

Single Photon Transistor

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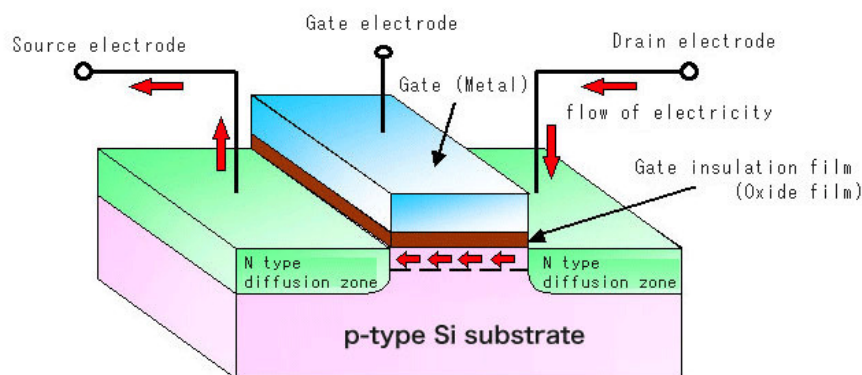
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Abstract

The concept of an optical transistor is not a new one. The difficulty with building optical devices that use light as a control, is that photons do not usually interact strongly. This makes it very difficult to make devices at very low power (numbers of photons needed) or very precise. This project is about the proposition of using strong coupling between optical emitters with propagating surface plasmons confined in a conducting nanowire to create strong non-linear reactions between individual photons. What this can physically translate to is that this can act as a two photon switch for incident photons moving in the nanowire. This concept can be extended to create an optical transistor device in which "current"(a flow of photons) between the "source" and "drain" can be controlled by the presence or absence of a single photon.

Electrical and Optical Transistors

A transistor in general is a 3 port device in which a control at one of those ports can manage the flow between the other 2 points. In an electrical transistor, you supply a voltage at the "gate" electrode and it will open a conduction channel between the "source" and "drain" electrode's (figure 1). The actual construction of an electrical transistor may vary greatly, but the idea is the same.



Construction of MOSFET

Figure 1

In order for this transistor to have any real use, for example as a switch, the operation between the gate-source voltage and the drain-source current should be non-linear. If it was linear, there would be no control of the flow; there would only be an amplification or attenuation to the control signal, and no switching behavior. In a common MOSFET transistor, there are 3 modes of operation. The first mode is the cutoff region and this region very important. For any gate-source voltage (V_{gs}) below some threshold voltage (V_{tn}), the channel is closed or off. Likewise for some V_{gs} above a saturation voltage (V_{sat}), the current across the channel is:

$$i_d = \frac{K_n}{2} (V_{gs} - V_{tn})^2 (1 + \lambda_{V_{ds}})$$

For all V_{gs} in between these two limits, the transistor is said to operate in a triode region and the current across the channel is:

$$i_d = K_n \left(V_{gs} - V_{tn} - \frac{V_{ds}}{2} \right) * V_{ds}$$

Both the triode region and the saturation region operate in a non-linear fashion. A more common way to look at the operation of the MOSFET transistor is to look at the current between the source and drain (i_{ds}) and the voltage between the source and drain (v_{ds}). The overall operation of the transistor through these regions is very much non-linear (Figure 2). In this image, the linear region is equivalent to the triode region. An important feature that will come back later is that the current through the channel is almost completely constant in the saturation region. Transistors can function as either switches or as amplifiers. As a switch this operates a simple on/off device. In order to do this, the transistor would be in either cutoff or saturation at all times. However when the transistor is in the triode or linear region, the output current is more or less amplified by the control voltage, and the device functions as an amplifier with a gain of :

$$g_m = \frac{2I_d}{V_{gs} - V_{tn}}$$

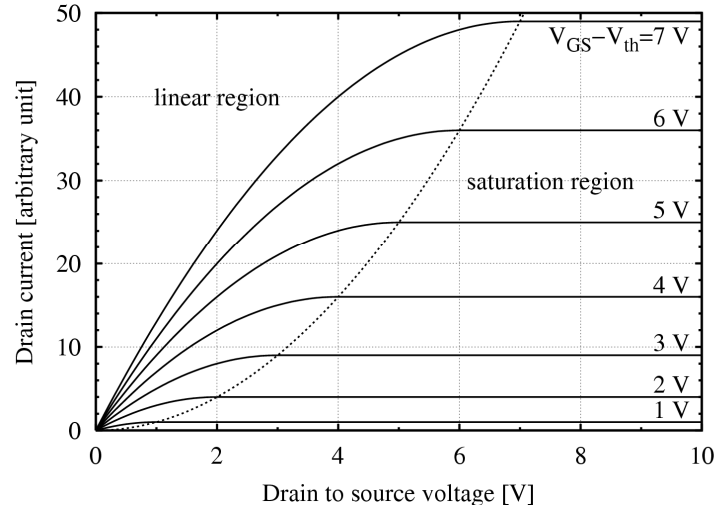


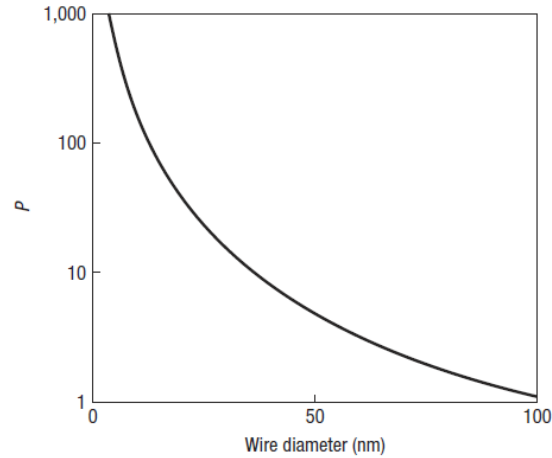
Figure 2

Now when considering the concept of an optical photonic transistor, the idea will be to control the flow of an optical “signal” by the presence of a control at an optical “gate”. Just as the electrical switch, in order for this to work, there must be a strong, non-linear optical interaction. In this case the absolute limit in scale for an optical transistor would be a transistor that can operate based on the presence or absence of a single photon at its gate. Unfortunately, the non-linear interaction between photons is very weak. It should be mentioned, that there are numerous concepts in development of creating a switch capable photon interaction, such as atomic ensembles, or Quantum Electrodynamics (QED). This is only one such model. The hopeful solution proposed here is to utilize a tight concentration of optical fields in conjunction with guided surface plasmons along a conducting nanowire in order to achieve strong non-linear interactions between multiple optical emitters.

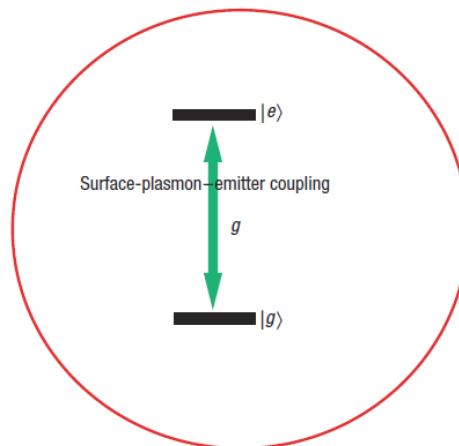
Plasmons and Nanowires

Surface plasmons are, “propagating electromagnetic modes confined to the surface of a conductor-dielectric interface”¹. Another way to look at surface

plasmons is to say that they occur when there is a strong interaction between a surface and light, which will end up resulting in a polariton. A polariton is a quasi-particle resulting from a photon that is strongly coupled with a plasmon. These plasmon will occur at the interface between a vacuum and a metal which means that they will occur and be confined to a metallic nanowire. One very unique and interesting property of the surface plasmon is that they can be confined in sub-wavelength dimensions. This means that even when the radius of the nanowire is below the optical wavelength, that good confinement and guiding can be achieved. As a result of this narrow confinement, there is a large coupling constant between the surface plasmons and any near by emitter(explained later) with a dipole-allowed transition. This coupling constant goes by: $g\alpha l / \sqrt{A_{eff}}$, where A_{eff} is transverse mode area, which will be small given the sub-wavelengths, and g is the coupling constant. This will also reduce the group velocity and respectively increase the density of states $D(\omega)\alpha l / R$, with R being the radius of the nanowire and $D(\omega)$ is the density distribution. The spontaneous emission ratio of an emitter into the plasmons goes by, $\Gamma_{pl} \sim g^2(\omega)D(\omega)$, substituting for the proportionalities developed earlier: $\Gamma_{pl}\alpha l / R^3$. This is going to be much larger than the emission rate (Γ^{\sim}) into any channel other than the nanowire. An interesting and useful metric to consider is the Purcell factor, which is “The enhanced radiative rate of an emitter within a microcavity relative to its value in free-space”², and also “Purcell factors greater than unity imply that the spontaneous emission rate is enhanced by the cavity”³. The Purcell factor (P) is defined as $\Gamma_{pl}/\Gamma^{\sim}$, which given this application can be very large; on the order of 10^3 (figure 3).

**Figure 3**

Because this strong coupling was arrived at by only geometrical means, it can be said that the coupling is broadband which is in direct contrast to other methods that attempt the same outcome such as QED (Quantum Electrodynamics). Now given these considerations, we can construct an emitter to use that is a simple two level configuration (figure 4). The emitter will be an atom in close proximity to the surface plasmons. This atom will assume has two distinct states; the ground state ($|g\rangle$) and the excited state ($|e\rangle$).

**Figure 4**

The Hamiltonian for this emitter is¹:

$$H = \hbar(\omega_{eg} - i\Gamma/2)\sigma_{ee} + \int dk \hbar c |k| \hat{a}_k^\dagger \hat{a}_k - \hbar g \int dk \left(\sigma_{eg} \hat{a}_k e^{ikz_a} + h.c. \right)$$

where $\sigma_{ij} = |i\rangle\langle j|$, and \hat{a}_k is the annihilation operator for the mode with wavevector k and z_a is the emitter position.

Single Emitter Properties

When an interaction occurs with the two level emitter discussed previously, the propagation of plasmons in the nanowire can be significantly altered. In this system, there are only two modes of operation. For low incident power levels, the emitter can act as a near perfect mirror, reflecting all incoming photons with almost unit probability. The second mode is saturation, which occur very quickly for higher power levels, and when the emitter saturates, most of the photons will be transmitted without modification. At this point, the emitter is desired to act as a mirror, so saturation is undesirable.

Consider a single photon being scattered (figure 5). The only surface plasmon modes in this case that are of interest are ones near the optical frequency (ω_{eg}), which means that left and right propagating plasmons can be ignored or treated like separate fields. The Reflection Coefficient can be obtained by solving for the scattering eigenstates of the system, and the approach can be generalized to finite P .^{1 & 8}

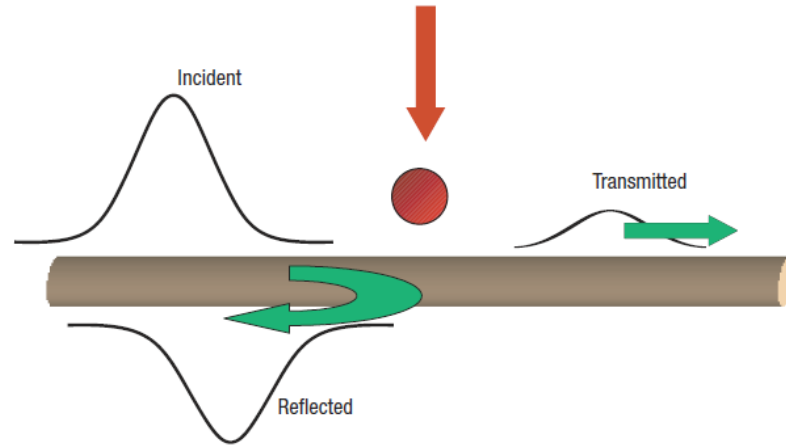


Figure 5

The reflection coefficient for an incoming photon of wavevector k is¹:

$$r(\delta_k) = -\frac{1}{1 + \Gamma^*/\Gamma_{pl} - 2i\delta_k/\Gamma_{pl}}$$

And transmission coefficient

$$t(\delta_k) = 1 + r(\delta_k)$$

On resonance, which occurs when the surface plasmons are excited by light, the reflection reduces to approximately $r \approx -1(1 - 1/P)$, which given the large Purcell factor shown earlier, will give an $r \approx -1$, which means that the emitter will act as an approximately perfect mirror, dependant on the Purcell factor. Since the coefficient is negative, it will also impart a phase shift equal to π . Earlier it was mentioned that the confinement to the nanowire was very good. Now, this can be evaluated. The probability that an incident photon is lost into the environment k is very low. $k \equiv 1 - \Re - \Im = 2\Re/P$. Once again given the large Purcell factor, this coefficient will be quite low. Given a P of about 100 (lower than predicted), the probability k is less than 2%.

The Ideal Single Photon Transistor

Consider modifying the current emitter concept that has an atom with just states $|g\rangle$ and $|e\rangle$, by adding an additional state $|s\rangle$. This new state will be decoupled from the surface plasmons, keeping in mind that $|g\rangle$ and $|e\rangle$ are coupled still via the surface plasmon modes. The new state $|s\rangle$ is metastable, and is resonantly coupled to $|e\rangle$, which can be done with an optical control field with a Rabi frequency of $\Omega(t)$. The Rabi frequency here is the frequency at which a Rabi flop occur from $|e\rangle$ to $|s\rangle$. Now using this emitter we can construct a system in which a single photon, or lack thereof, at the “gate” will completely control the “signal”, similar to that of a classic transistor (figure 6).

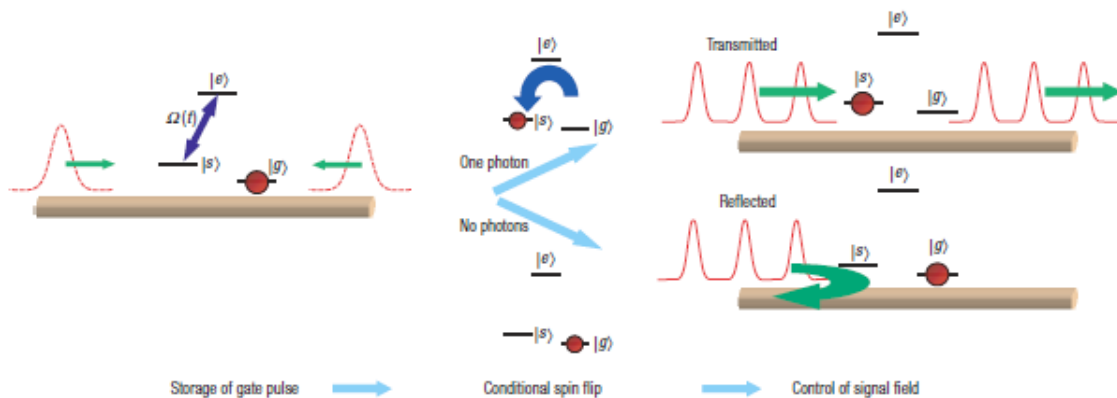


Figure 6

Now, in order to have this control, there must be a way of storing a single photon (or not storing the non-present photon), and thus creating an atomic memory of the gate field, which will allow the gate to interact or control subsequent signals. In order to accomplish this, the system is initialized in state $|g\rangle$ and then an optical control field with Rabi frequency $\Omega(t)$ is applied at the same instant that a photon arrives in the surface plasmon modes. This control must be impedance matched, and if it is, it will capture the incoming photon while also inducing a

spin flip from $|g\rangle$ to $|s\rangle$. If there was no photon present then the system remains unaltered in $|g\rangle$. The optimal storage can be obtained by splitting the incoming pulse and having it incident from both sides of the emitter simultaneously¹ (figure 3). This is come upon by comparing this storage operation to that of single photon generation in which, the emitter would be driven from $|s\rangle$ to $|g\rangle$. The storage efficiency for this system will correspond to the generation efficiency of the opposite system, and is given by $\sim 1-1/P$, and given the large Purcell factor possible, the efficiency becomes ~ 1 .

After the optical field is turned off, the system will be in either $|g\rangle$ or $|s\rangle$. When the emitter is in $|g\rangle$, it will act as described previously, which is that of a near perfect mirror. Because $|s\rangle$ is decoupled, when the emitter is in $|s\rangle$, it will have virtually no effect on incident signals. So, now there is a system in which, there will either be perfect reflection, or perfect transmission, dependant heavily on the internal state of the emitter, or the atomic memory. Now this system behaves similar to a transistor. The single photon pulse that is stored or not, is the control of the device. If a photon was present, the emitter flips to $|s\rangle$ and will allow signals to “flow”. If no photon was present, the system remains in $|g\rangle$ and will “block” incoming signals (figure 6).

There is a limitation however to this system. The performance of the reflection while in $|g\rangle$, has a limit. If the system is in $|g\rangle$ after the optical control field, and is left in $|g\rangle$, it can eventually be pump charged into $|s\rangle$, which would of course cause un-desirable behavior. The desired behavior is that the emitter remain in $|g\rangle$ and reflect incident photons, but in $|s\rangle$, it will transmit them. The number of photons that can be effectively reflected is dependant on branching ratio of decay rates from $|e\rangle$ and is on the order of P . This number corresponds to the effective gain of the transistor.

Applications and Outlook

One of the biggest fields that transistors are used for is integrated systems. Electrical transistors integrate very well for large systems. Unfortunately surface plasmons experience losses as they propagate along a nanowire, similar to the way an electrical signal can have losses through resistance in wires. In an electrical system, the resistance is higher for a small diameter wire, and thus there is more loss as you decrease the size of the wires. In a similar fashion, a very small nanowire may give you a high Purcell factor which is important for this systems operation, but the same narrow nanowire also increases dissipation or loss. This may seem like a deterrent, but there are schemes that have been designed for just this solution.

...if we can integrate surface-plasmon devices with low-loss dielectric waveguides. Here, the surface plasmons can be used to achieve strong nonlinear interactions over very short distances, but are rapidly in- and out-coupled to conventional waveguides for long distance transport. One such scheme is shown in Fig. 7, where excitations are transferred to and from the nanowire via an evanescently coupled, phase-matched dielectric waveguide. The losses will be small provided that the distance needed for the surface plasmons to be coupled in and out and interact with the emitter is smaller than the characteristic dissipation length, which can be accomplished using optimized surface-plasmon geometries (for example, tapered wires or nanotips^{15,28}) or periodic structures with engineered surface-plasmon dispersion relations³⁹. Coupling efficiencies of ~95%, for example, are predicted using simple systems²⁸.¹

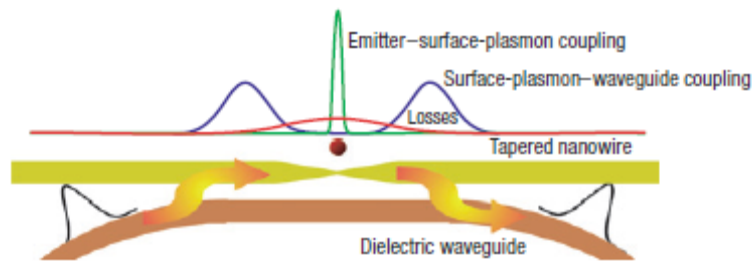


Figure 7

By these methods, large scale integrated photonic device become a possibility. In computing systems, both classical and quantum, the transistor that operates on light has been called “the holy grail of optical computation”⁵. In electrical systems the speed of communication of signals will always be limited by the speed of electrons moving in that system. Similarly the system described above would be limited to the speed of the photons in that system. The difference is that electrons in conductors will travel at a mere fraction of the speed of light, whereas photons in this system travel at the speed of light.

Another interesting feature of this system that was mentioned earlier is that it is broadband, which means that it will not require any special precise tuning of either the emitter or the nanowires. This makes surface plasmons a promising candidate for use with solid-state emitters such as quantum-dot nanocrystals or colour centres, in which the spectral properties will vary over different individual emitters. This scheme may also be used for the detection of individual photons, as the system interacts on a single photon basis. There is also a wide array of applications in the quantum field for this sort of device.

The last note here, is that as far as is publicly available, this is only a theoretical model and no hard experimental devices have been created.

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