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EXPERIMENT 4

IMPEDANCES of SOURCES and LOADS

I. PURPOSE

To familiarize with the input and output impedance of transistors

II. THEORETICAL CONSIDERATIONS

Circuit loading concept

A voltage source has intrinsically an internal resistance (r). Attaching to the circuit a load whose resistance R_L is less than or even comparable to the internal resistance will reduce the output voltage V_{ab} considerably as illustrated in the Fig 1.

$$V_{out} = V_{in} \frac{1}{1 + (r_{source} / R_{Load})} \qquad (1)$$

Solution: To make a stiff source, i.e. $R_{Load} >> r_{source}$

(rule of thumb $R_{Load} > 10\, r_{source}$)

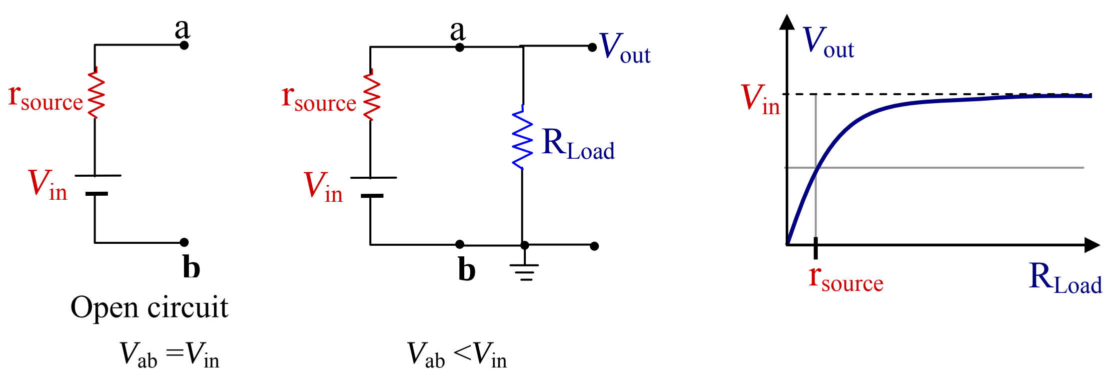


Fig. 1 Circuit loading, "the undesirable reduction of the open-circuit voltage V_{in} by the load."

In electronic circuits stages are connected one after another.

i) Sometimes it is OK to load the circuit, as far as we know how much the loaded is, and particularly if Z_{in} is going to be constant.

ii) Of course, it is always better to have a stiff source ($Z_{out} << Z_{in}$), so that signal levels do not change when a load is connected.

iii) However, there are situations in which it is rather required to have $Z_{out} = Z_{in}$. That is the case in radiofrequency circuits to avoid signal reflections.

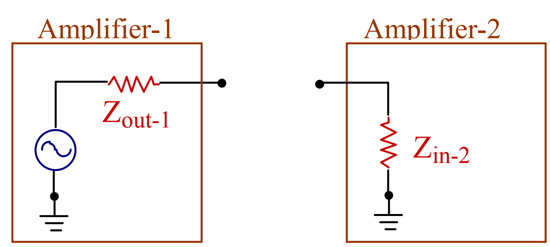


Fig. 2 Amplifiers are typically characterized by their effective output and input impedances. This is particularly important for analysis when cascading them one after another.

III. EXPERIMENTAL CONSIDERATIONS

III.1 Making stiffer sources: Emitter follower

III.1A Input impedance of the emitter-follower circuit

III.1B Output impedance of the emitter-follower circuit

III.2 Matching impedance: measuring the 50 Ω output impedance.

III.1 Emitter follower

The circuit shown in the figure is called an *emitter follower* because the output terminal (the emitter) follows the input (the base) except by a diode drop voltage:

$$V_E = V_B - 0.6V \quad (2)$$

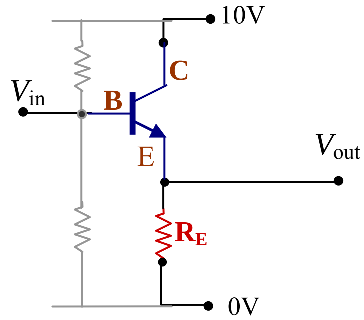


Fig. 3 Emitter follower. The details of the resistors for DC-bias are omitted.

At first glance this circuit may appear useless, until one realizes that it has input impedance much larger than the output impedance. This means,

- The circuit requires less power from the signal source to drive a given load than would be the case if the signal source were to drive the load directly.

Or equivalently

- A signal of some internal impedance can drive a load of comparable or even lower impedance without loss of amplitude (that is, without the detrimental effects of the voltage divider effects.)

How to arrive to such conclusion constitutes the subject of study in this section.

Method of analysis

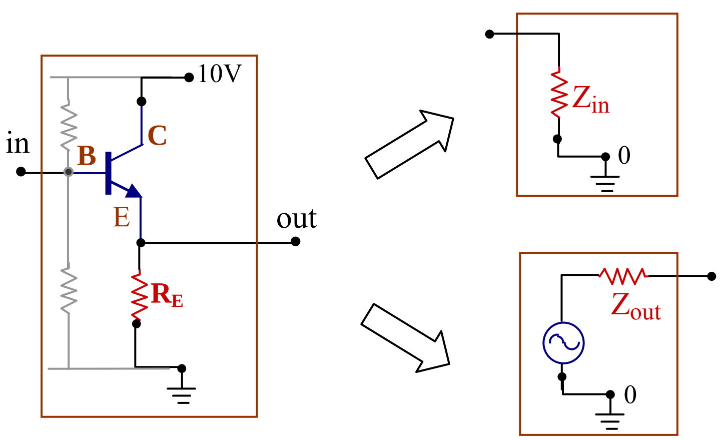


Fig. 4 Emitter follower and its black-box representation highlighting its effective input and output impedances.

Underlying our method of analysis is to consider the emitter follower as a black box, which to the effect of measuring its effective load it will be considering having input impedance Z_{in}, and to the effect of driving a subsequent circuit stage it will be considered having output impedance Z_{out}. Our objective in this lab is to **calculate** and **measure** these two impedances (of the same emitter follower circuit.)

III.1A Input impedance of the emitter-follower circuit

Experimental procedure

Fig. 5 (diagram on the left) shows in a very straightforward manner that the input impedance can be determined experimentally by measuring the base-voltage Δv_B and the input base-current Δi_B. The diagram on the right shows one way of implementing the measurement using an oscilloscope (see the accessory circuit.) For Δv_{in} use an ac-voltage of amplitude ~ 20 mV. Notice an additional resistance R_x has been introduced for the purpose of measuring Δi_B. (it plays also the role of a base resistor, as used in previous bias transistor setups. Together with the +5V battery, it properly bias the transistor.) Similar to the method used in experiments 1 and 2, you will have to implement a floating ground and use the secondary output voltage of the transformer as the voltage source.

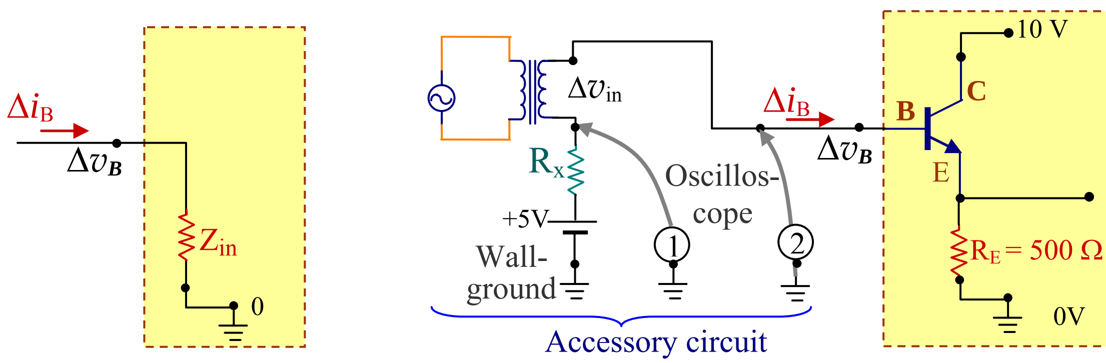


Fig. 5 Left: Black box diagram representing the emitter-follower. Measuring $\Delta v_B / \Delta i_B$ gives the value of Z_{in}. **Right:** Optional experimental implementation for measuring Δv_B and Δi_B. Set the oscilloscope inputs to ac-mode measurements and, during the measurement, verify if Δv_{out} follows Δv_B (i.e. ensure the transistor is working in the active region.)

[An alternative experimental setup to more properly bias the emitter-follower circuit presented at the end of these notes.]

Calculation-1

Objective: to predict theoretically the value of Z_{in}.

$I_E = I_B + I_C$ implies

$$\Delta i_E = \Delta i_B + \Delta i_C$$

In experiment #3 we found, $I_C = \beta I_B$; hence

$$\Delta i_E = (\beta+1)\Delta i_B \quad (3)$$

On the other hand, from (2) one obtains $\Delta v_E = \Delta v_B$; hence

$$\Delta i_E = \Delta v_E / R_E = \Delta v_B / R_E \quad (4)$$

From (3) and (4) $\Delta v_B / R_E = (\beta+1)\Delta i_B$, which gives

$$Z_{in} \equiv \frac{\Delta v_B}{\Delta i_B} = (\beta+1)R_E \quad (5)$$

Since β is typically of the order of 100, then Z_{in} is ~ 100 times greater than R_E. For R_E = 0.5 kΩ,

$Z_{in} \sim 50\ \text{k}\Omega$

[Notice the mathematical derivation above is independent of the particular accessory circuit (see Fig. 5) that may be used to experimentally measure Δi_B and Δv_B. It only requires that the transistor is properly biased (i.e. working in the active region, as to justify the use of $I_C = \beta I_B$.]

TASK: Verify if the predicted value given in (5) is close to the experimental value you measured in the section above.

Calculation-2 (Here we present an alternative method for obtaining Z_{in}.) This section is offered for completeness of the topic. No experimental task are assigned to this section.

How much does the follower-emitter circuit load a given source (in the sense described in Fig 1 above)?

To answer this question, let's find out how much does Δv_{in} differ from Δv_{out}.

Fig. 6 shows an arrangement to calculate the loading effect of the emitter-follower circuit. Here we assume the emitter-follower id being cascade with a circuit of equivalent Thevenin voltage source Δv_{in} and equivalent impedance r_{source}. (For simplicity, the details of the resistors used for DC bias the transistor are omitted.)

Notice:

$$\Delta v_{in} - r_{source}\,\Delta i_B - \Delta v_E = 0$$

Since $\Delta v_E = \Delta v_{out}$,

$$\Delta v_{in} - \Delta v_{out} = r_{source}\,\Delta i_B$$

Using (5) $\Delta i_B = \dfrac{\Delta v_B}{(\beta+1)R_E}$, and since $\Delta v_B = \Delta v_E = \Delta v_{out}$, one obtains $\Delta i_B = \dfrac{\Delta v_{out}}{(\beta+1)R_E}$. Hence,

$$\Delta v_{in} - \Delta v_{out} = r_{source}\,\frac{\Delta v_{out}}{(\beta+1)R_E}$$

$$\Delta v_{in} = \Delta v_{out}\left(1 + \frac{r_{source}}{(\beta+1)R_E}\right)$$

$$\Delta v_{out} = \frac{1}{1 + \dfrac{r_{source}}{R_E}\dfrac{1}{(\beta+1)}} \qquad (6)$$

Compare this last expression with the case expression (1) in which the transistor circuit is not used. We realize that as part of the emitter-follower circuit the effective impedance is $R_E(\beta+1)$.

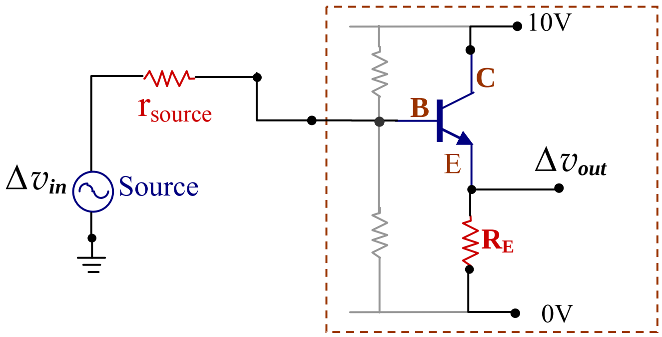


Fig. 6 Setup to analyze the loading effect of the emitter-follower. The details of the resistors for DC-bias are omitted.

In short

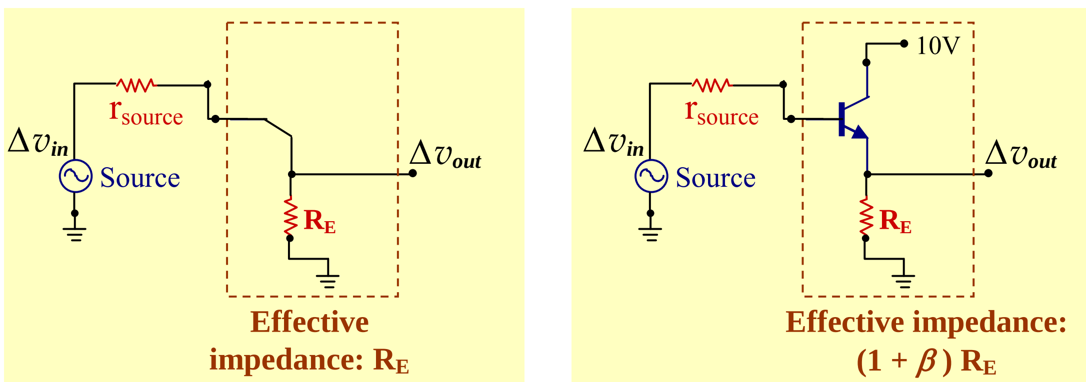


Fig. 7 Schematic comparison of the effective input impedance caused by R_E, depending on whether or not a transistor is used. The higher impedance of the latter is an advantage feature.

III.1B Output impedance of the emitter-follower circuit

Similarly to the procedure for finding the input impedance, we will consider the emitter follower circuit (Fig. 8, left diagram) as a black box (right diagram) for the purpose of finding its equivalent output impedance. A variable resistance R_{LOAD} will do the trick.

Experimental procedure

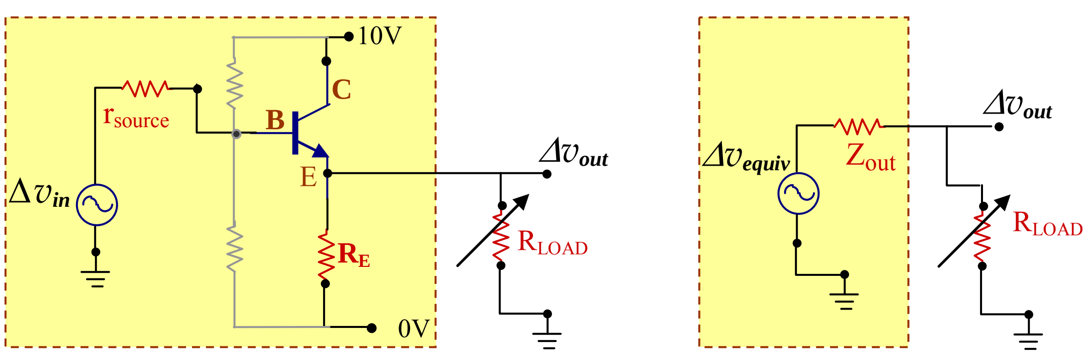


Fig. 8 The follower emitter circuit (left) and its equivalent circuit (right)

In this setup, the output voltage across R_{LOAD} is given by,

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv} \qquad (7)$$

By choosing two different values, R_1 and R_2, for R_{LOAD} one obtains,

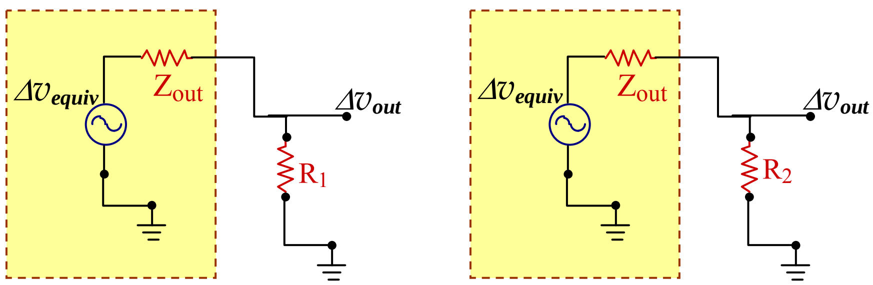


Fig. 9 The equivalent follower emitter circuit hooket to two different output loads

$$\Delta v_{out;1} = \frac{R_1}{Z_{out} + R_1} \Delta v_{equiv} \quad \text{and} \quad \Delta v_{out;2} = \frac{R_2}{Z_{out} + R_2} \Delta v_{equiv}$$

Solving for Z_{out} (see proof at the end of these notes),

$$Z_{out} = R_2 \frac{\left[\dfrac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right]}{\left[1 - \dfrac{R_2}{R_1} \dfrac{\Delta v_{out;1}}{\Delta v_{out;2}} \right]} \qquad (8)$$

As we expect Z_{out} to be very low ($\leq 100\Omega$) compared to the input impedance, you may use

- R_1 and R_2 to be both low.
- Alternatively, R_1 can be the input impedance of the oscilloscope (that is, you just hook the output of the emitter-follower to the oscilloscope. Take R_1 equal to infinity. Then use a low value resistance R_2.

Theoretical prediction

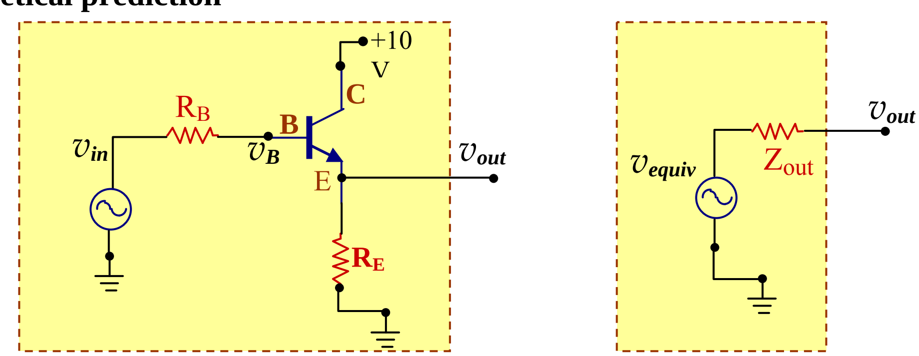


Fig. 10 The follower emitter circuit with a direct DC bias using R_B (left) and its equivalent circuit (right.)

$$i_B = \frac{v_{in} - v_B}{R_B}$$

Since $i_E = (\beta+1)i_B$, it follows $i_E = (\beta+1)\frac{v_{in} - v_B}{R_B}$

In Fig 10, notice $v_{out} = R_E i_E$. Hence,

$$v_{out} = (\beta+1)\frac{R_E}{R_B}(v_{in} - v_B)$$

Since we will be mainly interested in the ac signals, we can take $v_{out} = v_B$ (its difference being ~ 0.7 volts.) Thus,

$$v_{out} = (\beta+1)\frac{R_E}{R_B}(v_{in} - v_{out})$$

$$v_{out}\left[1 + (\beta+1)\frac{R_E}{R_B}\right] = (\beta+1)\frac{R_E}{R_B}v_{in}$$

$$v_{out} = \frac{(\beta+1)\frac{R_E}{R_B}}{1 + (\beta+1)\frac{R_E}{R_B}}\ v_{in} \qquad (9)$$

Notice in Fig. 10 that,

- In the open loop state, the right side circuit gives $v_{out}^{open\ circuit} = v_{equiv}$. In the circuit on the left, $v_{out}^{open\ circuit} = v_{out}$. Since these two circuits must be equivalent, one obtains,

$$v_{equiv} = \frac{(\beta+1)\frac{R_E}{R_B}}{1 + (\beta+1)\frac{R_E}{R_B}}\ v_{in} \qquad (10)$$

- When in the short circuit state, the right side circuit gives $i^{short\ circuit} = v_{equiv} / Z_{out}$. In the circuit on the left, connecting v_{out} to ground (short circuit) causes to bypass R_E. The current through the wire that causes the short circuit would be $i^{short\ circuit} = (\beta+1)i_B = (\beta+1)\frac{v_{in}}{R_B}$

  Since these two circuits must be equivalent, one obtains,

$$v_{equiv} / Z_{out} = (\beta+1)\frac{v_{in}}{R_B}$$

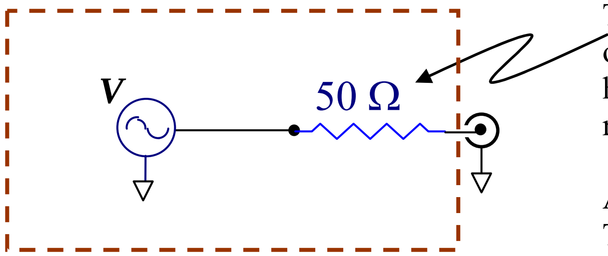
$$Z_{out} = \frac{R_B}{(\beta+1)}\frac{v_{equiv}}{v_{in}} \quad (11)$$

Using (10),

$$Z_{out} = R_E \frac{1}{1+(\beta+1)\frac{R_E}{R_B}} \quad (12)$$

IV. Matching Impedance

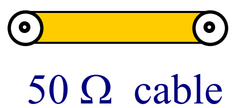
50 Ω Output Impedance



This is a real resistor, physical placed at the output (as literally drawn in the figure.) A high quality resistor, with the lowest reactance possible, is preferred.

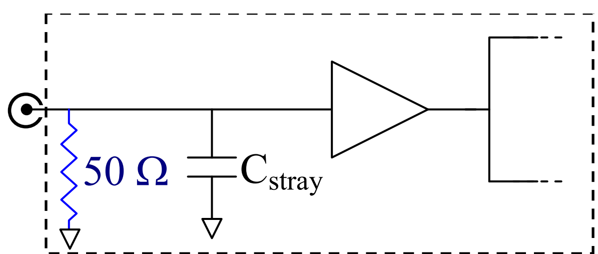
Alternatively, you can think this is the Thevenin's equivalent circuit.

50 Ω Impedance Cable

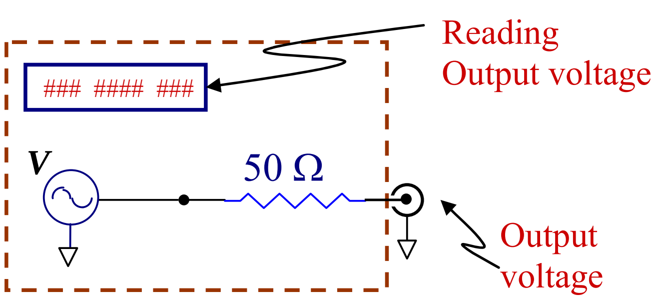


This is a 50 Ω complex impedance cable

50 Ω Input Impedance

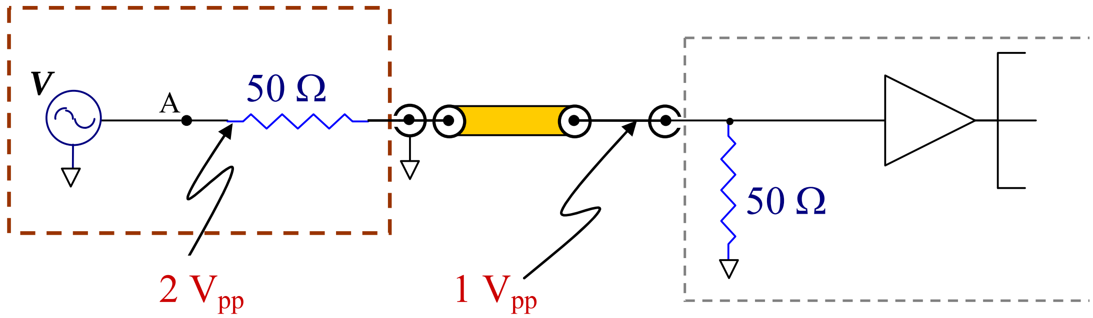


MEASUREMENTS



Some signal generators are specified to have 50 Ω output impedance AND expect to be connected to 50Ω input impedance devices. ONLY if the latter requirement is satisfied, the reading of the output voltage will coincide with the actual output voltage.

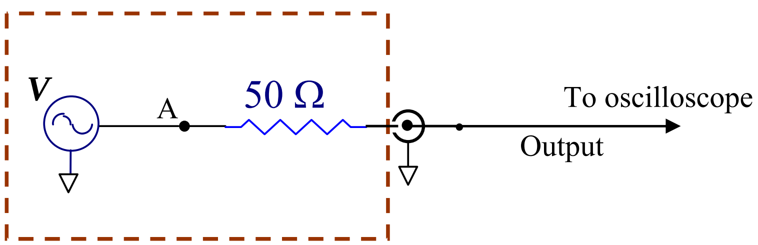
Notice in the case above that if, for example the output voltage were 1 Vpp (1 volt peak-to-peak) the voltage at A would be 2 Vpp



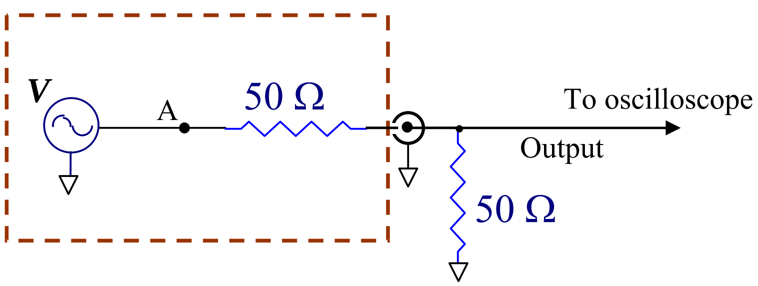
The input and output impedance create a voltage divider, hence the voltage readings (2 V_{pp} and 1V_{pp}) shown in the figure.

HOW TO TEST IS AN EQUIPMENT HAS A 50Ω OUTPUT IMPEDANCE OR NOT?

Task: Verify whether or not your signal generator has a 50Ω output impedance.



Expected output voltage: V_A



Expected output voltage: (1/2) V_A

Appendix

Proof of expression (8):

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv}$$

$$\Delta v_{out;1} = \frac{R_1}{Z_{out} + R_1} \Delta v_{equiv} \quad \text{and} \quad \Delta v_{out;2} = \frac{R_2}{Z_{out} + R_2} \Delta v_{equiv}$$

$$\frac{\Delta v_{out;1}}{\Delta v_{out;2}} = \frac{R_1}{R_2} \frac{(Z_{out} + R_2)}{(Z_{out} + R_1)}$$

$$(Z_{out} + R_1) \frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} = (Z_{out} + R_2)$$

$$Z_{out} \left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right] + R_1 \frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} = R_2$$

$$Z_{out} \left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right] = R_2 \left[1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}} \right]$$

$$Z_{out} = R_2 \frac{\left[1 - \frac{\Delta v_{out;1}}{\Delta v_{out;2}} \right]}{\left[\frac{R_2}{R_1} \frac{\Delta v_{out;1}}{\Delta v_{out;2}} - 1 \right]}$$

Alternative case of sing the oscilloscope as one of the impedance to compare.
Connect the output to the oscilloscope (assumed here to have $R_{LOAD} = \infty$). This allows measuring the amplitude of Δv_{equiv}.

$$\Delta v_{out} = \frac{R_{LOAD}}{Z_{out} + R_{LOAD}} \Delta v_{equiv} \xrightarrow[R_{LOAD} \to \infty]{} \Delta v_{equiv}$$

Then, using an arbitrary resistance R_{ext} (typically low values work better) as R_{LOAD} measure the corresponding Δv_{out}^{ext}. Then we solve Z_{out} from (7)

$$\Delta v_{out}^{ext} = \frac{R_{ext}}{Z_{out} + R_{ext}} \Delta v_{equiv}$$

$$(Z_{out} + R_{ext}) = \frac{R_{ext}\Delta v_{equiv}}{\Delta v_{out}^{ext}}$$

$$Z_{out} = R_{ext}\left[\frac{\Delta v_{equiv}}{\Delta v_{out}^{ext}} - 1\right]$$