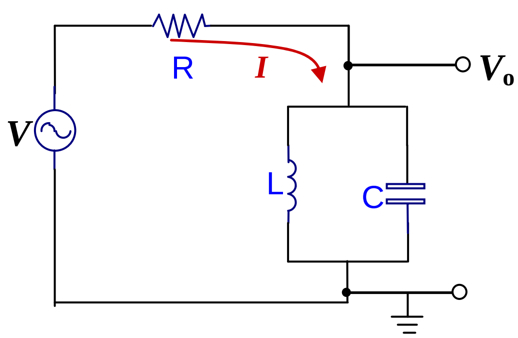
EXPERIMENT 2
PARALLEL RESONANCE
(Complex impedance.)

PURPOSE:
To study the dependence of impedance as a function of frequency when capacitors and inductors are connected in parallel.

THEORETICAL CONSIDERATIONS:
Similar to the series connection counterpart, resonance also occurs in a RLC parallel circuit. In the former, the current peaks at the resonance frequency; in the latter, the voltage across the inductor or the capacitor is the quantity that peaks. To analyze the circuit, let's calculate the impedances and current across the circuit.



- The impedance Z_1 across the inductance and the capacitor (connected in parallel) is calculated through the expression,

$$\frac{1}{Z_1} = \frac{1}{j\omega L} + \frac{1}{1/(j\omega C)} = \frac{1-\omega^2 LC}{j\omega L}$$. Hence,

$$Z_1 = j\frac{\omega L}{1-\omega^2 LC} \quad \text{(across LC)} \qquad (1)$$

Notice, this impedance is very large when

$$\omega = \omega_o \equiv \frac{1}{\sqrt{LC}}$$

The total impedance of the circuit is: $Z = R + j\dfrac{\omega L}{1-\omega^2 LC}$ (2)

- For an applied harmonic voltage $V = V_p\, e^{j\omega t}$ the resulting total current is:

$$I = \frac{V}{R + j\dfrac{\omega L}{1-\omega^2 LC}} \qquad (3)$$

Notice the current is zero at resonance (where the impedance becomes infinity.)

- The voltage V_o is equal to $I\,Z_1$,

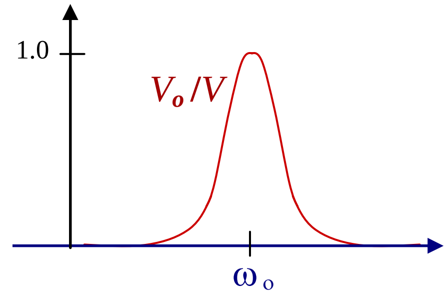
$$V_o = I\,Z_1 = \frac{V}{R + j\dfrac{\omega L}{1-\omega^2 LC}}\;\frac{j\omega L}{1-\omega^2 LC} = \frac{j\omega L}{R\,(1-\omega^2 LC\,) + j\omega L}\,V$$

An equivalent expression can be worked out for V_o.

$$V_o = \frac{1 \;+\; j\,(R/\omega L)\,(1-\omega^2 LC\,)}{1 \;+\; (R/\omega L)^2\,(1-\omega^2 LC\,)^2}\,V \qquad (4)$$

A relationship between the magnitudes of and results,

$$\left|\frac{V_o}{V}\right| = \frac{1}{\sqrt{1 + (R/\omega L)^2 \, (1-\omega^2 LC)^2}} \qquad (5)$$



At $\omega = \omega_o \equiv \dfrac{1}{\sqrt{LC}}$:

- The current I is zero
- The output voltage V_o across the LC combination becomes equal the input voltage V, independent of the value of R.
- The currents along the capacitor and inductance are $I_C = \dfrac{V}{1/(j\omega_o C)} = j\omega_o CV$ and $I_L = \dfrac{V}{j\omega_o L} = -j\dfrac{1}{\omega_o L}V$, respectively. These two currents are equal in magnitude, but out of phase (they cancel each other so no current I passes through the resistor.) Accordingly, a circulating current exists in the parallel LC combination at resonance.

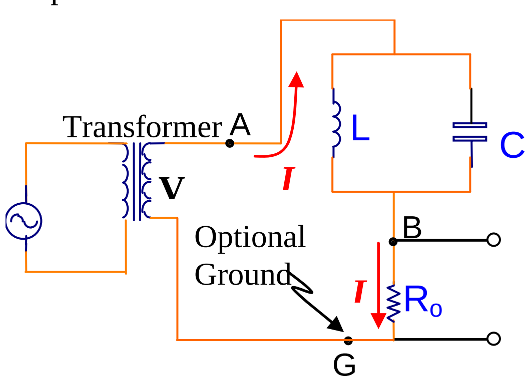
Parallel resonance is commonly used in electronic circuits to achieve a high impedance (which develops an appreciable signal at resonance.)
Parallel resonance is also used to emphasize a narrow frequency bandwidth over other frequency ranges: by changing the value of, for example, the capacitance, the circuit is tuned to different frequencies (radio, TV applications.)

EXPERIMENTAL CONSIDERATIONS:

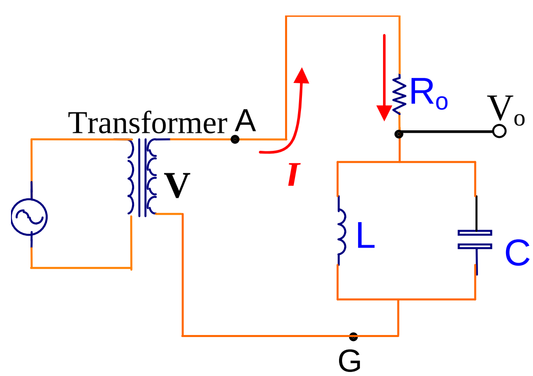
The basic "signal generator" is the same as that used in Experiment-1. The oscilloscope connections are also the same initially.

MEASUREMENTS:

a) Take measurements similar to those of in Experiment-1 to determine the impedance for a circuit composed of the inductor and capacitor in parallel. Compare your results to theoretical predictions.

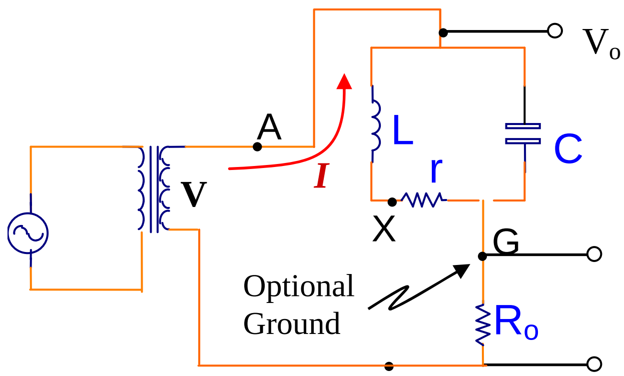


Circuit for measuring V and I (hence, the impedance.)



Circuit for measuring V and V_o.

In order to understand how the circuit operates more fully, it will be instructive to add the small resistor *r*. This will allow for several additional measurements:



b) By shifting the oscilloscope input normally connected to A to point X, it will be possible to compare the amplitude and phase of the currents internal and external to the parallel resonance circuit for a variety of frequencies

c) As additional measurements, you may wish to shift the resistor r to be in series with the capacitor instead.