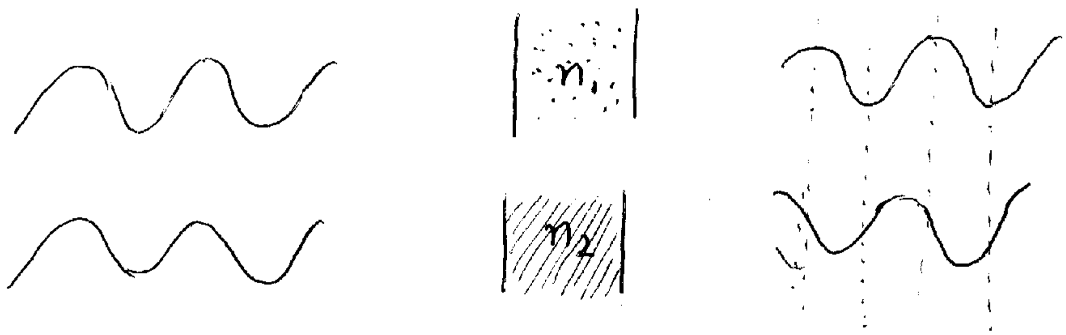
This lecture will not be included in the final exam

How to calculate the PHASE DIFFERENCE between two waves that have travelled through different media

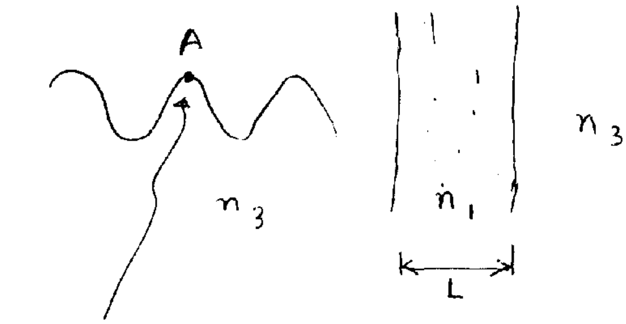


Method 1: To follow in time how a wavefront (wavefront A for example) travels through the different media and find out with which wavefront of wave-2 (it might be wavefront P) it will interfere when the waves come out from the slab. The initial phase difference (between A and P) will be the phase difference of the wave coming out from the slab.

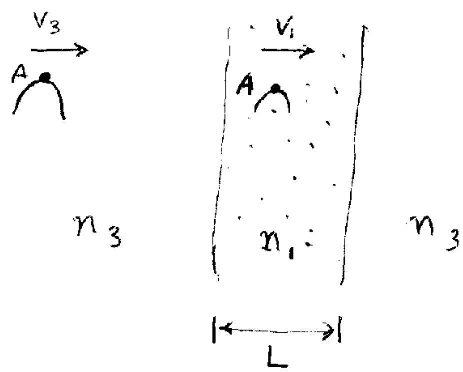
Method 2: a) To simply take a snapshot of the waves at a arbitrary time t.

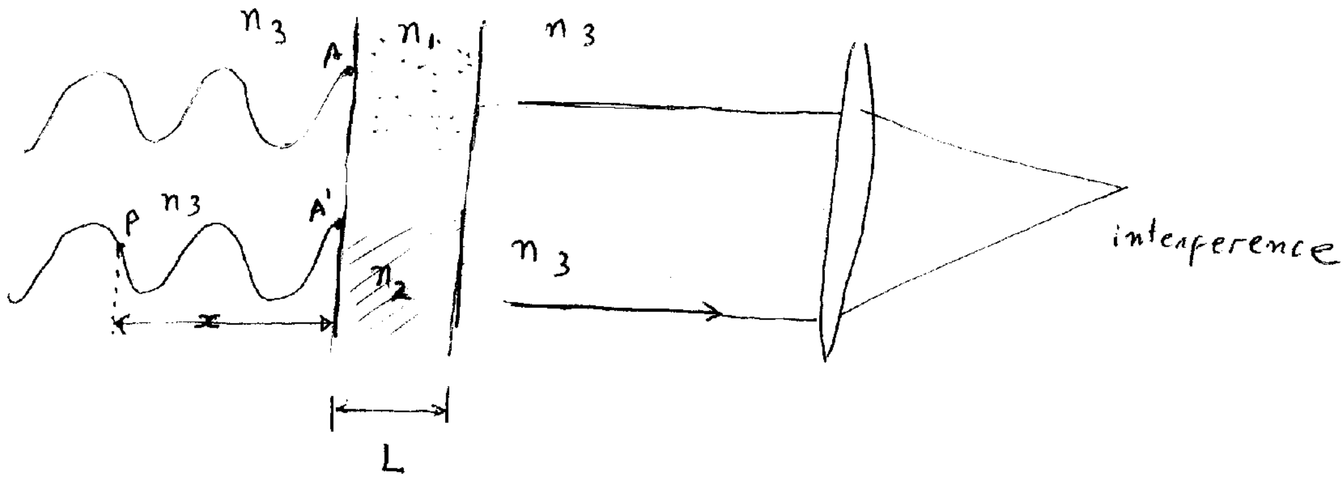
b) Evaluate the phase difference between the points located at the side-walls of the slabs.

METHOD - 1

- 

This crest will travel with speed V_3 (while in the medium n_3) and will continue (still being a crest) traveling with a speed V_1 (while inside the medium n_1)

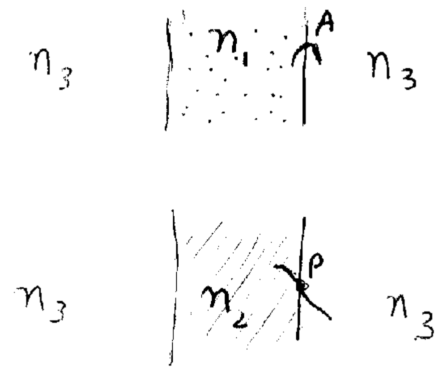


- 

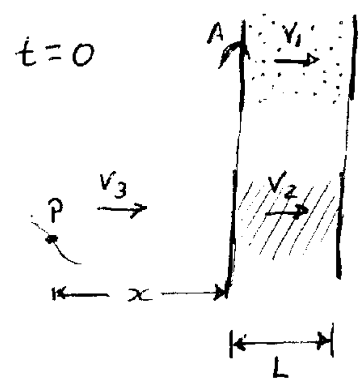
crest A will not interfere with crest A' (because they travel at different speed through the slab of thickness L)

The frontwave A rather will interfere with a wavefront P (of the other wave) located a distance "x" back.

The wavefront P is such that it will arrive at the right side of the slab simultaneously with the wavefront A



We determine "x" through the following condition:



Simultaneous arrival at the right side of the slab implies (assuming $v_2 > v_1$)

$$\frac{L}{V_1} = \frac{L}{V_2} + \frac{x}{V_3}$$

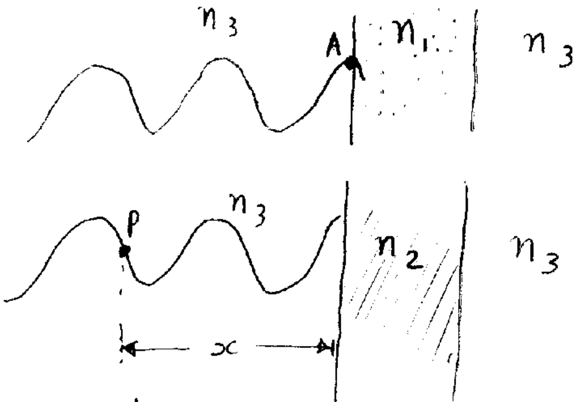
$$\Rightarrow x = V_3 L \left(\frac{1}{V_1} - \frac{1}{V_2} \right)$$

OR (using $k = \frac{2\pi}{\lambda} = \frac{2\pi}{v/f}$ $\quad k = \frac{w}{v}$)

$$x = \frac{w}{k_3} L \left(\frac{k_1}{w} - \frac{k_2}{w} \right)$$

$$\boxed{x = \frac{L}{k_3} (k_1 - k_2)}$$

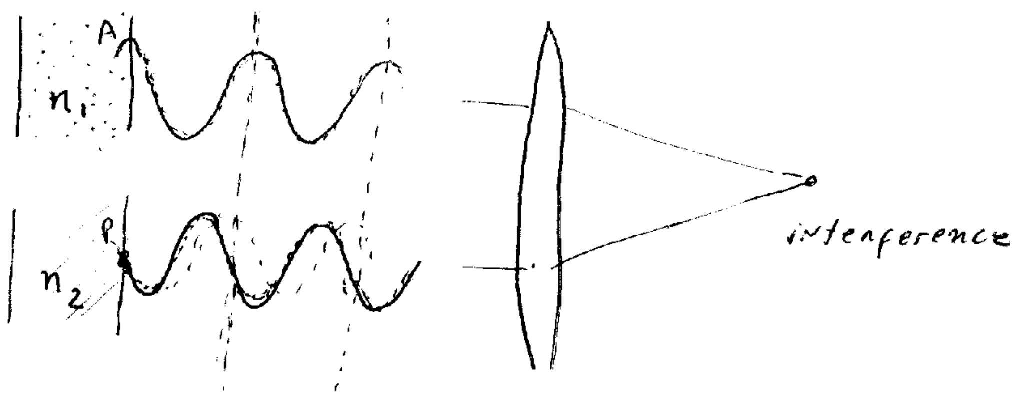
At $t=0$



$x = \frac{L}{k_3}(k_1 - k_2)$

Notice: the initial ($t=0$) phase difference between the wavefronts A and P will be the phase difference of the waves that interfere

At $t = \frac{L}{v_1}$



the phase difference between A and P, at $t=0$, is $\Lambda = k_3 x$

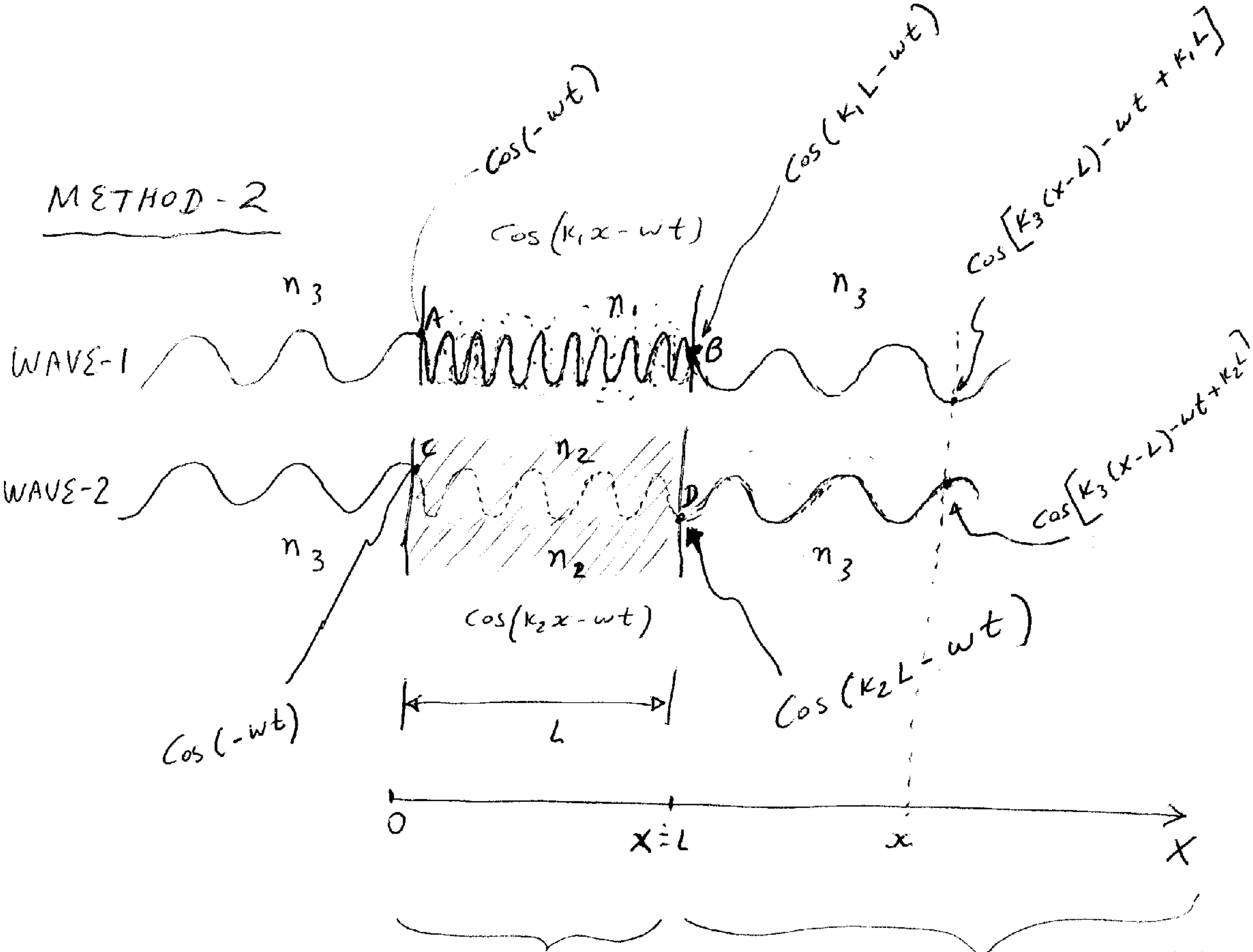
$$\Lambda = k_3 x$$

$$= k_3 \frac{L}{k_3}(k_1 - k_2) = L(k_1 - k_2)$$

$$\boxed{\Lambda = L(k_1 - k_2)}$$

Notice that, in this method, all what mattered was how fast a wavefront was traveling through the different media, regardless of the "shape" of the waves.

METHOD - 2

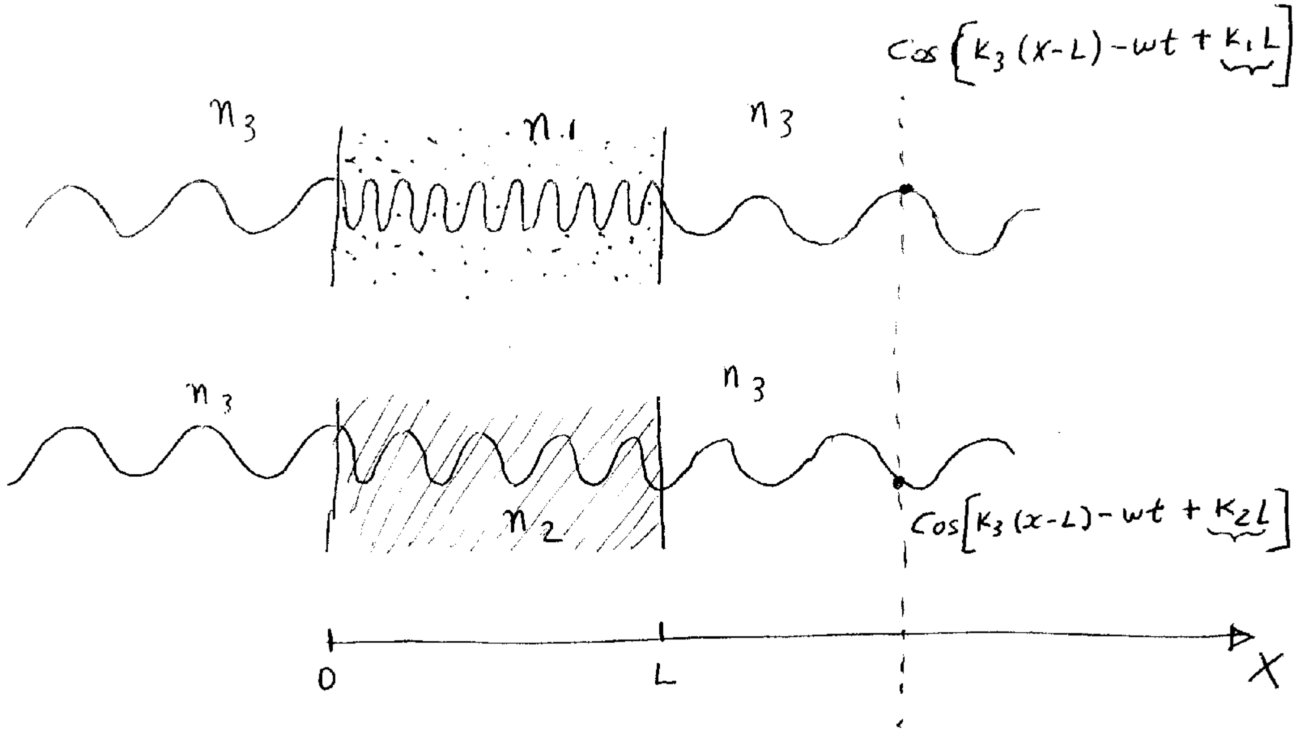


The electric fields at A and B are vibrating at the same frequency but with a phase difference $k_1 L$

The electric fields at C and D are vibrating at the same frequency but with a phase difference $k_2 L$

In this region both waves take the form $\cos(k_3 x - \omega t + \delta)$

But each wave is expressed mathematically in such a way that it reflects its phase at points B and D respectively.



thus, the phase difference is $\delta = L(k_1 - k_2)$

$$\boxed{v_1 < v_2} \Rightarrow \frac{c}{n_1} < \frac{c}{n_2} \Rightarrow \boxed{n_2 < n_1}$$

$$k = \frac{\omega}{v} \Rightarrow \boxed{k_1 > k_2}$$