Superlenses

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Abstract

When we study an emitting source, depending on the distance from the source we may talk of near and far field. The near field contains the finest details of an object. Nowadays, our imaging systems reproduce only the far field, which is enough for imaging macroscopic objects, but as sizes become really small, the resolution is limited and it becomes critical to manipulate it. In this paper I will give an overview of the state of our knowledge.

Contents

1 Introduction

Human beings love pictures. Pictures are often the quickest way to describe a situation or to understand it. As scientists, our purpose is to try to understand and predict how nature behaves, no wonder why we developed instruments like telescopes, microscopes... The subject of superlenses is especially interesting to me, because it could eventually allow us to increase the resolution of our imaging systems and thus would help us to understand better the behavior of matter at nanoscale.
In this paper I will deal the maximum resolution of an image, then I will introduce the concept of perfect lens using metamaterials. To conclude I will deal with a sub-wavelength imaging experience.

2 Maximum resolution of an image

When we want to observe really small objects we face diffraction [7]. That is why, the maximal resolution of an image is limited. Let's consider for instance a plan-wave traveling in the vacuum and meeting an interface with another material, as represented in figure 1.

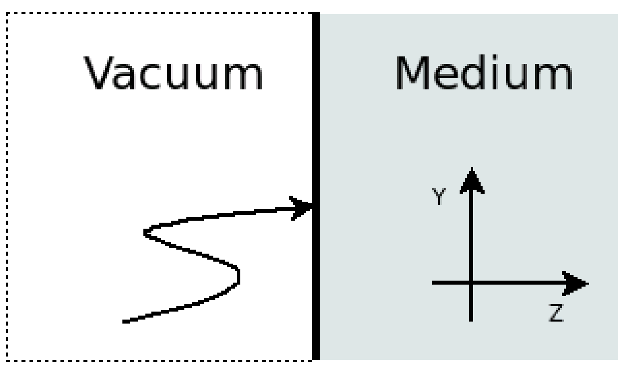


Figure 1: Scheme of an vacuum-material interface

If we assume the electric field to be:

$$\overrightarrow{E}(x,y,t) = \overrightarrow{E_0}\exp(jk_x x + jk_y y + jk_z z - jwt)$$

Using the wave equation:

$$-w^2\overrightarrow{E} = v^2(-{k_x}^2\overrightarrow{E} - {k_y}^2\overrightarrow{E} - {k_z}^2\overrightarrow{E})$$

Rearranging terms we obtain the main equation:

$$\boxed{k_z = \pm\sqrt{\frac{\mu\varepsilon w^2}{c^2} - k_x^2 - k_y^2}} \tag{1}$$

where k_i is the wave-vector in the i-direction, μ is the relative magnetic permeability, ε is the relative permittivity, ω the angular frequency of the wave and c the speed of light. If the geometry of the interface imposes large values of k_x, for a slit smaller than the wavelength for example $k_x^2 + k_y^2 > \frac{\mu\varepsilon\omega^2}{c^2}$ and we need to use the complex square-root so for small wave-lengths:

$$k_z = j\sqrt{k_x^2 + k_y^2 - \frac{\mu\varepsilon w^2}{c^2}} \tag{2}$$

and the electric field becomes

$$\overrightarrow{E}(x, y, t) = \overrightarrow{E_0}\exp\left(-\sqrt{k_x^2 + k_y^2 - \frac{\mu\varepsilon w^2}{c^2}}z\right)\exp(jk_x x + jk_y y - jwt)$$

In figure 2 below, are modeled two functions representing a wave traveling along the z axis. It gives a qualitative view of the phenomena and has no quantitative value.

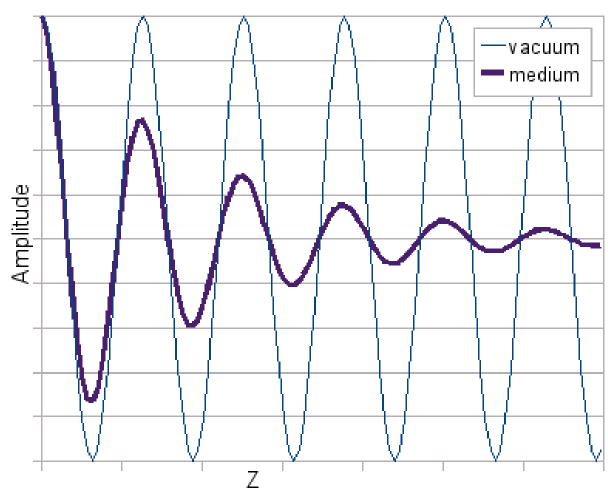


Figure 2: Wave traveling in vacuum or in a material with a complex k_z.

As we see, the amplitude of the electric field decays exponentially. If we consider only a two dimensional case ($k_y = 0$), we can say that it happens for k_z real, that is to say:

$$\frac{\mu\varepsilon w^2}{c^2} < k_x^2 \Leftrightarrow \frac{4\pi^2}{\lambda^2} < \left(\frac{2\pi}{d}\right)^2$$

Which is the case roughly for $\lambda > d$ where d could be the size of a slit or a detail to be resolved. All the waves whose wavelength are greater than d, will see their amplitude decay, removing them from the image, thus we loose resolution.[3]

3 Negative refraction index

3.1 Problem settings

Let's imagine for a moment that we can use a material with strange characteristics such as a negative permittivity and a negative magnetic permeability. This could be a rectangular material with $\epsilon_r = -1$, $\mu_r = -1$ and thickness d. Let's consider now a S-polarized incident electric field meeting an interface. It means that the electric is perpendicular to the plane of propagation. Figure 3 shows the geometry of the problem.

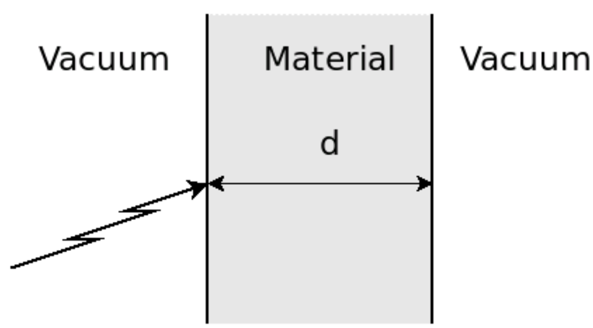


Figure 3: vacuum-material and then a material-vacuum interface.

The expression of the incident electric field is:

$$E_i(x, y, t) = \overrightarrow{E_0} \exp(jk_x x + jk_z z - jwt)$$

We're interested by the case where $k_z = j\sqrt{k_x^2 - k_y^2 - \frac{\omega^2}{c^2}}$, that is the geometry of the interface imposes k_z to be a complex number, as we've seen in the previous section.

3.2 Amplitude reflexion and transmission at each interface

At the interface, the incident electric field is partially reflected, partially transmitted. The reflected electric field is:

$$E_r(x, y, t) = r\overrightarrow{E_0}\exp(jk_x x - jk_z z - jwt)$$

where r is the amplitude reflection coefficient.
The transmitted electric field is:

$$E_t(x, y, t) = t\overrightarrow{E_0}\exp(jk_x x + jk_z^{'} z - jwt)$$

with t, the amplitude transmission coefficient and $k_z^{'} = j\sqrt{k_x^2 - k_y^2 - \frac{\mu\epsilon\omega^2}{c^2}}$
One can show with the Maxwell equations at boundary conditions[6] that at the first interface we have $1 + r = t$ and $\frac{1}{\mu}k_z' = k_z + rk_z$, thus it is possible to derive

$$t = \frac{2\mu k_z}{\mu k_z + k_z'} \text{ and } r = \frac{\mu k_z - k_z'}{\mu k_z + k_z'} \tag{3}$$

Similarly, at the second interface, we have:

$$t' = \frac{2k_z\prime}{\mu k_z + k_z'} \text{ and } r' = \frac{k_z' - \mu k_z}{\mu k_z + k_z'} \tag{4}$$

Thus, from these coefficients, we should be able to calculate an overall coefficient.

3.3 Overall transmission

Then, we need to consider that light may either be directly transmitted from the left side to right side, or be reflected several times in the slab, as shown in figure 4.

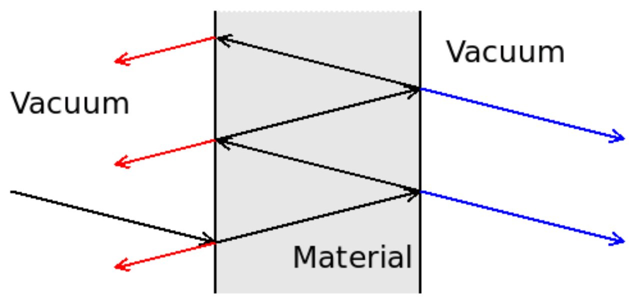


Figure 4: Transmission and reflexion inside the slab.

So the total amplitude of the electric field is:

$$T_s = tt' \exp(jk'_z d) + tt'r'^2 \exp(3jk'_z d) + tt'r'^4 \exp(5jk'_z d) + ...$$

$$T_s = tt' \exp(jk'_z d) \sum_{n=0}^{\infty} (r'^2 \exp(2jk'_z d))^n$$

$$T_s = \boxed{\frac{tt' \exp(jk'_z)}{1 - r'^2 \exp(2jk'_z d)}} \tag{5}$$

Substituting in (5) with (3) and (4) and taking the limit of T_s when $\mu \to -1$ and $\epsilon \to -1$

$$T_s = \lim_{\substack{\epsilon \to -1 \\ \mu \to -1}} \frac{2\mu k_z}{\mu k_z + k'_z} \frac{2k_z\prime}{\mu k_z + k'_z} \frac{\exp(jk'_z)}{1 - \frac{k'_z - \mu k_z}{\mu k_z + k'_z}^2 \exp(2jk'_z d)}$$

$$T_s = \boxed{\exp(-jk_z d)} \tag{6}$$

One can show that the same results apply for an electric field oscillating in the plane of propagation, also called a P-polarized electric field[3].

3.4 Overall reflexion

Here again, we need to sum the multiple scattering events:

$$
\begin{aligned}
R_s &= r + tt'r'^3 \exp(4jk'_z d) + tt'r'^5 \exp(6jk'_z d) + ... \\
R_s &= r + tt' \exp(2jk'_z d) \sum_{n=0}^{\infty} (r'^2 \exp(2jk'_z d))^n \\
R_s &= \boxed{r + \frac{tt'r' \exp(2jk'_z d)}{1 - r'^2 \exp(2jk'^2_z d)}}
\end{aligned} \tag{7}
$$

Substituting in (7) with (3) and (4)

$$
\begin{aligned}
R_s &= \lim_{\substack{\epsilon \to -1 \\ \mu \to -1}} r + \frac{2\mu k_z}{\mu k_z + k'_z} \frac{2k_z\prime}{\mu k_z + k'_z} \frac{k'_z - \mu k_z}{\mu k_z + k'_z} \frac{\exp(2jk'_z d)}{1 - \frac{k'_z - \mu k_z}{\mu k_z + k'_z}^2 \exp(2jk'_z d)} \\
R_s &= \boxed{0}
\end{aligned} \tag{8}
$$

3.5 Results

We've found, that the overall transmission coefficient of this slab is $\exp(-jk_z d)$, this means that the transmitted electric field is:

$$
\begin{aligned}
\overrightarrow{E_t}(x, y, t) &= T_s \overrightarrow{E_0} \exp(jk_x x + jk_y y + jk_z z - jwt) \\
\overrightarrow{E_t}(x, y, z) &= \overrightarrow{E_0} \exp(jk_x x + jk_y y - jwt)
\end{aligned}
$$

Since the amplitude of the electric field is no longer depending on k_z, the exponential decay is compensated by our material. Another noteworthy property is that the reflexion coefficient equals 0, this means that there is no loss at the interface.
So there is no reflexion, all the waves are transmitted and the exponential decays is compensated, thus there is no theoretical limit to the maximum resolution. Such properties are the ones we would like to achieve when imaging an object. This material could make a perfect lens[3].

4 From theory to experiment

4.1 Metamaterials

In the previous section, I introduced a new kind of materials with properties not found in nature such as a negative permittivity and a negative permeability. It turns out, that thanks to engineering progresses, we are now able to manufacture this kind of material for a given frequency. These materials are designed and man-made, we call them metamaterials.

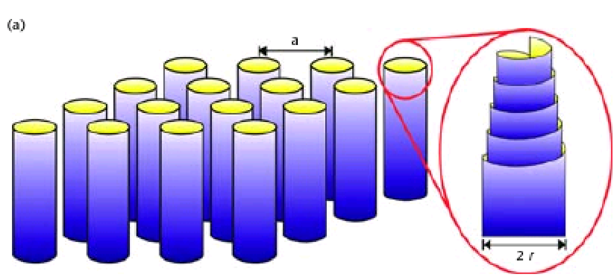
There are a lot of types of metamaterials, but most of them use surface plasmons. Some of them, mostly metals, behave like plasmas. Specialists think it is due to the fact that in metals, there are a lot of free electrons like in plasmas there are a lot of free particles. It turns out that in such metamaterials, we have the following relationship:

$$\epsilon = 1 - \frac{\omega_{ep}^2}{\omega^2} \text{ and } \mu = 1 - \frac{\omega_{mp}^2}{\omega^2}$$

So by tuning these parameters, it is possible to create a material with the desired values.

4.2 Swiss Rolls

The Swiss roll is a metamaterial which consists of a sheet of conductor coated with an insulator around a cylinder. Since there is an inductance in a coil and a capacitance between the inner turn and the outer turn, it behaves like a resonant system for a given frequency in the GHz range. A set of these Swiss Rolls behaves like a metamaterial.



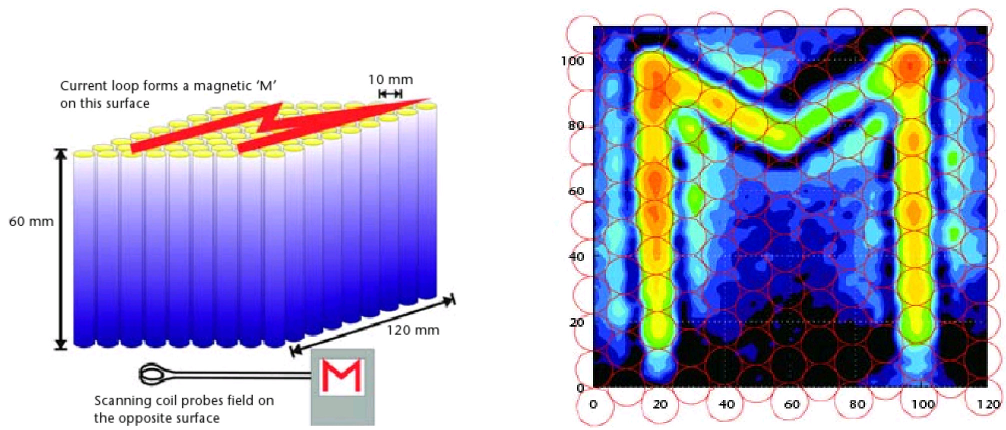
(a) scheme

(b) picture

Figure 5: Swiss roll[4]

4.3 Experiment of Mike Wiltshire

Mike Wiltshire realized a sub-wavelength imaging experiment in 2003. He designed a set of coils formed into the shape of a “M” emitting a magnetic wave of wavelength $\lambda = 15$ m. He put it on a stack of Swiss rolls of diameter $D = 10mm$ and length $L = 60mm$ and scanned the opposite surface with a probe coil[4]. Since $D << \lambda$, we would expect the magnitude of the magnetic wave to decay in the material. Figure 6 show the results.



(a) scheme experiment (b) results

Figure 6: Wiltshire Experiment[4]

The results are really astonishing. We can clearly see the M-shape. The Swiss rolls do transmit the wave, and there’s no decay. This experiment was an important step, because the negative refraction and sub-wavelength imaging theories were controversial.

4.4 Limits of metamaterials

Developing metamaterials is a really new field of research. Specialists are facing two main problems that prevent us from manufacturing superlenses today.

The first one is that dispersion, the refraction index, permittivity and permeability depend on the frequency used. And yet in visible light there are a lot of frequencies not just one. And for each of them, a different response of the metamaterial.

The second one, is engineering limits, we cannot manufacture nowadays metamaterials working in the visible light range even though the theory is already developed. Some experiments have been done in the THz range. The fast development of these materials allow us to think that the infrared is not far behind[4].

5 Conclusion

To conclude, I would like to say that the interest for superlenses is very new. If the theory seems to be ready, we still face difficulties to manufacture the materials we need. But this field has been progressing really fast for the last ten years allowing us to think that super lenses will soon be a reality. This could have a huge impact, especially in microscopy, and could help us to keep on studying matter deeper and deeper.

References

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