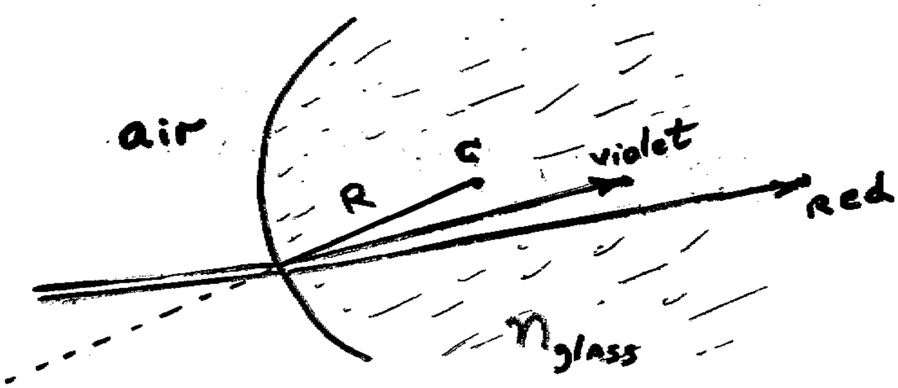
That is, different colors refract at different angles



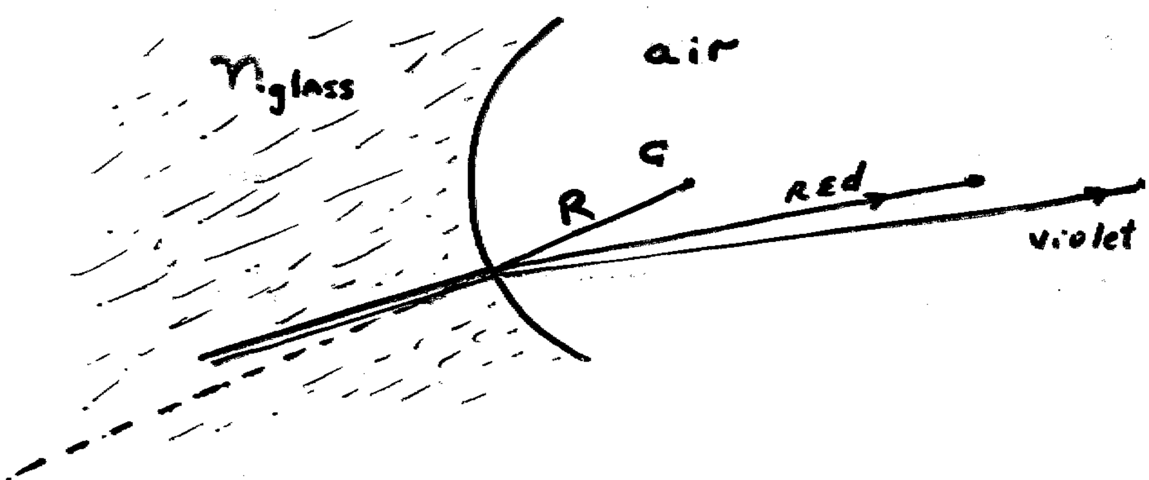
thus

focal length $f = f(\lambda)$

Realizing that



a convex spherical surface makes the violet beam to bend more, and



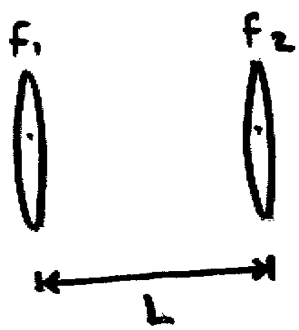
a concave spherical surface make the violet beam to deviate more,

chromatic aberration is eliminated by making use of multiple refracting elements of opposite power

Common solution: the achromatic doublet,

consisting of a convex and concave lens, of different glasses, cemented together

Another solution to minimize chromatic aberration



is to use two sepaante lenses of the <u>same glass</u>

Since the effective power of the combination, $\frac{1}{f_{eff}}$, will depend on both the index of refraction n and the distance separation L, it is possible that a proper choice of L can make $\frac{d}{dn}\left(\frac{1}{f_{eff}}\right) = 0$

Let's see.

$$\frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{L}{f_1 f_2}$$

(we'll justify later on the use of this formula)

$$\frac{1}{f_1} = (n-1)\left(\frac{1}{R_{11}} - \frac{1}{R_{21}}\right) = (n-1)K_1$$

n: index of refraction

$$\frac{1}{f_2} = (n-1)\left(\frac{1}{R_{12}} - \frac{1}{R_{22}}\right) = (n-1)K_2$$

$\Rightarrow$

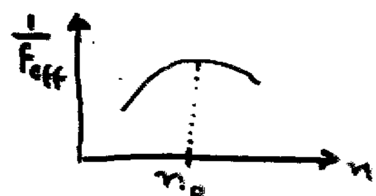
$$\frac{1}{f_{eff}} = (n-1)K_1 + (n-1)K_2 - (n-1)^2 L K_1 K_2$$

$$\frac{d}{dn}\left(\frac{1}{f_{eff}}\right) = K_1 + K_2 - 2(n-1)L K_1 K_2$$

For a given color (a given λ_o) we will have a corresponding index of refraction $n(\lambda_o) = n_o$

For such a value n_o we'll choose an L that will make the expression above equal to zero

$$K_1 + K_2 - 2(n_o - 1)L K_1 K_2 = 0$$



this will ensure that when we use a slightly different wavelength $\lambda \neq \lambda_o$, the value of $\frac{1}{f_{eff}}$ will not change but to the order of $(\Delta n)^2$

thus, the value for L that we need to minimize chromatic aberration is given by

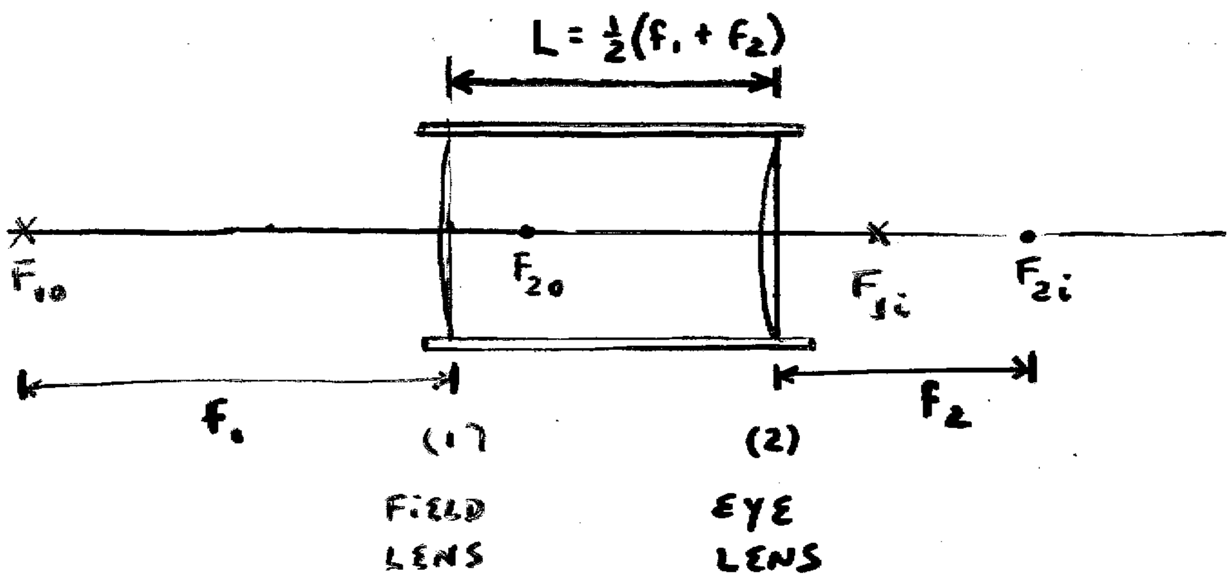
$$L = \frac{1}{2(n_0-1)}\left(\frac{1}{K_2}+\frac{1}{K_1}\right) = \frac{1}{2}\left(f_2+f_1\right)$$

$n_0 = n(\lambda_0)$

λ_0 = wavelength around which we want to minimize chromatic aberration

The Huygens eyepiece

It incorporates the design feature required by the expression above



× focal points of FL

• focal points of EL

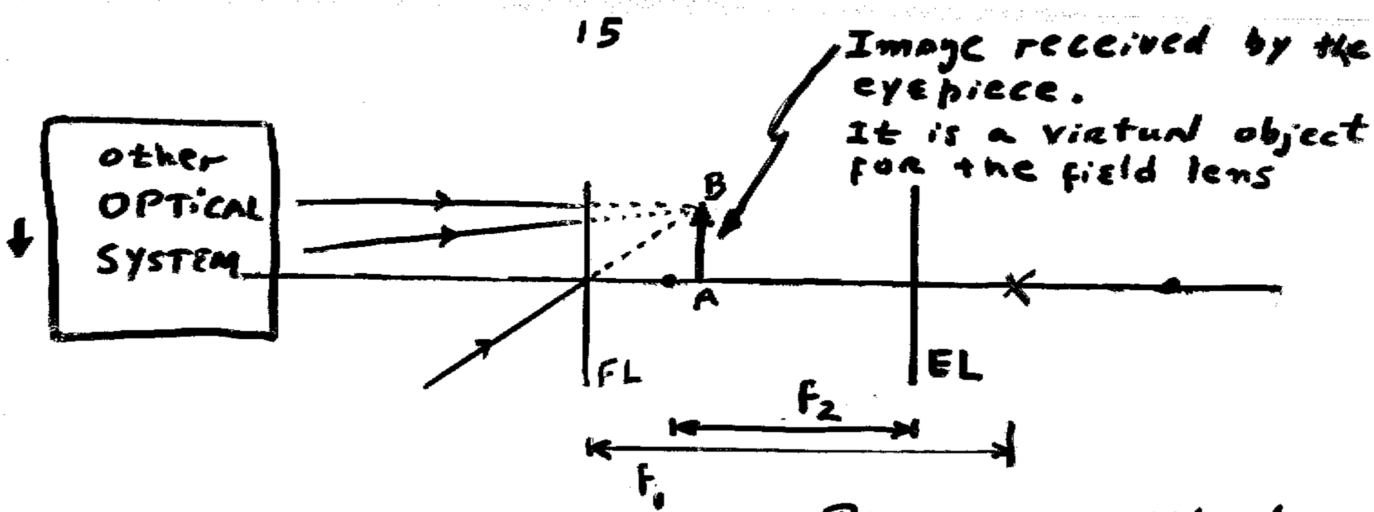
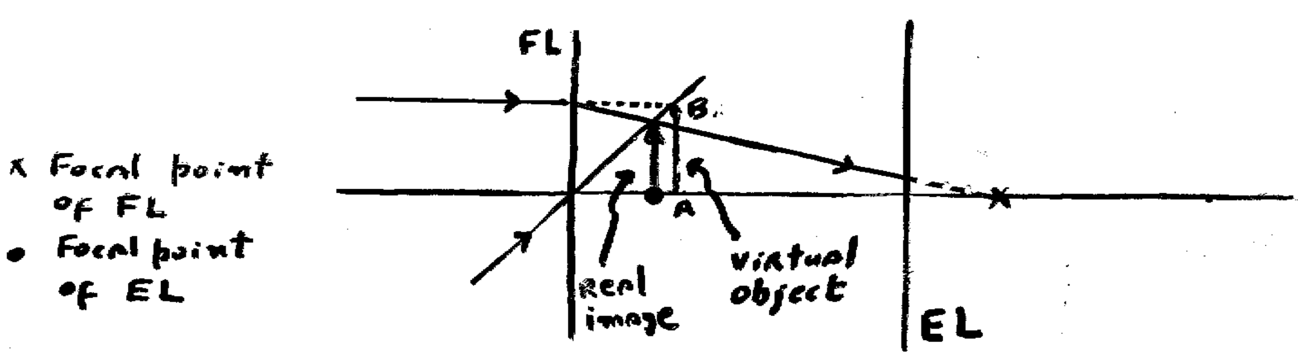
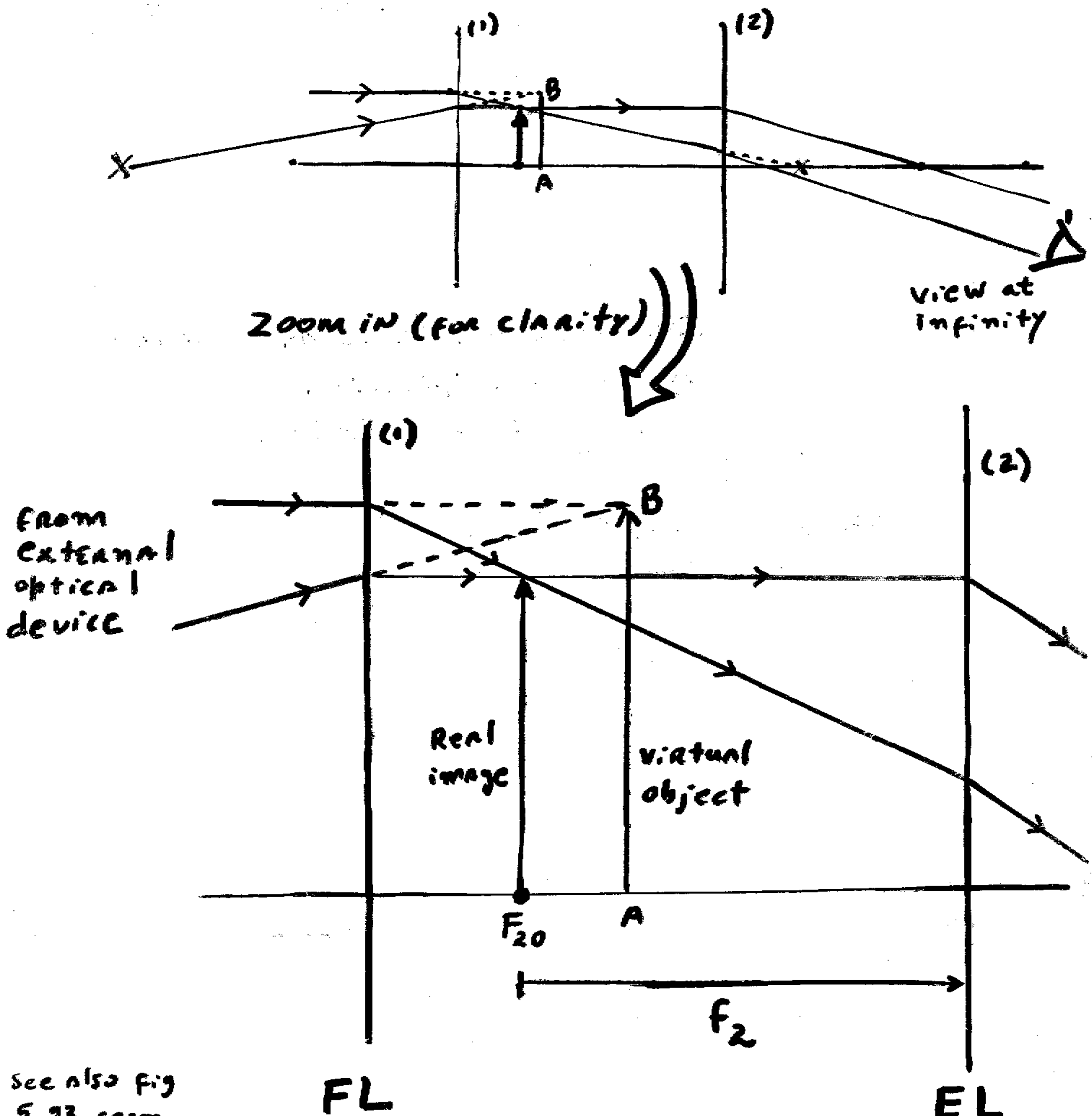


Diagram correspond to $f_1 \approx 1.7\ f_2$

The FL will form a real image out of the virtual object AB. In that process, the eyepiece is positioned relative to the "other optical system" in such a way that the image of AB ends up located at the focal point of the EL



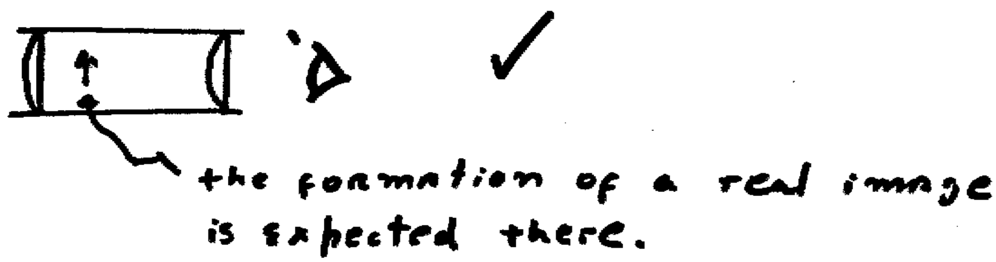


See also fig 5.93 from your textbook (page 212)

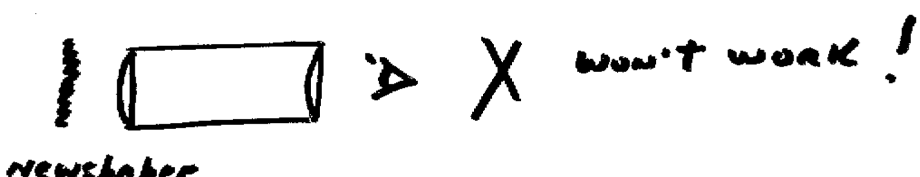
Huygens eyepiece

- The Huygens eyepice can not work as a simple magnifier

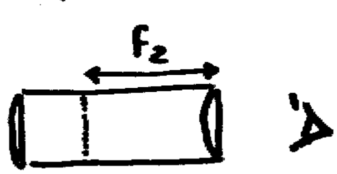
The reason is that a real image is expected inside the eyepiecE,
or, equivalently,
because of the <u>internal</u> position of the EL's focal point (F_{20} in the previous graph) where a real image should be located



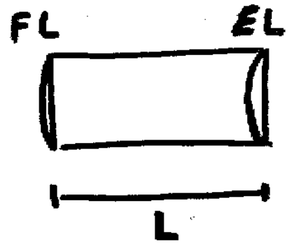
So



- For quantitative measurements:
If a reticle with scale is used, it will have to be placed internally, at the EL's focal point



THE RAMSDEN EYEPIECE



Let's make $f_1 = f_2$ and call it f

Also,

Let's keep $L = \frac{1}{2}(f_1 + f_2) = f$

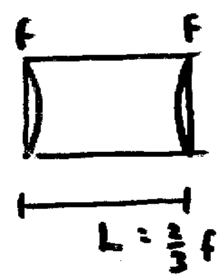
Notice, then, that if a real object falls at the position of the FL, the EL will form an image of that object at infinity, thus serving the purpose as an eyepiece

A disadvantage of this arrangement is that the surface of the FL is also in focus, including dust. An alternative is to reduce the separation distance a little bit (although this will compromise chromatic aberration)

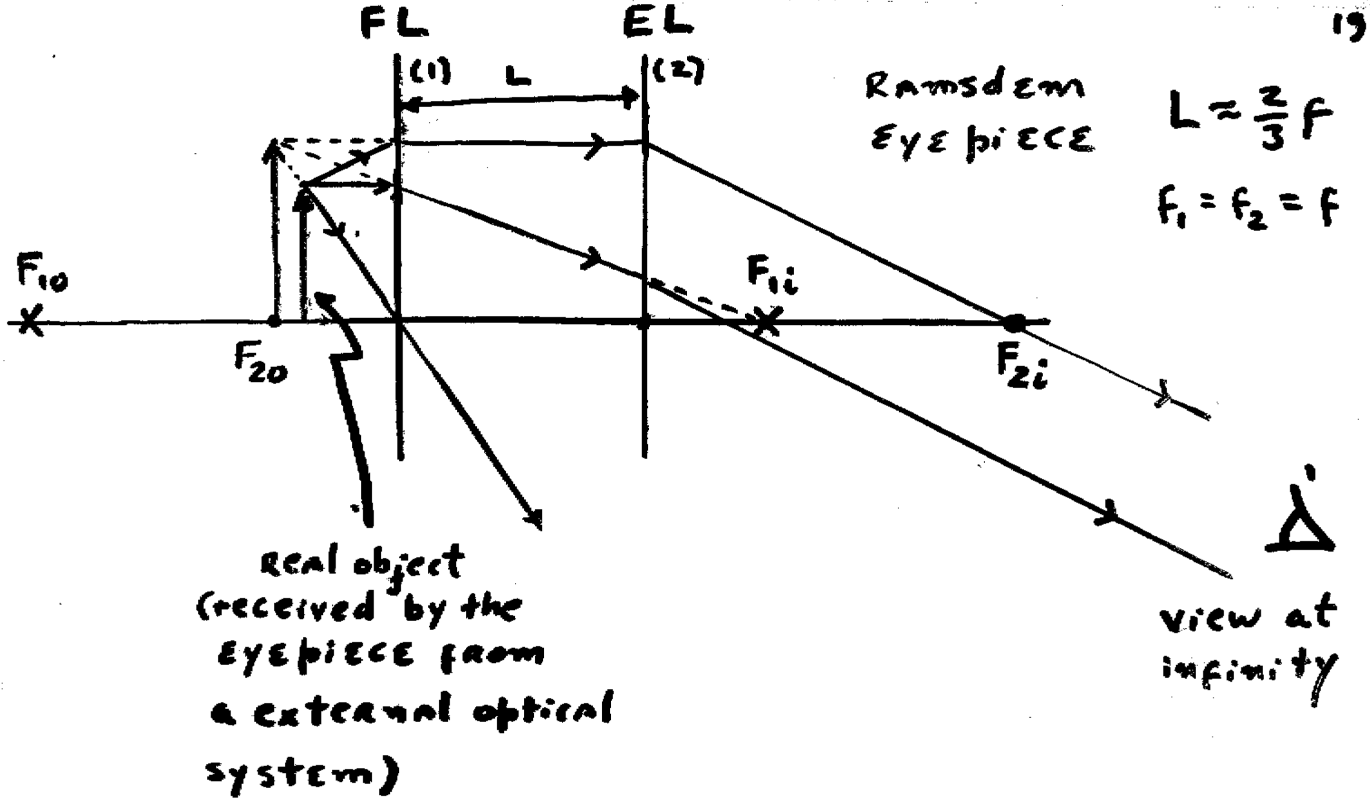
Ramsden eyepiece

$$f_1 = f_2 \equiv f$$

$$L \approx \frac{f_1 + f_2}{3} = \frac{2}{3} f$$



Notice, all the focal points will be located externally

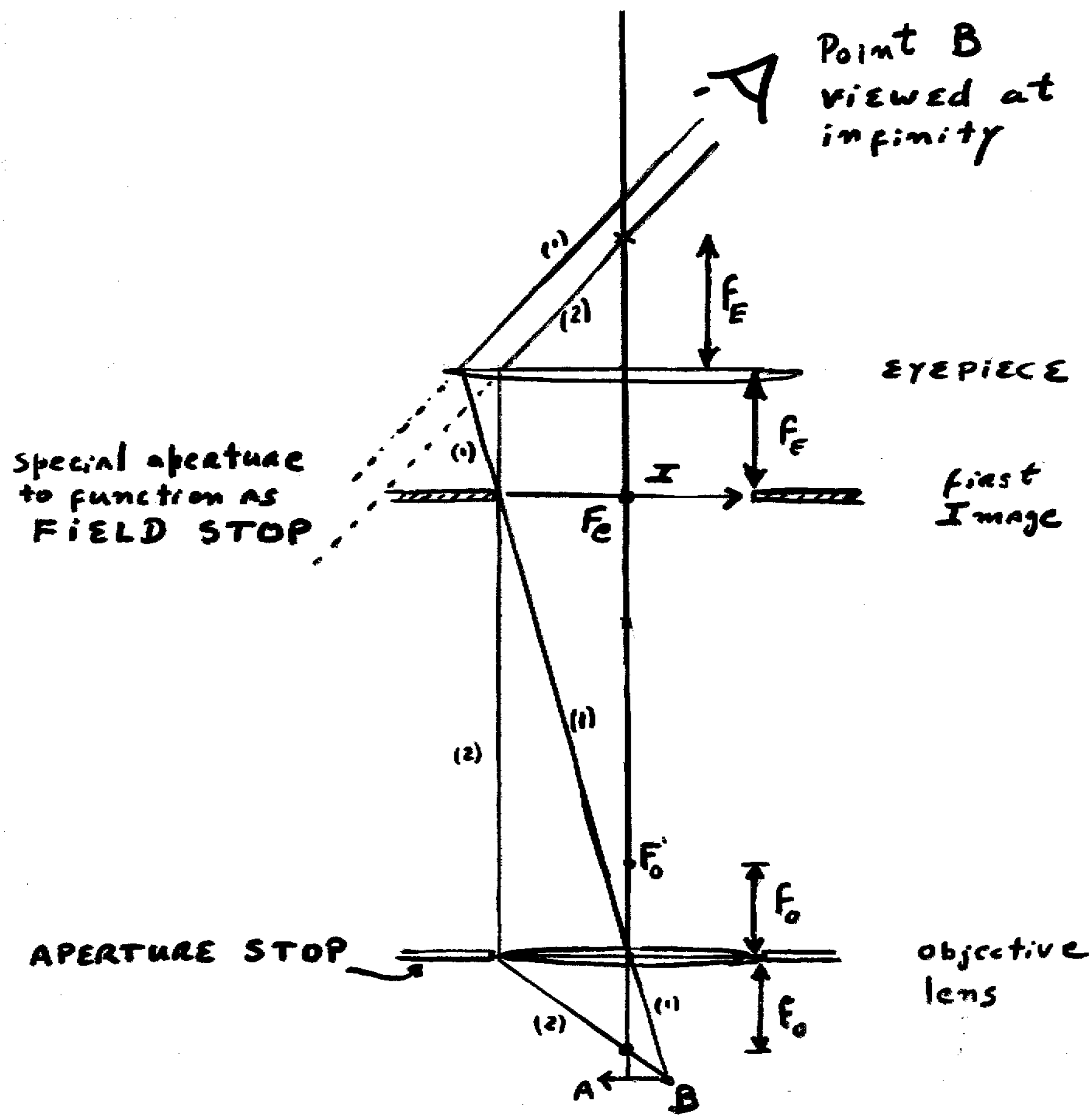


KELLNER EYEPIECE

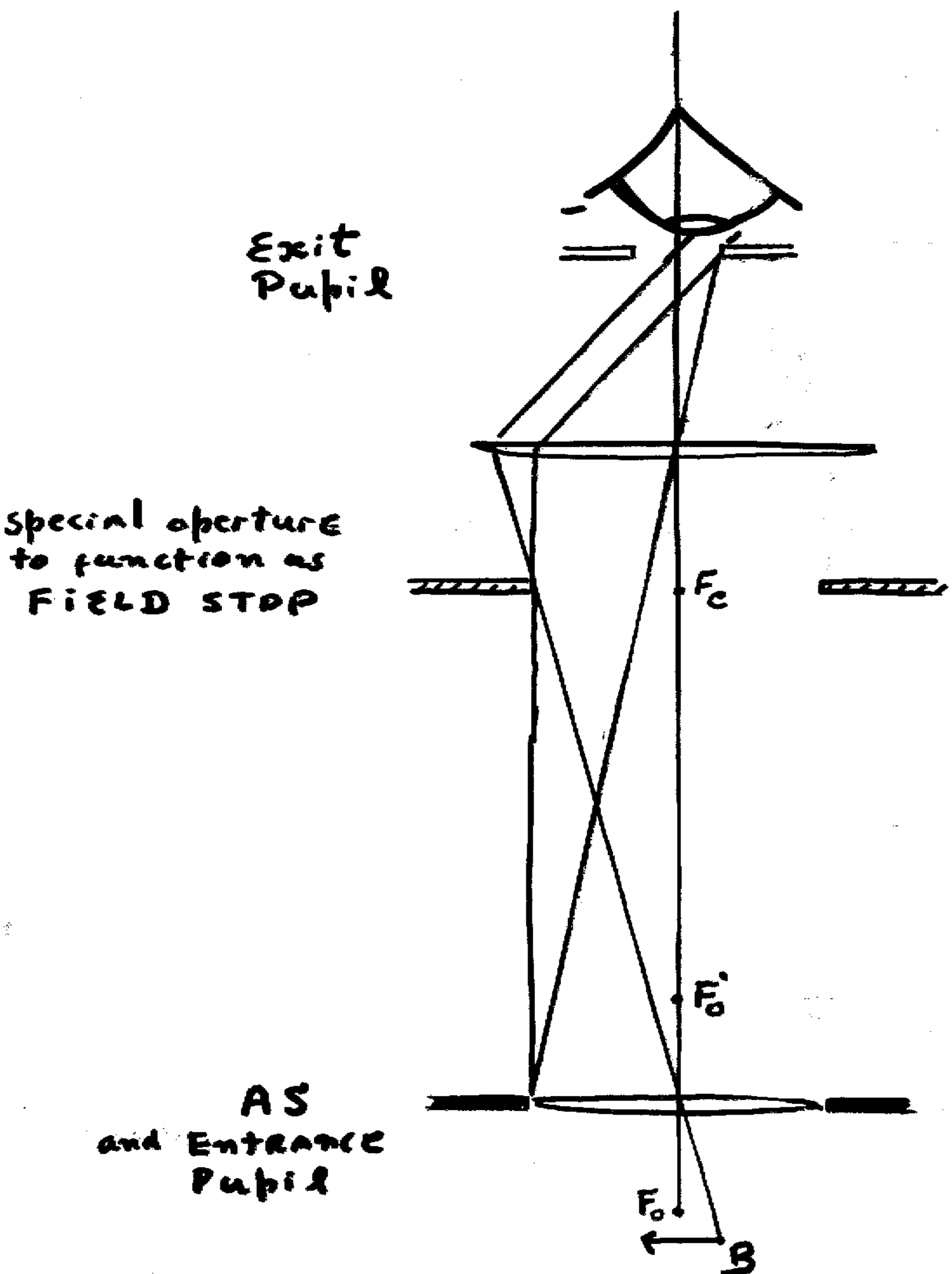
In the Ramsdem eyepiece, the condition for minimizing chromatic aberration $L = \frac{1}{2}(f_1 + f_2)$ has been violated.

To restore correction for chromatic aberration, the eye lens is replaced with a achromatic doublet

MICROSCOPES



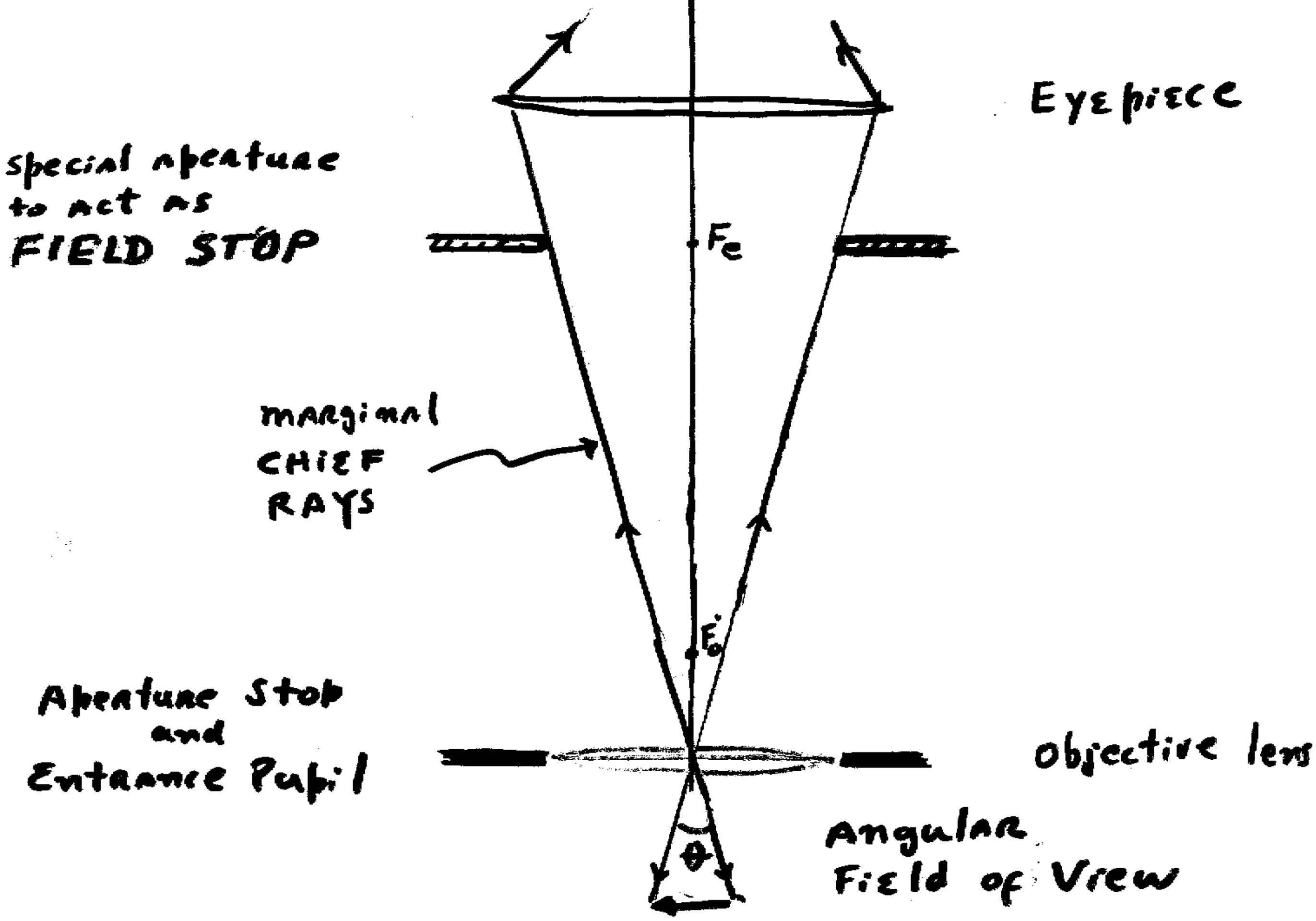
Identifying the EXIT PUPIL



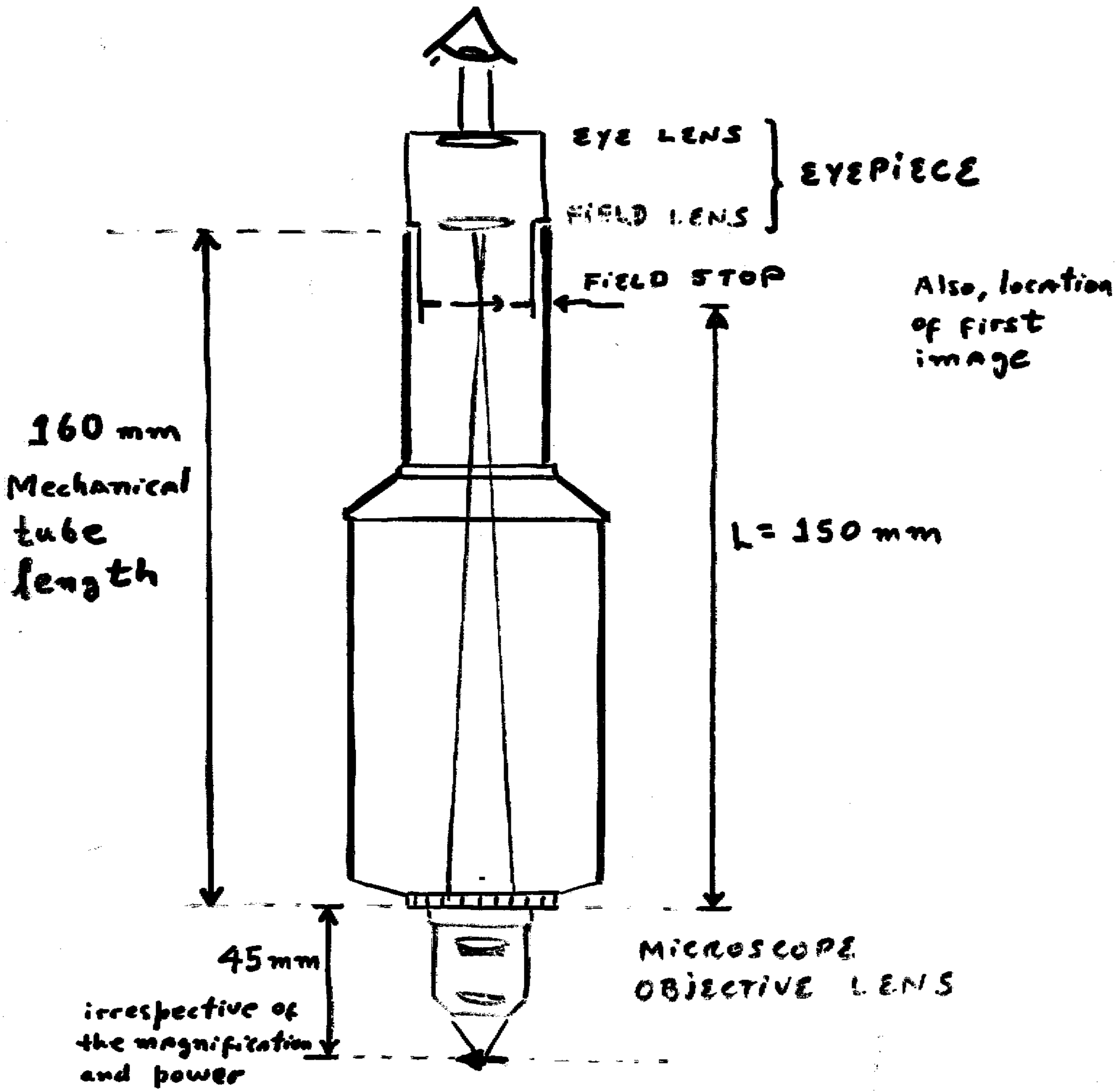
Identifying the Field Stop and the Angular Field of View

The FIELD STOP limits the extent of the largest object that can be viewed

Angular Field of View is the angle subtended by the marginal chief rays at the Entrance Pupil



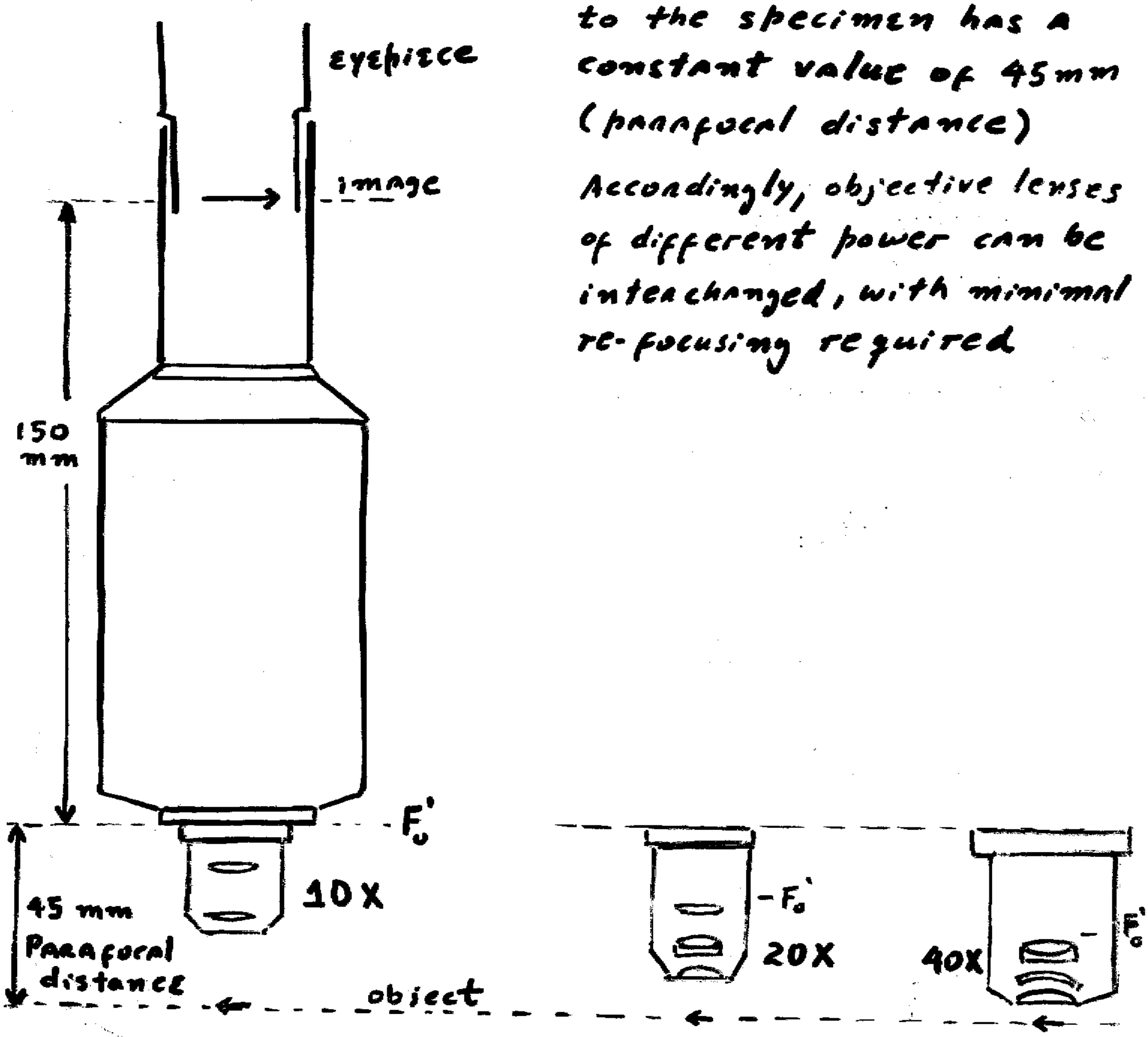
Standard dimensions in commercial microscopes



Parfocality design:

- Image to object distance is always 195 mm irrespective of the objective lens power
- Distance from the objective lens shoulder to the specimen has a constant value of 45 mm (parafocal distance)

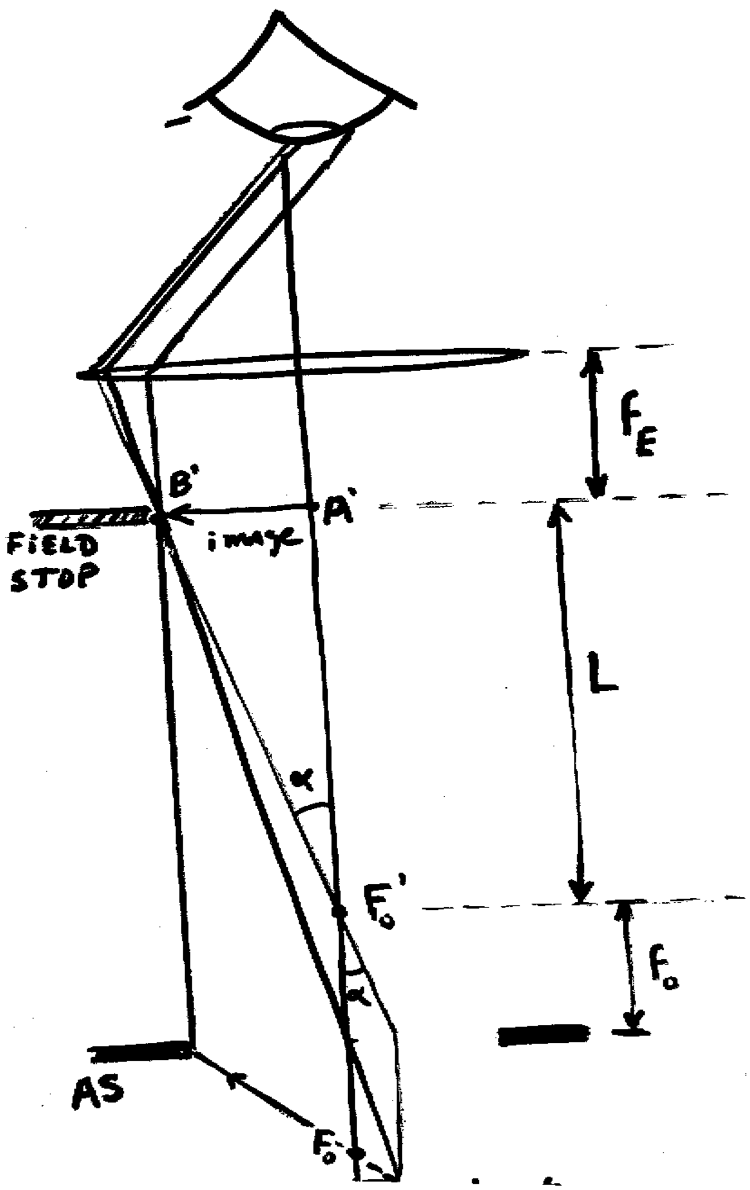
Accordingly, objective lenses of different power can be interchanged, with minimal re-focusing required



MAGNIFICATION

$$M = \text{angular magnification} \times \text{transverse magnification}$$

$$= \frac{250\,mm}{f_E} \times \frac{L}{f_o}$$



Angular Magnification by the Eyepiece

$$\boxed{M_{A,E} = \frac{250\,mm}{f_E}}$$

Image seen at ∞

$$\tan\alpha = \frac{A'B'}{L}$$

$$\tan\alpha = \frac{AB}{f_o}$$

$$\Rightarrow \frac{A'B'}{AB} = \frac{L}{f_o}$$

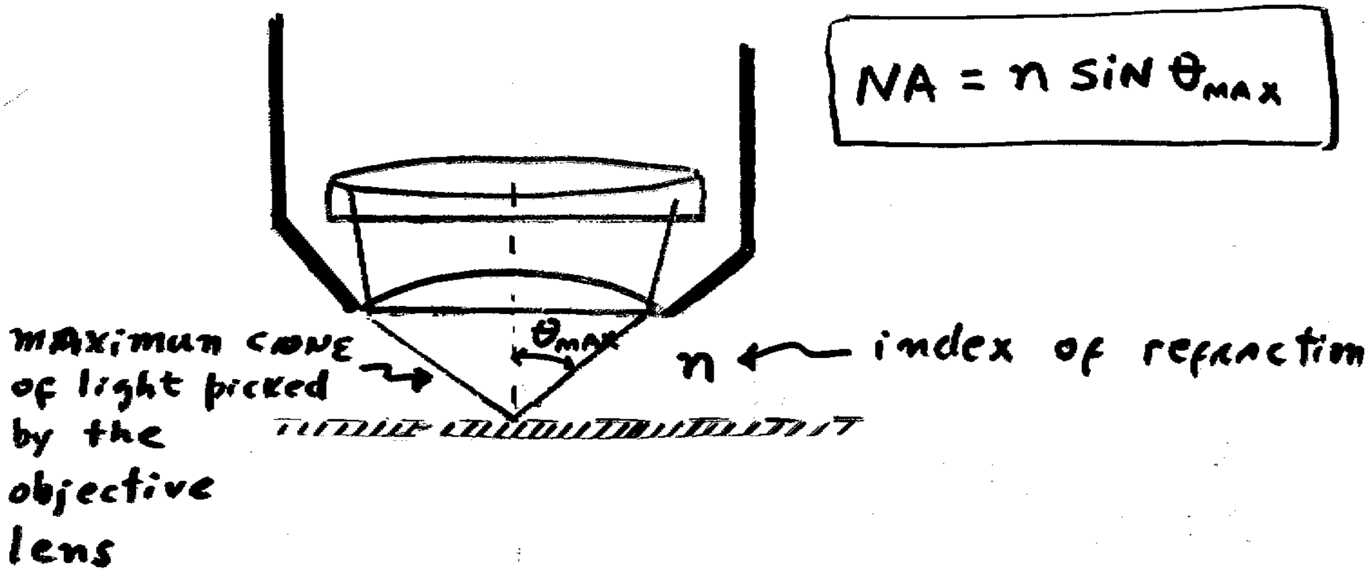
thus

Transverse Magnification by the Objective Lens

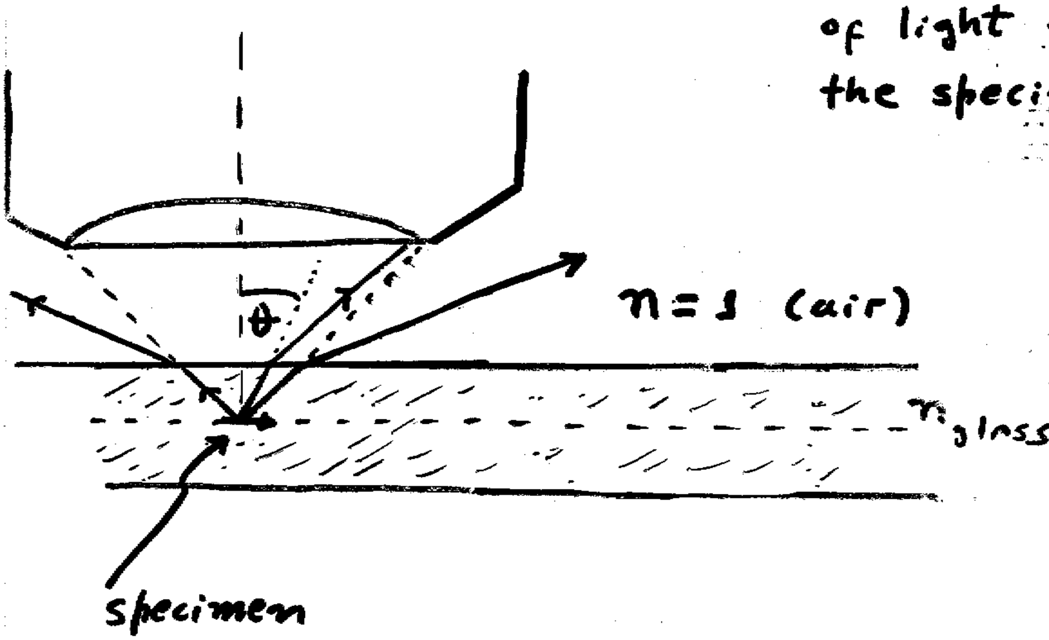
$$\boxed{M_{T,O} = \frac{L}{f_o}}$$

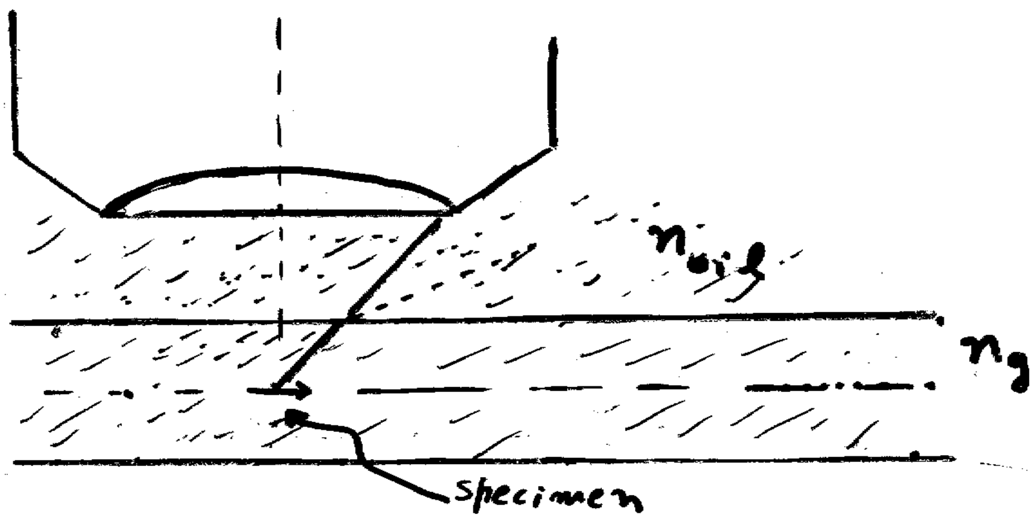
Numerical Aperture NA

$$\boxed{NA = n \sin \theta_{MAX}}$$



Looking through a cover slide decreases the cone of light gathered from the specimen





$n_{oil} = n_{glass}$

Filling the space between the Objective Lens and the cover glass with a substance (i.e. oil) that matches the index of refraction of the glass helps to increase the cone of light gathered by the objective lens from the specimen

RESOLVING POWER

$$R \sim \frac{\lambda}{2 \times NA}$$

maximum possible resolution

The smaller R the better resolution

For green light $\lambda = 500\,nm$

R is of the order of 250 nm

$\Longleftrightarrow$ 4000 $\frac{lines}{mm}$ at best

This would be the best we could do with the most expensive conventional optical microscope available on the world.

Looking at thing smaller than 100 nm are out of reach for a $\lambda = 500\,nm$ microscope.

In the Geometrical Optics approximation (i.e. $\lambda = 0$) ray tracing would have given $R = 0$ (an extraordinary resolution!) Unfortunately $R = 0$ with $\lambda \neq 0$ does not happen.

The limited resolution can be explained in WAVE OPTICS