EXPERIMENT 6

LOW-PASS FILTERS, 3dB BREAKPOINT and COMPARATORS

I. PURPOSE:

This laboratory session pursues two main objectives. First, to build a low-pass RC filter and measure the output voltage (magnitude and phase) as a function of its frequency. A log-log plot (Bode plot) of the output signal vs frequency will help familiarize with the concepts of "3dB breaking point" and "decrease of output levels per octave and per decade." Second, familiarize with the functioning of the operational amplifiers, and its application in comparator circuits. Comparators that uses no feedback and a Schmitt Trigger comparator (that uses positive feedback) will be built.

II. THEORETICAL CONSIDERATIONS

II.1. The scale of decibels

Comparison of signal amplitudes

Let's consider two signals of amplitudes A_1 and A_2 respectively.

The ratio of these two signals is: $\frac{A_1}{A_2}$

Because we often we deal with ratios that change by many order of magnitude during a testing procedure, the decibel scale is frequently used. By definition,

$$\left(\frac{A_1}{A_2}\right)_{dB} \equiv 20\ \log_{10}\frac{A_1}{A_2} \qquad (1)$$

$\log_{10} 1 = 0$, $\log_{10} 2 = 0.3010$ $\log_{10} 3 = 0.477$,

$\log_{10} 5 = 0.6989$ $\log_{10} A^n = n \log_{10} A$, $\log_{10} AB = \log_{10} A + \log_{10} B$

Example: $\frac{A_1}{A_2} = 2$ is equivalent to +6 dB

because $\left(\frac{A_1}{A_2}\right)_{dB} \equiv 20 \log_{10} 2 = 6$

We say, A_1 is +6 dB relative to A_2.

Example: A signal A_1 10 times as large as A_2 is + 20 dB
A signal A_1 one-tenth as large as A_2 is - 20 dB

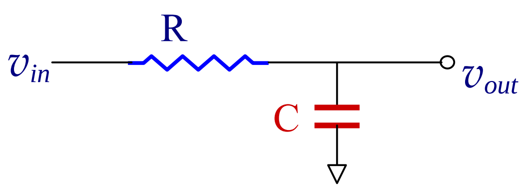
Comparison of Power levels

If P_1 and P_2 represent the power of two signal levels, their ratio in decibels is defined by,

$$\left(\frac{P_1}{P_2}\right)_{dB} \equiv 10 \log_{10} \frac{P_1}{P_2} \qquad (2)$$

II.2 Low–pass filter

If P_1 and P_2 represent the power of two signal levels, their ratio in decibels is defined



Output voltage (magnitude and phase)

$$v_{out} = \frac{v_{in}}{R + Z_C} Z_C = \frac{v_{in}}{R + \frac{1}{j\omega C}} \frac{1}{j\omega C} = \frac{1}{j\omega CR + 1} v_{in}$$

$$v_{out} = \frac{1}{\sqrt{(\omega RC)^2 + 1}} e^{j\varphi} v_{in} \quad \text{where} \quad \varphi = \tan^{-1}(-\omega RC) \qquad (3)$$

That is, the output voltage v_{out} lags the input voltage v_{in}.

The ratio of the input and output voltage magnitude is given by

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{1}{\sqrt{(\omega RC)^2 + 1}} \qquad (4)$$

II.3 Bode-plots and the 3dB breakpoint

RC is the characteristic time response of the low-pass RC circuit.

At $\omega = 1/RC$:

- The ratio $|v_{out}/v_{in}|$ drops to $1/\sqrt{2} = 0.7$
  In the decibels scale, this ratio is equal to $20\ \log_{10}(0.7) = -3$;
  that is $|v_{out}/v_{in}|_{dB} = -3$ dB
- The change in phase is 45^{o}.

  The frequency at which the output voltage drops by -3 dB is referred to as the "-3 dB breakpoint" of a filter (or of any circuit that behaves as a filter.).

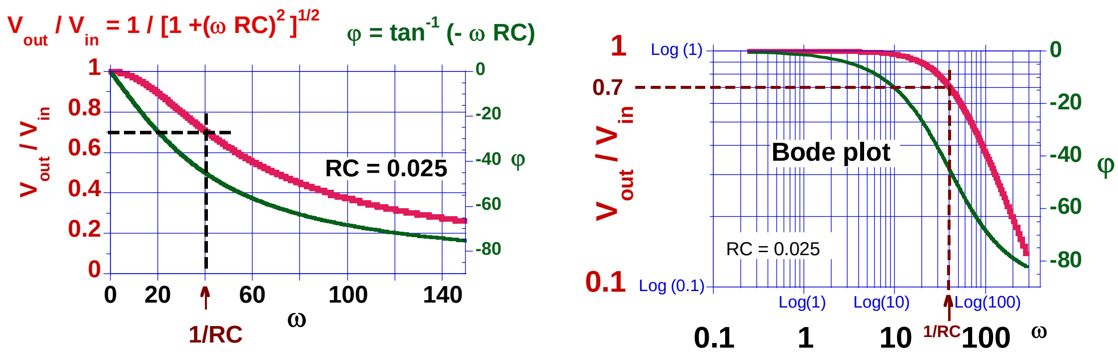


Fig. 1 Left: Frequency response of a low pass filter. **Right:** The right figure displays the same data but in a logarithmic scale.

II.4 Voltage drop at high frequencies (in decibels)

Let's express the ratio of voltages in decibels. From expression (4) we obtain,

$$\left|\frac{v_{out}}{v_{in}}\right|_{dB} = 20\log_{10}\frac{|v_{out}|}{|v_{in}|} = 20\log_{10}\frac{1}{\sqrt{(\omega RC)^2+1}} = -10\log_{10}\left[(\omega RC)^2+1\right]$$

At high frequencies (ω>1/RC),

$$\left|\frac{v_{out}}{v_{in}}\right|_{dB} = 20\log_{10}\frac{|v_{out}|}{|v_{in}|} \xrightarrow[\text{large }\omega]{} = -20\log_{10}(\omega RC)$$

$$\underbrace{\left|\frac{v_{out}}{v_{in}}\right|_{dB}} \xrightarrow[\text{large }\omega]{} -20\ \underbrace{\log_{10}(\omega)} - 20\log_{10}(RC) \qquad (5)$$

We expect, then, that at large frequencies (i.e. for ω > 1/RC), a plot of $|v_{out}/v_{in}|_{dB}$ vs $\log_{10}(\omega)$ should be a linear decreasing, with a slope of 20. This is in fact displayed in the figure below.

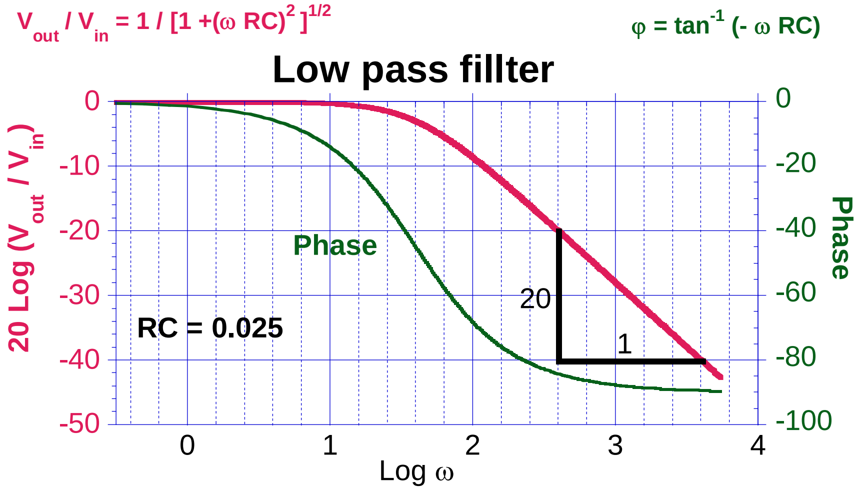


Fig. 2 Linear decrease of the output signal (in decibels) at high frequencies.

II.4A Rolloff per octave

As in music,

in **one octave** the **frequency is doubled**. (6)

How much does the voltage ration changes (in decibels) per octave?
Answer:

In one octave the horizontal axis in Fig. 3 changes by
$\left[\log_{10}(2\omega) - \log_{10}(\omega)\right] = \log_{10}(2) = 0.3010$.

Since the slope is -20, a change in the horizontal axis by 0.30 will give a vertical change equal to -20 x 0.30 = -6 dB

Accordingly, we say:

In a RC low-pass filter the output voltage drops -6 dB per octave (7)

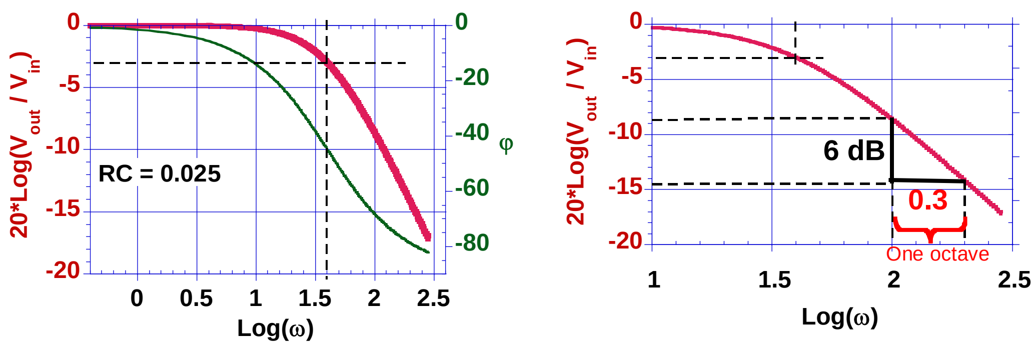


Fig. 3 Left: *Linear drop of the output/input voltage ration (in decibels) at high frequencies in a logarithmic scale.* ***Right:*** *Same data but in a zoom-in scale.*

II.4B Decibels per decade

In a **decade**, the **frequency changes by a factor of 10.** (8)

The corresponding change in the horizontal axis of Fig. 2 above is

$$[\log_{10}(10\omega) - \log_{10}(\omega)] = \log_{10}(10) = 1$$

Since the slope is -20, a change of 1 in the horizontal axis will correspond to a vertical axis equal to -20 x 1 = -20 dB

Accordingly, we say:

In a RC low-pass filter the output voltage drops -20 dB per decade (9)

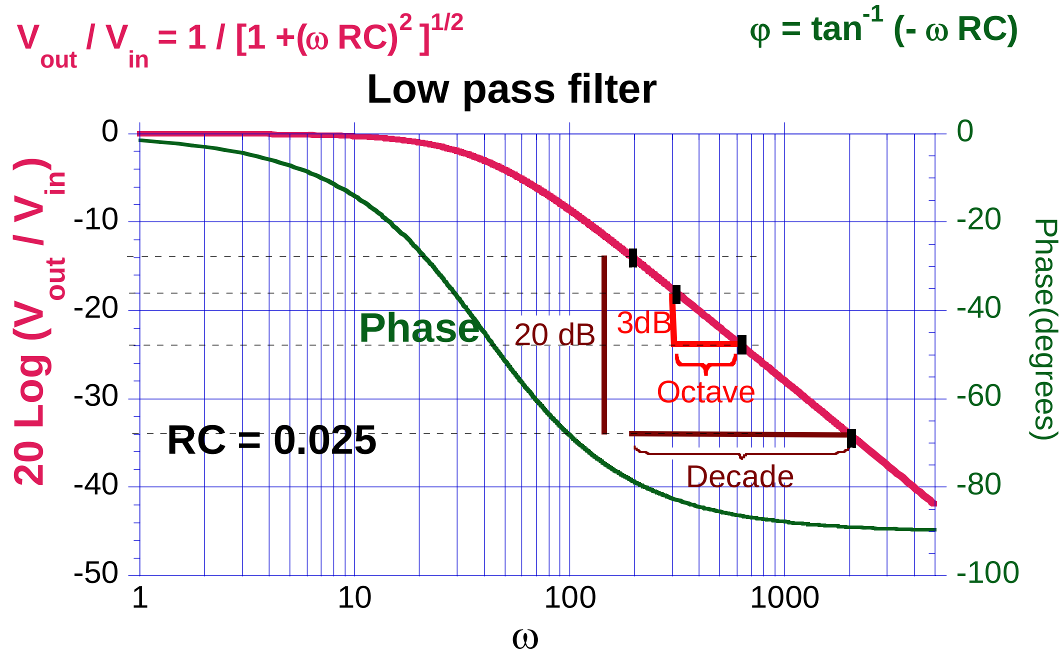


Fig. 4 Response from a single-stage low-pass filter

II.5 Dual-stage low–pass filter

TASK: Derive an expression for v_{out} in terms of v_{in}.

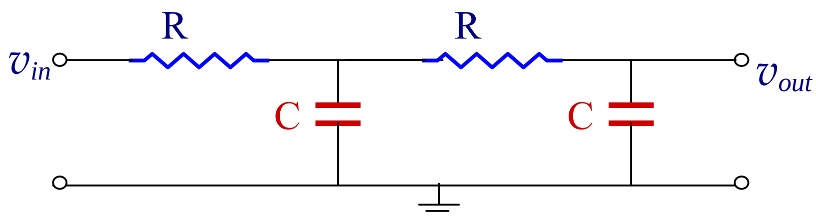


Fig. 5 Two-stage low-pass filter

III. EXPERIMENTAL CONSIDERATIONS

III.1 RC FILTER

III.1A Single-stage filter

Implement a RC Low-Pass filter. (You may use C= 0.22 μF and R = 1 KΩ; of course, feel free to try other values.) Use a signal generator to provide a sinusoidal input signal v_{in} of ~ 400 mV amplitude, and check with the oscilloscope whether the output voltage leads or lags the input voltage. Suggestion: Monitor v_{in} and v_{out} in the oscilloscope's channels 1 and 2, respectively.

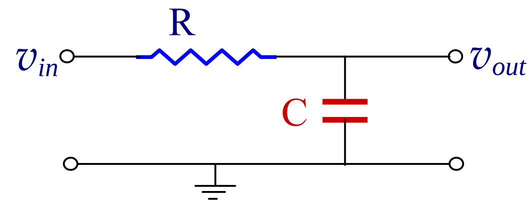


Fig. 6 Low-pass RC filter

- Make a Bode plot of the magnitude and phase of v_{out}/v_{in} as a function of the angular frequency. Make sure to get data over several decades of frequency values.
  To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.

III.1B Two-stage low-pass filter.

Implement two RC Low-Pass filters in cascade. Repeat the same measurement requested in part 1A.

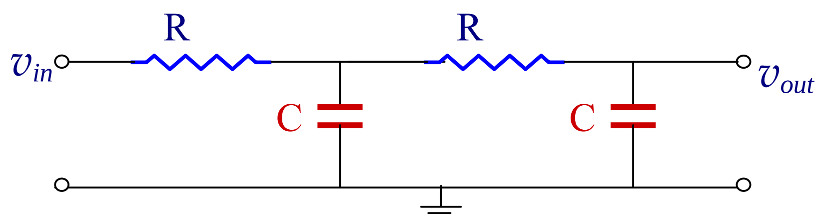


Fig. 7 Two low-pass RC filters.

- Make a Bode plot of the magnitude and phase of v_{out}/v_{in} as a function of the angular frequency. Make sure to get data over several decades of frequency values.
  To measure the phase, use the same technique used in Experiment-1.
- Determine experimentally the frequency at which the 3dB breaking point occurs. Compare your experimental results with the expected theoretical value.
- Determine experimentally the change of the output signal in dB per octave and per decade.

III.2 OPERATIONAL AMPLIFIERS and COMPARATORS

III.2A Comparator (with no feedback)

III.2B Comparator with positive feedback: Schmitt trigger

Op amps are widely used to amplify dc or ac signals. In this lab session, we will use LM358AP whose terminal connections are shown in Fig. 3.

$$v_{out} = A\,(v_{in(+)} - v_{in(-)})$$

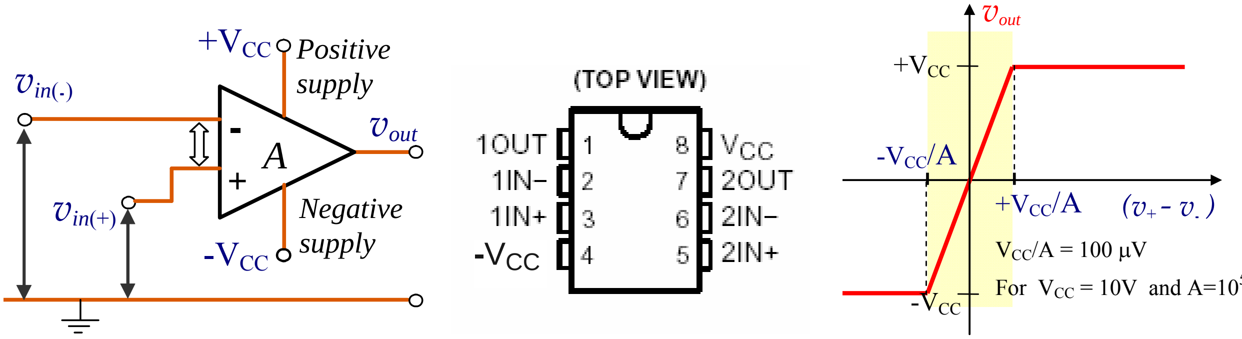


Fig. 3 Left: Op-amp terminals. **Center:** Pin assignment for the dual op-amp LM358AP. Use Vcc=12V. **Right:** Input/output volatge characteristics.

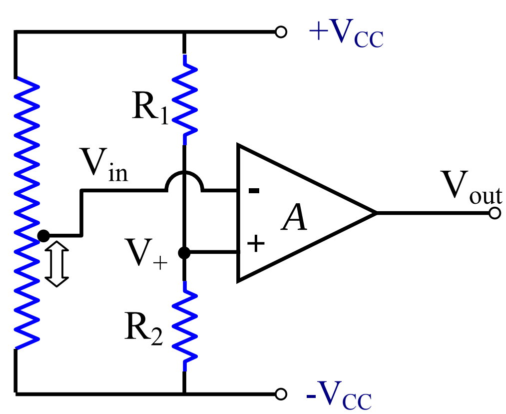
APPLICATIONS: COMPARATORS and SCHMITT TRIGGER

Many applications require knowing which or two signals is larger, upon which a terminal may need to be activated (or deactivated). In a digital voltmeter, for example, in order to convert an analog signal to a digital number, the unknown voltage is applied to one input of a comparator, while a linear ramp is applied to the other input. A digital counter counts the cycles of a clock while the ramp is less than the unknown voltage and display the result when both signals are equal. The resultant count is proportional to the unknown voltage. In this session we will build first a simpler comparator and then a more reliable Schmitt Trigger.

III.2A Comparator (with no feedback)

The circuit shown in the figure compares two voltages (V_{in} and V_+) and amplifies the difference between them. The resistors R_1 and R_2 establish the switching point at which the output will change state (V_{CC} to $-V_{CC}$, or vice versa.

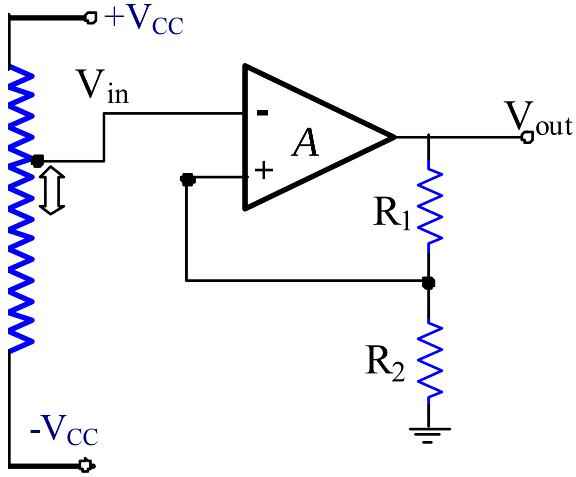
Set up such a circuit using resistors in the 1KΩ to 100 KΩ range, determine the trigger point voltage, and examine the levels at the output and inputs. Sketch a plot of V_{out} vs V_{in}.
How would you use that circuit to measure a resistor?

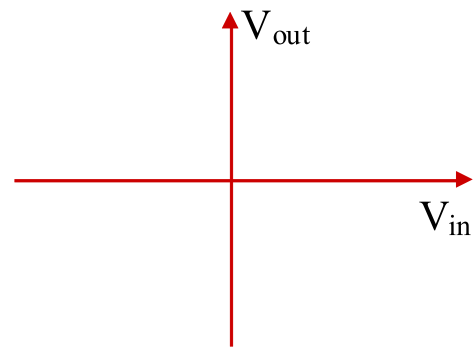


Comparator with positive feedback: Schmitt trigger

The simple comparator presents disadvantage when the input voltage is noisy, causing the output to make several transitions as the input passes through the trigger point. This problem can be solved by using *positive feedback*.

The effect of R_1 and R_2 is to make the circuit have two thresholds, depending of the output state.
Set up such a circuit using resistors in the 1KΩ to 30 KΩ range, determine the trigger point voltage, and examine the levels at the output and inputs. Sketch a plot of V_{out} vs V_{in}.
Verify that the output depends both on the input voltage and on its recent history, an effect called **hysteresis**.


+VCC
Vin
-
A
+
Vout
R1
R2
-VCC


Vout
Vin