

08/19/09

PHYSICS 223

Exam-2

NAME _____

Please write down your name **also** on the back side of this exam

1. A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s.
- 1A.** How far apart (in units of cm) are two points x_1 and x_2 that, at a given time t_0 , differ in phase by $\pi/3$ rad?
- a) 11.7 b) 8.3 c) 23.4 d) 4.1 e) NA
- 1B.** What is the **phase difference** between two displacements Y_1 and Y_2 (at a certain point x_0) at times 0.5 ms apart? (Notice, you are not being asked for the values of Y_1 nor Y_2 ; just their relative phase difference.)
- a) π b) $\pi/2$ c) 23.4 d) 4.1 e) NA

Solution

$v = \lambda f$, $f = 500$ Hz and $v = 350$ m/s implies, $\lambda = 350/500 = 0.7$ m
 $Y = A \cos(kx - \omega t + \phi)$

a) Things happen at $t = t_0$

$\text{Phase}_1 = kx_1 - \omega t_0 + \phi$ $\text{Phase}_2 = kx_2 - \omega t_0 + \phi$
 $\text{Phase}_1 - \text{Phase}_2 = [kx_1 - \omega t_0 + \phi] - [kx_2 - \omega t_0 + \phi] = [k(x_1 - x_2)]$
 $\pi/3 \text{ rad} = [k(x_1 - x_2)]$ implies $x_1 - x_2 = (\pi/3)/k = (\pi/3) \lambda / 2\pi = \lambda/6 = 0.7/6 = 11.7 \text{ cm}$

b) Things happen at $x = x_0$ $\text{Phase}_1 = kx_0 - \omega t_1 + \phi$ $\text{Phase}_2 = kx_0 - \omega t_2 + \phi$

$\text{Phase}_1 - \text{Phase}_2 = [kx_0 - \omega t_1 + \phi] - [kx_0 - \omega t_2 + \phi] = [\omega(t_2 - t_1)] = [2\pi f(t_2 - t_1)]$
 $(t_2 - t_1) = 0.5 \text{ ms}$ implies, $\text{Phase}_1 - \text{Phase}_2 = [2\pi \cdot 500 \text{ Hz} \cdot (0.5 \times 10^{-3} \text{ s})] = \pi/2$

-
2. Waves along a particular string of uniform linear mass density are described by the following Wave Equation $\frac{\partial^2 y}{\partial x^2} - \frac{1}{4 \text{ m}^2/\text{s}^2} \frac{\partial^2 y}{\partial t^2} = 0$, where all the quantities are expressed in MKS system.

2A If the tension and the linear mass density of the string are kept constant, which of the following expression(s) is (are) true?

- a) Waves of higher frequency travel at the same speed than waves of lower frequency.
- b) The greater the wavelength, the slower speed of the wave
- c) The greater the wavelength the faster speed of the wave.
- d) All the statements above are incorrect.

2B Which of the following waves is (are) solutions of the wave equation given above? (All the quantities below are expressed in MKS units.)

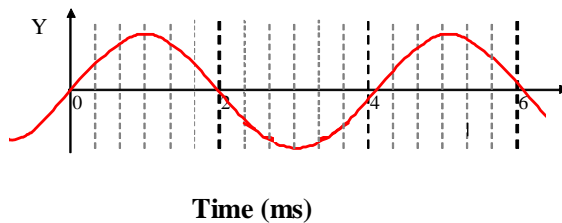
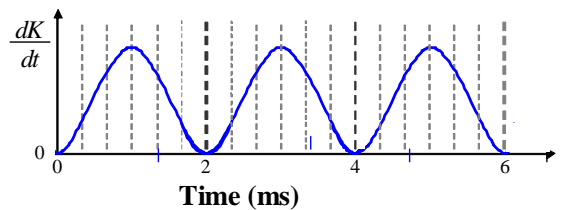
- a) $y_1 = 12 \sin(3x - 6t)$,
- b) $y_2 = 5 \cos(x - 4t)$,

c) $y_3 = \text{SIN}(2x + 1.0t)$

d) None of the above waves satisfy the given wave equation

3. A harmonic wave $y = A\text{SIN}(kx - \omega t)$ of wavelength $\lambda = 0.1 \text{ m}$ is set along a string that has a linear mass density of $\mu = 2 \text{ g/m}$. The rate at which the kinetic energy is carried along by the wave is given by $\frac{dK}{dt} = \frac{1}{2}\mu v \omega^2 A^2 \text{COS}(kx - \omega t)$, where v stands for the velocity of the wave.

- 3A The figure below is a sketch the rate $\frac{dK}{dt}$ at which kinetic energy passes through a particular mass element (at a particular position), plotted as a function of time. Use the second graph below to sketch y vs $time$ (corresponding to a segment at an arbitrary particular position.)



- 3B Calculate the speed v of the wave (in m/s).

a) 25 b) 50 c) 12 d) 100 e) NA

4. 4A. Two connected strings with linear mass densities that are related by $\mu_1 = 4 \mu_2$ are under some tension. (Assume the wires are very long so we can omit the effects of the reflected wave from the right end)

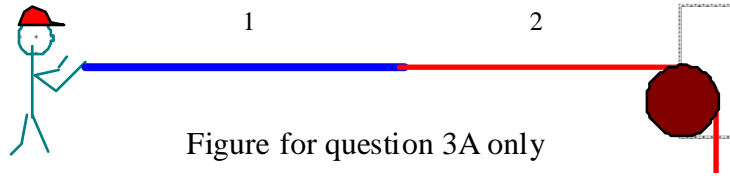


Figure for question 3A only

A wave of frequency 120 Hz and wavelength $\lambda = 10$ cm is set on the left wire

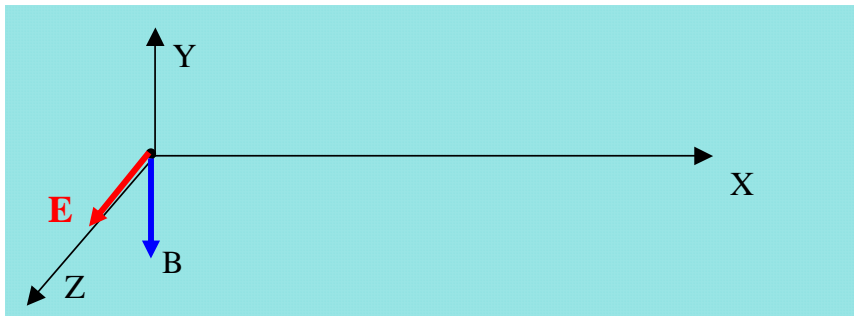
- a) The wavelength on the right side will be 40 cm.
- b) The velocity of the wave will be the same in both sides
- c) The wavelength of the wave is the same at both sides.
- d) The tension on the wire at the left side is twice the tension on the left side.
- e) NA

4B. Write an equation describing a sinusoidal transverse wave traveling on a cord in the $(-X)$ direction with a wavelength of 10 cm, a frequency of 120 Hz and an amplitude of 2 cm. Assume the cord has a uniform linear mass density. Write clearly and include the proper units.

ANSWER
$$y(x, t) = 0.02m \text{ SIN} \left(\frac{2\pi}{0.1m} x + 2\pi \frac{120}{\text{sec}} t \right)$$

5. The electric field of a certain electromagnetic field is given by

$$E_x = 0, \quad E_y = 0, \quad E_z = 48 \text{ V/m COS} (10^7 \text{ m}^{-1} \mathbf{x} - \omega \mathbf{t})$$



5A. In the diagram above draw the electric field and the magnetic field vectors at the position $x=0$ and at the instant $t=0$. In addition, write down below the expressions for the component of the magnetic field of this wave

$$B_x = 0 \qquad B_z = 0$$

$$B_y = -1.6 \times 10^{-7} \text{ T COS} (10^7 \text{ m}^{-1} \mathbf{x} - \omega \mathbf{t})$$

5B. The wavelength λ of the electromagnetic wave is,

- a) 10^7 m b) 628 nm c) 10^{-7} m d) 209 nm e) NA
-

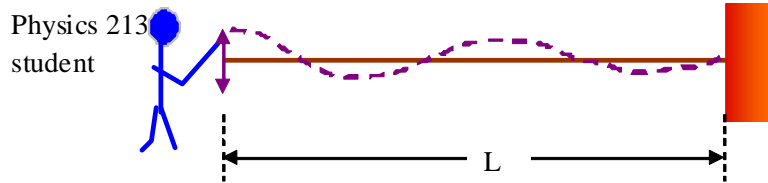
6. 6A What phase difference between two otherwise identical traveling waves, moving in the same direction along a stretched string, will result in the combined wave having an amplitude of the two combining waves? (Give your answer in degrees)

- a) 70 b) 120 c) 0 d) 45 e) NA

6B. Two identical traveling waves, moving in the same direction, are out of phase by $\pi/2$ rad. What is the amplitude of the resultant wave in terms of the common amplitude y_m of the two combining waves? :

- a) $1.4 y_m$ b) $1.9 y_m$ c) $1.0 x y_m$ d) 0 e) NA
-

7. The figure below shows a 0.5 meter long string (with one end attached to the wall) of linear mass density 100 g/m, under a tension of 1.6 N. At one end a person shakes the string up and down with an amplitude of 4 cm.



7A The speed of the reflected waves traveling along the string is:

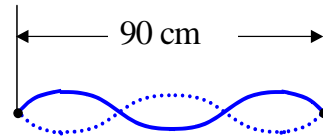
- a) 4 m/s b) 0.12 m/s c) 5 m/s d) 2 m/s e) NA

7B The person shakes the string with either 5 Hz or 6 hertz (one case at a time). Indicate which expression is correct:

- a) Both cases satisfy the conditions to establish resonant standing waves in the string
b) Since the reflected waves travel at a different speed than the incident wave, no resonance pattern is possible to establish along the string.
c) Since only one end of the string is fixed, no resonant standing wave can be set in the string.
d) Only one case satisfies the conditions for a resonant standing wave
e) NA
-

8. **8A** A string is fixed at both ends. The following list is missing some of its resonance frequencies: 150 Hz, 225 Hz, 300 Hz, 375 Hz, 525 Hz. One of the missing frequencies is:
- a) 100 Hz b) 50 Hz c) 450 Hz d) 400 Hz e) NA

- 8B.** A nylon guitar string has a linear density of 10 g/m and is under a tension of 145 N. The fixed supports are 90 cm apart. The string is oscillating in the standing wave pattern shown in the figure. Calculate the frequency of the traveling waves whose superposition gives this standing wave.



- a) 240 Hz b) 100 Hz c) 200 Hz d) 23 Hz e) NA

9. **9A** The A string of a violin is a little too tight stretched. Six beats per second are heard when the string is sounded together with a tuning fork that is oscillating accurately at concert A (440 Hz). What is the frequency of the violin string oscillation?

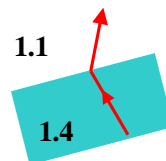
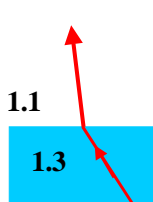
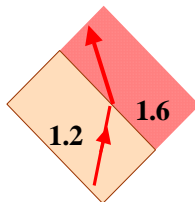
- a) 446 Hz b) 3 Hz c) 452 Hz d) 437 Hz e) NA

- 9B.** A tuning fork (fork #1) of unknown frequency makes four beats per second with a standard fork (fork #2) of frequency 384 Hz. The beat frequency decreases when a small piece of wax is put on a prong of fork #1. What is the frequency of fork #1?

- a) 382 Hz b) 380 Hz c) 386 Hz d) 388 Hz e) NA

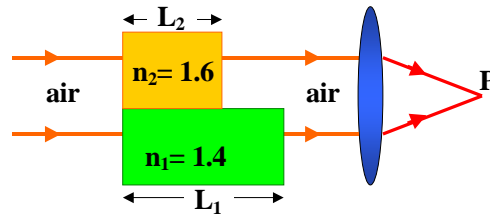
BONUS QUESTION

- B1. (5 points)** Choose the correct answer to indicate if the following 3 drawings show physically possible refraction. (F = false, T = true.)



- a) F T F
 b) F F T
 c) T F F
 d) F F T
 e) NA

B2. (5 points) Two waves of light in air of wavelength 600 nm are initially in phase. They then travel through plastic layers as shown in the figure, with $L_1 = 4 \mu\text{m}$, $L_2 = 3.5 \mu\text{m}$, $n_1=1.4$ and $n_2=1.6$.



Which of the following expression is correct?

- a) Light travels faster in medium 2 than in medium 1.
- b) The frequency of the electric field oscillations is higher in medium 2 than in medium 1.
- c) The wavelength in medium 1 is the same than in medium 2.
- d) Light propagates with the same speed regardless it travels in medium-1, medium-2 or in air.
- e) NA.

1 hertz = 1 Hz = 1 oscillation per second

$$\text{nm} = 10^{-9} \text{ m}, \quad \mu\text{m} = 10^{-6} \text{ m}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$$

$$c = 3 \times 10^8 \text{ m/s}$$

Identity:

$$\begin{aligned} E_m \cos(A) + E_m \cos(B) &= \\ &= 2E_m \cos\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \end{aligned}$$

$$\begin{aligned} E_m \sin(A) + E_m \sin(B) &= \\ &= 2E_m \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right) \end{aligned}$$

Waves traveling along a string: $v = (T/\mu)^{1/2}$

$$\text{Doppler Effect: } f_{\text{observer}} = f_{\text{source}} \frac{V_{\text{sound}} \pm v_{\text{observer}}}{V_{\text{sound}} \pm v_{\text{source}}}$$

Beat frequency:

$$\text{Let } \omega_2 = \omega_1 + \Delta\omega$$

$$\begin{aligned} p &= p_0 \cos(\omega_2 t) + p_0 \cos(\omega_1 t) \\ &= 2p_0 \cos\left(\frac{\Delta\omega}{2} t\right) \cos\left(\frac{\omega_1 + \omega_2}{2}\right) \end{aligned}$$

Electromagnetic waves:

$$E_y = E_m \cos(kx - \omega t)$$

$$B_z = B_m \cos(kx - \omega t)$$

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad B_m = \frac{E_m}{c}$$

$$\text{Poynting vector} \quad \vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$$

$$\text{Magnitude of the Poynting vector} \quad |S| = \epsilon_0 c E^2$$

$$\text{Average:} \quad \langle S \rangle = \frac{1}{2} \epsilon_0 c E_m^2$$

Waves

$$y = A_m \cos(kx - \omega t + \delta)$$

A_m = amplitude,

$(\omega t + \phi)$ = phase of the motion,

δ is the phase constant, and ω = angular frequency.

$$v = \lambda f$$

$$T = \frac{1}{f} \quad (\text{Period})$$

$$\omega = \frac{2\pi}{T} = 2\pi f \quad (\text{angular frequency})$$

$$k = \frac{2\pi}{\lambda}$$

$$\text{Snell's law } n_1 \sin \theta_1 = n_2 \sin \theta_2$$

$$\text{Index of refraction: } n = \frac{c}{v}$$

$$\text{Phase shift: } \Phi = kL$$

