PH-315 Winter 2009 **La Rosa**

EXPERIMENT 1
RLC SERIES CIRCUIT RESONANCE
(Complex impedance)

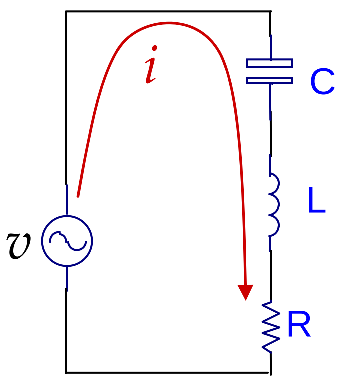
PURPOSE

To use an oscilloscope to make AC measurements of voltage and current.

To observe the frequency-dependence of the impedance and the phase in an AC circuit.

To use series resonance to determine the inductance of a coil using a known capacitor.

THEORETICAL CONSIDERATION



The total impedance for the series combination is

$$Z = R + j\omega L + \frac{1}{j\omega C} = R + j(\omega L - \frac{1}{\omega C}) \qquad (1)$$

If the input voltage is given by

$$v = V_o e^{j\omega t} \qquad (2)$$

then the current is given by,

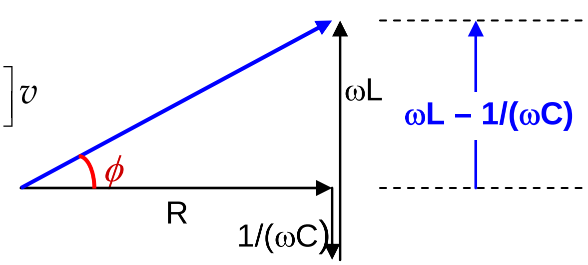
$$I = \frac{v}{Z} = \frac{1}{R + j(\omega L - \frac{1}{\omega C})} v \qquad (3)$$

Multiplying and dividing by the complex conjugate, this expression can be rewritten as

$$I = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \left[\frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} - j \frac{(\omega L - \frac{1}{\omega C})}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \right] v$$

Notice the quantity in brackets has magnitude equal to 1, and can be expressed in terms of complex number of phase ϕ:

$$I = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} \left[COS(\phi) \ - \ jCOS(\phi) \right] v$$



$$I = \frac{1}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} e^{-j\phi} v$$

Using (2),

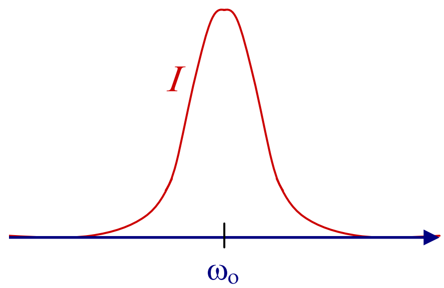
$$I = \frac{V_o}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} e^{j(\omega t - \phi)} \qquad (4)$$

Notice form (1) and (4) that when $\omega L = 1/(\omega C)$,

The impedance is minimum

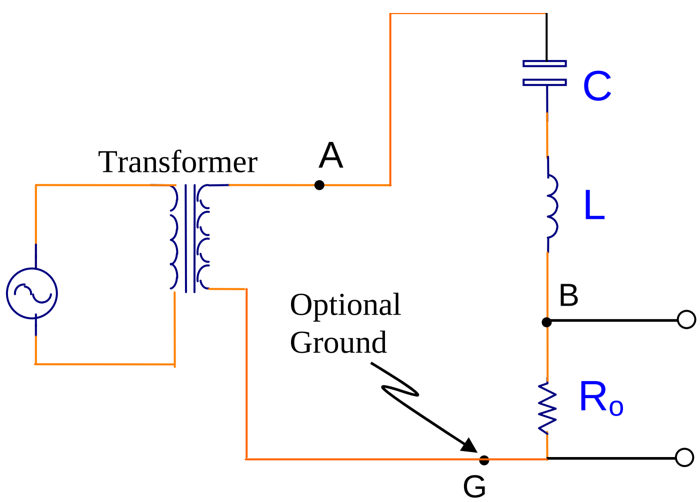
The current is maximum

Hence $\omega = \omega_o = \sqrt{\frac{1}{LC}}$ is called the resonance frequency.



EXPERIMENTAL CONSIDERATIONS

The oscilloscope is our basic measuring tool in alternating current (AC) circuits. We will be employing oscilloscopes that are equipped with two inputs, and which switch rapidly between them to display both. The method of switching is selectable (alternate or chopped), and which to use depends on the frequency you are observing.



Since the oscilloscope is basically a voltage measuring device, and we will want to measure both voltage and current amplitudes, it will be convenient to use several elements to make our "signal generator."

It is often of advantage to couple out of the oscillator with a transformer to provide **ground isolation** (so you can choose any point in the circuit to be the ground). In the circuit in the figure above, point G has been chosen as the ground. The resistor R_o is a known resistor (whose value is on the order of 100 Ω). The ground of the oscilloscope will be connected to G, while the two inputs will be to points A and B.

GA will measure the input voltage $\mathbf{V_A}$, while

GB will measure $\mathbf{V_R} = \mathbf{I}R_0$ from which the current **I** can be deduced.

Notes: In atypical capacitor, labeled, for example 473G, it means C = $47\text{x}10^3$ picoFarads.

Measure the frequency of the input voltage using the oscilloscope. Do not trust the reading from the signal generator knob.

MEASUREMENTS

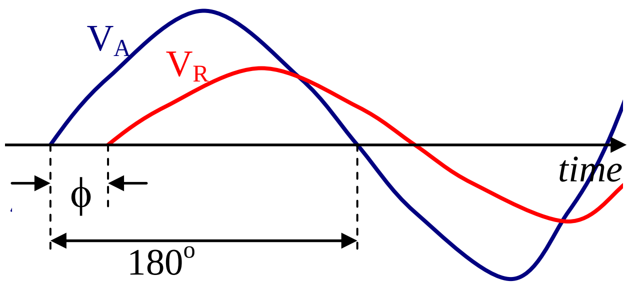
a) The magnitude of the impedance **Z** of the test circuit is determined from the ratio of amplitudes of the two signals, $\mathbf{V_A}$ and **I**.

That is, plot $|\mathbf{Z}|$ as a function of ω.

b) The phase can be measured as a distance between the points where the two traces cross the horizontal axis, and converted to degrees by comparing the half (or full) wavelength as shown on the oscilloscope. See figure below.

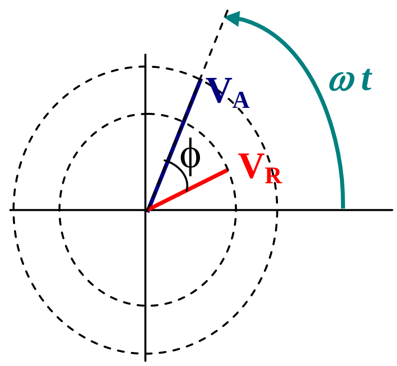
Pay attention during the measurements to verify if ϕ is positive or negative. That is, whether the $\mathbf{V_R}$ is lagging or ahead of $\mathbf{V_A}$.

Plot $\phi = \phi(\omega)$



$V_A = V_o \cos(\omega t)$; $V_R = V_o \cos(\omega t - \phi)$

The traces show V_R lagging V_A by ϕ.



Phasors $\mathbf{V_A}$ and $\mathbf{V_R}$ rotating with angular velocity ω . $\mathbf{V_R}$ lags $\mathbf{V_A}$ by ϕ.

c) During the course of measurements you should take enough data to make a graph of both impedance and phase as a function of frequency. How does the curve changes when using a higher value of the resistance R_o?

In order to compare this to theory, you will need to determine tire value of the inductance L. This is best done early in the exercise by locating the resonance frequency, where the impedance is minimum and where the **phase** between $\mathbf{V_A}$ and $\mathbf{V_R}$ **is zero**. This is obviously the frequency region where a majority of your data needs to be taken. The quality of your plot of impedance and phase is more dependent upon taking data that spans the region of interest than it is on the volume of data alone.

By rearranging the circuit elements and shifting the ground it is possible to compare the signals across C and R, and across L and R.

d) At resonance condition, measure the voltage across the three elements. Do these values add up?

e) Choose a frequency at which I is ~50% of $I_{resonance}$. Verify experimentally and corroborate theoretically the validity of Kirchhoff law (measure the complex values for V_c, V_L, V_R?

OVERCOMING the SHORTCOMINGS ENCOUNTERED in the EXPERIMENT

Problem: Variability of the input voltage amplitude

Correction: By the normalization method.

While sweeping the frequency of the input voltage $\mathbf{V_A}$ around the resonance frequency, it is observed that the amplitude of the driving voltage also changes. Ideally, it would be desirable that this amplitude remains constant.

The instability of the $\mathbf{V_A}$'s amplitude makes somewhat inaccurate the procedure of locating the resonance frequency by monitoring the frequency at which the current is maximum. (When you do this, you may find that at the frequency where the current is maximum, $\mathbf{V_A}$ and $\boldsymbol{I}$ are not in phase.) Still this is still a good preliminary step, since it helps to identify the frequency range where we have to take a closer look.

Next, sweep the frequency a bit, until you find that the phase between $\mathbf{V_A}$ and $\boldsymbol{I}$ is zero.

The suggested following step is to manually normalize the input voltage $\mathbf{V_A}$. By this we mean to choose a fixed amplitude value for $\mathbf{V_A}$, let's call it $\mathbf{V_{A,\ fixed}}$.

For a given frequency, record the current amplitude and phase.

Move to another frequency (you may notice that the amplitude of $\mathbf{V_A}$ has changed.) Turn the knob that controls the amplitude of $\mathbf{V_A}$ until you obtain back the predetermined fixed value $\mathbf{V_{A,\ fixed}}$. Then record the current amplitude and phase.

And so on, repeat the procedure for each frequency around the resonance frequency.