Generation, Propagation and Detection of Electromagnetic fields.

The 3rd and 4th Maxwell equations are necessary

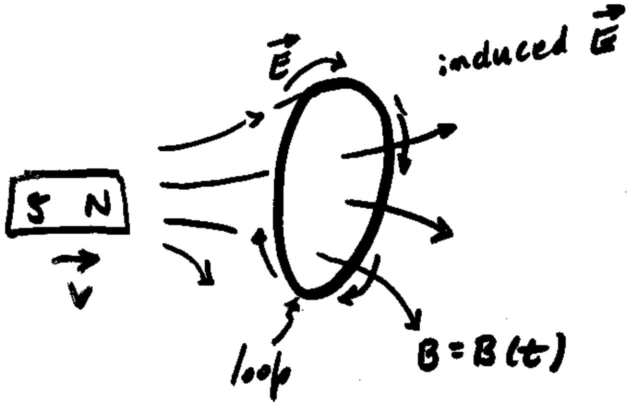
not only for generating the $\vec{E}$ and $\vec{B}$ fields from electrical currents ($\vec{j}$)

but also to sustain the propagation of these fields in source-free ($\vec{j}=0$) regions of space.

In this section we'll review

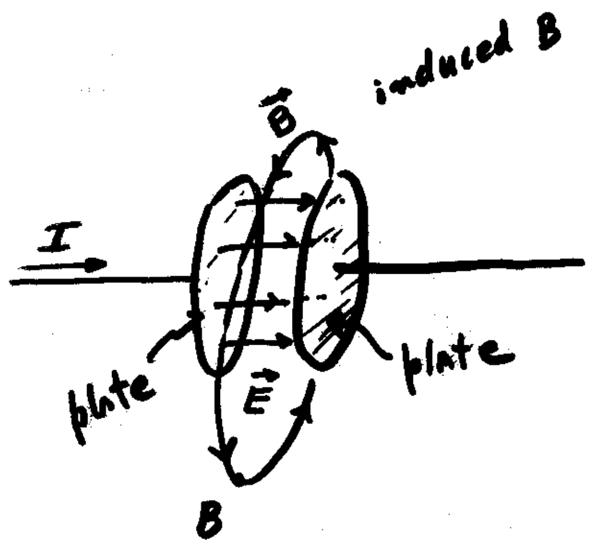
- Generation of EM waves
- Self-sustain propagation of EM waves
  the wave equation
- the Heinrich Hertz experiment
  Detection of EM waves

Generating electromagnetic waves
based on the time-dependent MAXWELL EQUATIONS



$$\int_{loop} \vec{E}\cdot d\vec{l} = -\frac{d}{dt}\int_{surface} \vec{B}\cdot d\vec{A}$$

You have control over $\vec{B}(t)$ (move the magnet) $\Rightarrow$ to generate $\vec{E}$

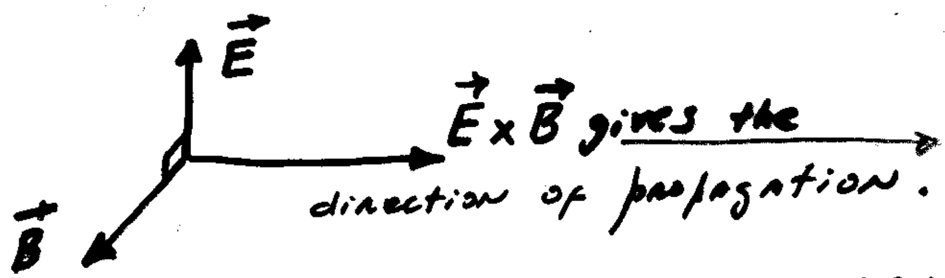


$$\int_{loop} \vec{B}\cdot dl = \mu_0 I + \mu_0 \frac{d}{dt}\epsilon_0 \int_{surface} \vec{E}\cdot dA$$

You have control over $\vec{E}(t)$ (change the current) $\Rightarrow$ to generate $\vec{B}$

Notice:

- The previous two graphs show that
  $\vec{E} \perp \vec{B}$
- We will not do it here, but it can be mathematically justified that the propagation of electromagnetic can be pictured in the following way



TRANSVERSE WAVE

QUESTIONS

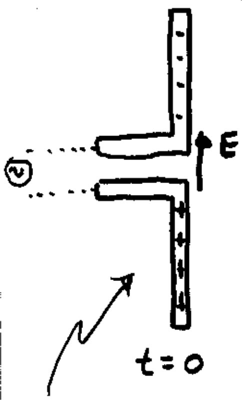
How do electromagnetic waves propagate?
(Sound waves need "substance" to propagate. You can NOT hear in vacuum. Do e-m waves need a medium to propagate?)

What is their speed?

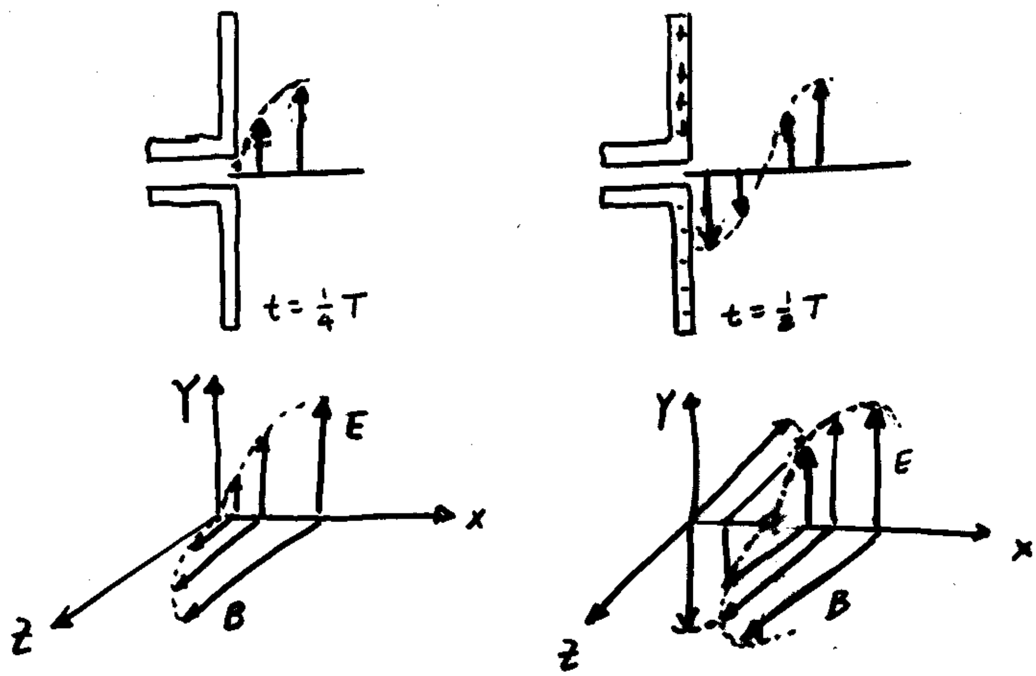
How to prove their existence?

Electromagnetic waves

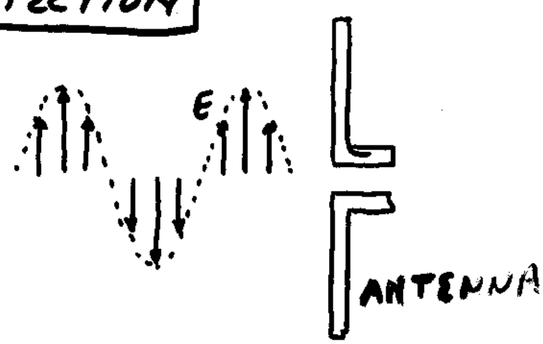
GENERATION



ANTENNA FED by an alternating-current generator

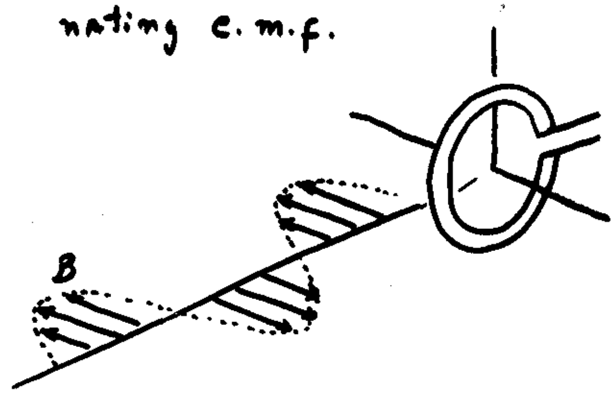


DETECTION

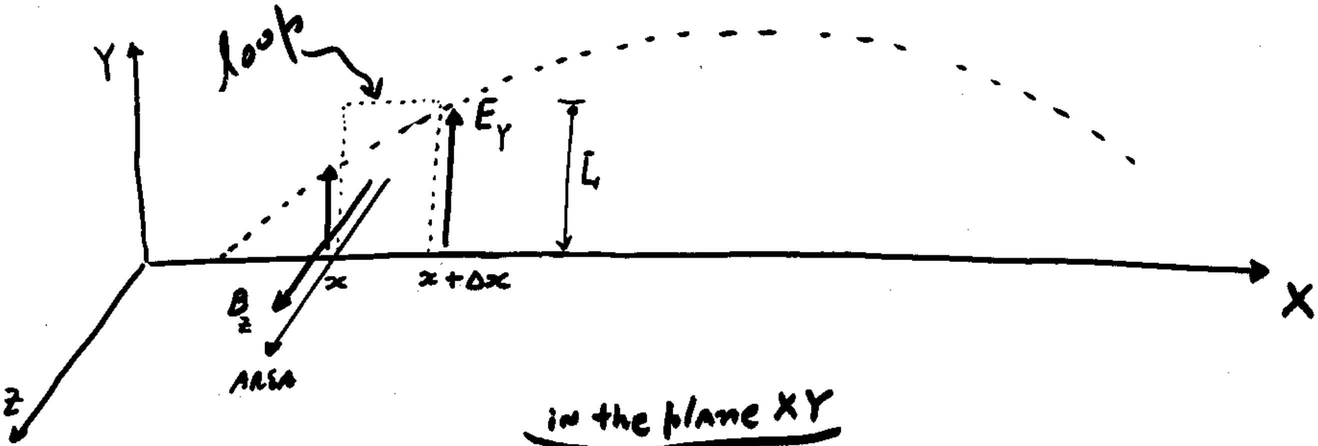


the alternating electric field of the wave produces an alternating urrent in the ntenna

the alternating magnetic field of the wave produces an alternating magnetic flux in the loop antenna, i.e. an alternating e.m.f.



Selfsustained propagation of electromagnetic waves



Let's consider a $\underbrace{\text{loop}}_{\text{in the plane XY}}$ of width $= \Delta x$ and height $= L$,

and apply Faraday's law:

$$\mathcal{E} = -\frac{d}{dt}\Phi_M$$

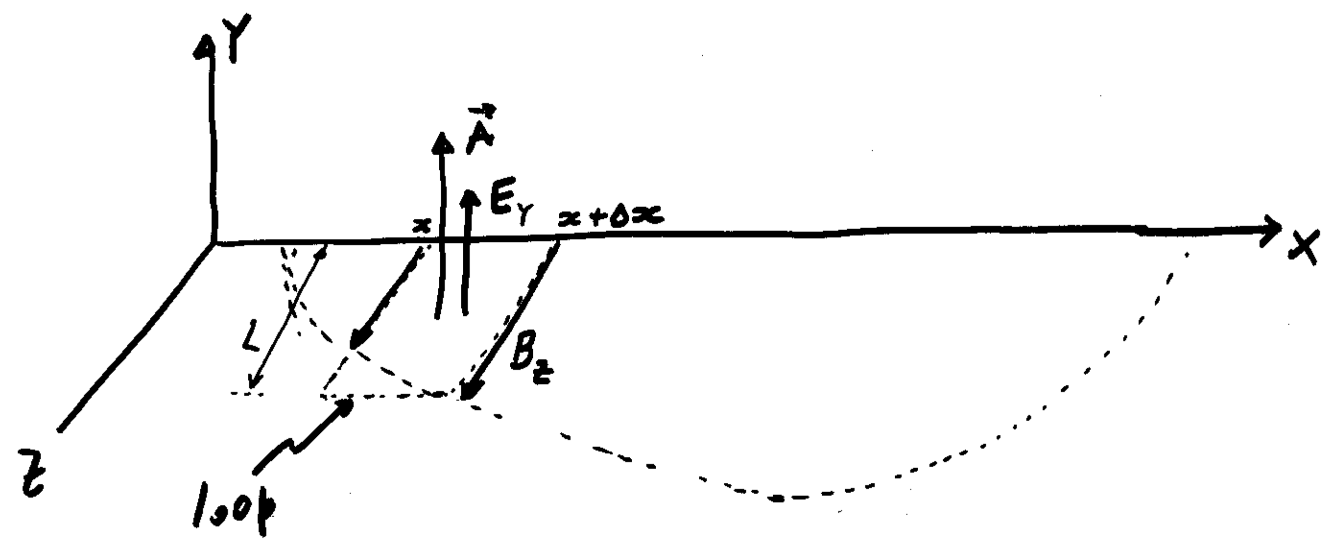
$$\mathcal{E} = \int_{loop} \vec{E}\cdot d\ell = L\,\underbrace{E_Y(x+\Delta x)} - \underbrace{L\,E_Y(x)}$$

$$\Phi_M = B_z A = B_z L\,\Delta x$$

$$L\left(E_Y(x+\Delta x) - E_Y(x)\right) = -L\,\Delta x\,\frac{\partial B_z}{\partial t}$$

$$\frac{E_Y(x+\Delta x) - E_Y(x)}{\Delta x} = -\frac{\partial B_z}{\partial t}$$

$$\Rightarrow \quad \frac{\partial E_Y}{\partial x} = -\frac{\partial B_z}{\partial t} \qquad (1)$$



Let's consider a loop of width Δx and length $= L$, and apply the modified Ampere's law (the one that includes the displacement current).

$$\underbrace{\int_{loop} \vec{B}\cdot d\vec{l}}_{} = \underbrace{\mu_0 i}_{=0} + \mu_0 \epsilon_0 \underbrace{\frac{d}{dt}\phi_E}_{}$$

because there is no current in the free space

$$= \frac{d}{dt} E_Y A = L\,\Delta x \frac{\partial E_Y}{\partial t}$$

$$L B_z(x) - L B_z(x+\Delta x)$$

Thus, we obtain:

$$-\frac{B_z(x+\Delta x) - B_z(x)}{\Delta x} = \frac{\partial E_Y}{\partial t} \Rightarrow \frac{\partial B_z}{\partial x} = -\mu_0 \epsilon_0 \frac{\partial E_Y}{\partial t} \qquad (2)$$

Let's combine ① and ②

From ① $\Rightarrow \dfrac{\partial^2 E_Y}{\partial x^2} = -\dfrac{\partial^2 B_z}{\partial x \partial t}$

From ② $\Rightarrow \dfrac{\partial^2 B_z}{\partial t \partial x} = -\mu_0 \epsilon_0 \dfrac{\partial^2 E_Y}{\partial t^2}$

$\Rightarrow \dfrac{\partial^2 E_Y}{\partial x^2} - \mu_0 \epsilon_0 \dfrac{\partial^2 E_Y}{\partial t^2} = 0$

The resulting equation can be writen as

$$\boxed{\frac{\partial^2 E_Y}{\partial x^2} - \frac{1}{\left(\frac{1}{\mu_0 \epsilon_0}\right)} \frac{\partial^2 E_Y}{\partial t^2} = 0}$$

Equation to be satisfied by the electric field E_Y

When compared with the wave equation

$$\frac{\partial^2 y}{\partial x^2} - \frac{1}{v^2} \frac{\partial^2 y}{\partial t^2} = 0$$

which has solutions of the form $y = y(x \pm vt)$

we realize that:

THE ELECTRIC FIELD E_Y constitutes a wave that propagates with velocity

$$v = \sqrt{\frac{1}{\mu_0 \epsilon_0}}$$

this is,

$$E_y = E_y\,(x \pm v\,t)$$

* EXERCISE: • Use equations ① and ② to obtain

$$\frac{\partial^2 B_z}{\partial x^2} - \frac{1}{\left(\frac{1}{\mu_0 \epsilon_0}\right)} \frac{\partial^2 B_z}{\partial t^2} = 0$$

- Verify that, in particular, the following wave

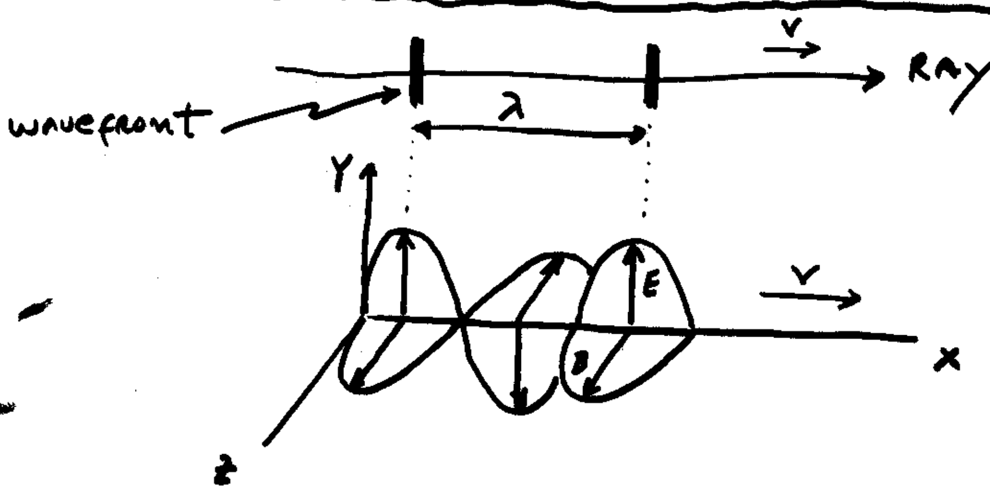
  $B_z = B_z\,(x - v\,t)$, where $v = \sqrt{1/\mu_0 \epsilon_0}$

  satisfies the equation above.

- Given the wave

  $B_z = \underbrace{1.6 \times 10^{-7}\,\text{Tesla}}_{\text{const}} \cos\,(kx - \omega t)$

  what should be the value of ω/k if we are told that B_z is the magnetic field of a propagating electromagnetic wave.



Pictorial representation of a sinusoidal electromagnetic wave

Do we know the values of ϵ_0 and μ_0?

Yes, indeed.

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m} \qquad \mu_0 = 1.26 \times 10^{-6}\ H/m$$

Let's calculate the velocity of an electromagnetic wave

$$\epsilon_0 \mu_0 = 11.1 \times 10^{-18}\ s^2/m^2$$

$$\frac{1}{\epsilon_0 \mu_0} = 0.09 \times 10^{18}\ m^2/s^2$$

$$\sqrt{\frac{1}{\epsilon_0 \mu_0}} = 0.3 \times 10^{9}\ m/s = 300{,}000\ Km/s$$

the speed of light!

With excitement Maxwell wrote:

"We can scarcely avoid the inference that light consists in the transverse undulation of the same medium which is the cause of electric and magnetic phenomena"

Notation $\quad c = 300{,}000\ Km/s$

Summary

MAXWELL EQUATIONS in VACUUM

$$\int_{\text{surface}} \vec{E}\cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\int_{\text{surface}} \vec{B}\cdot d\vec{A} = 0$$

$$\int_{\text{loop}} \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_{\text{surf}} \vec{B}\cdot d\vec{A}$$

$$\int_{\text{loop}} \vec{B}\cdot d\vec{\ell} = \mu_0 \int \vec{j}\cdot d\vec{A} + \mu_0 \epsilon_0 \frac{d}{dt}\int_{\text{surf}} \vec{E}\cdot d\vec{A}$$

E : electric field

B : magnetic field

$\vec{j}$: current density

$E = E(t) \xrightarrow[\text{produces}]{} B$

$B = B(t) \longrightarrow E$

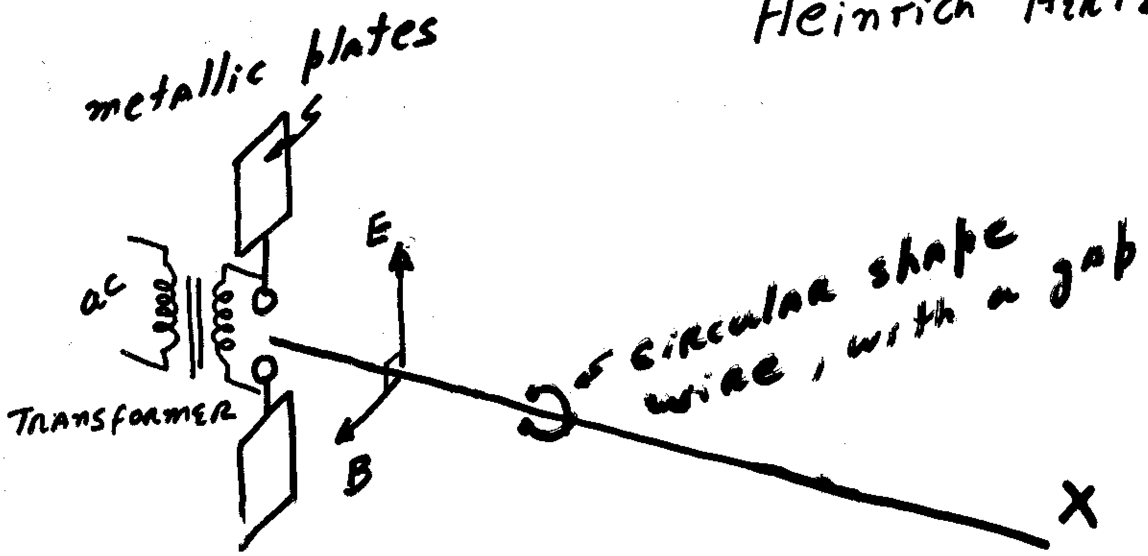
$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} = 0$$

$$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \qquad c = 300{,}000\ \text{km/s}$$

$E = E(x \pm ct)$ $\qquad$ $B = B(x \pm ct)$

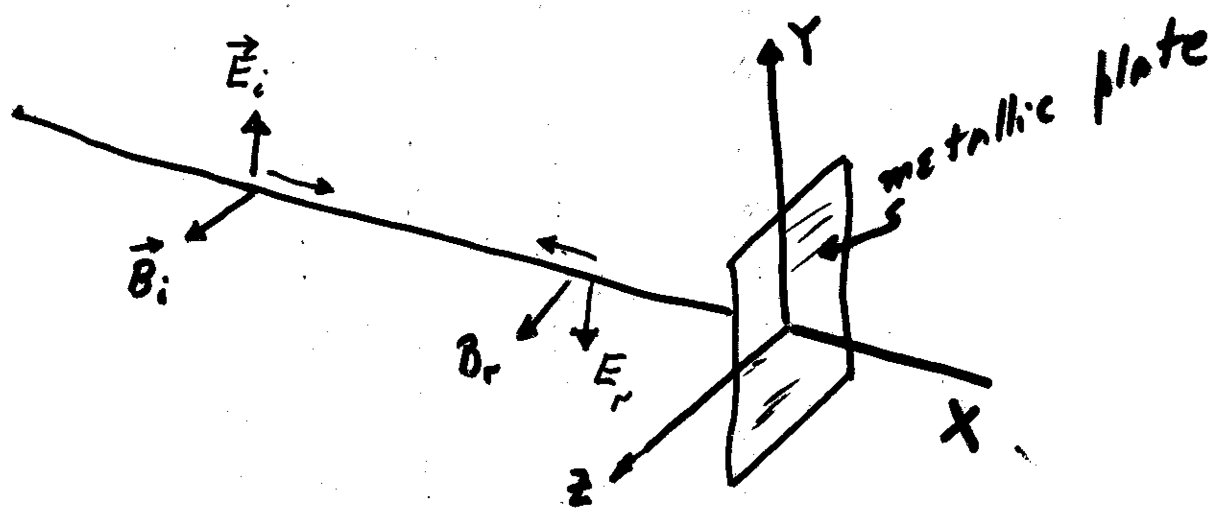
DETECTION of EM WAVES

Heinrich Hertz, 1888



$$E = E_m \sin(kx - wt)$$

w given by the transformer



$\rightarrow \quad E_i = E_m \sin(kx - wt)$

$\leftarrow \quad E_r = E_m \sin(kx + wt)$

E total: $E_y = 2E_m \sin kx \cos \omega t$

$\left(k = \frac{2\pi}{\lambda}\right)$

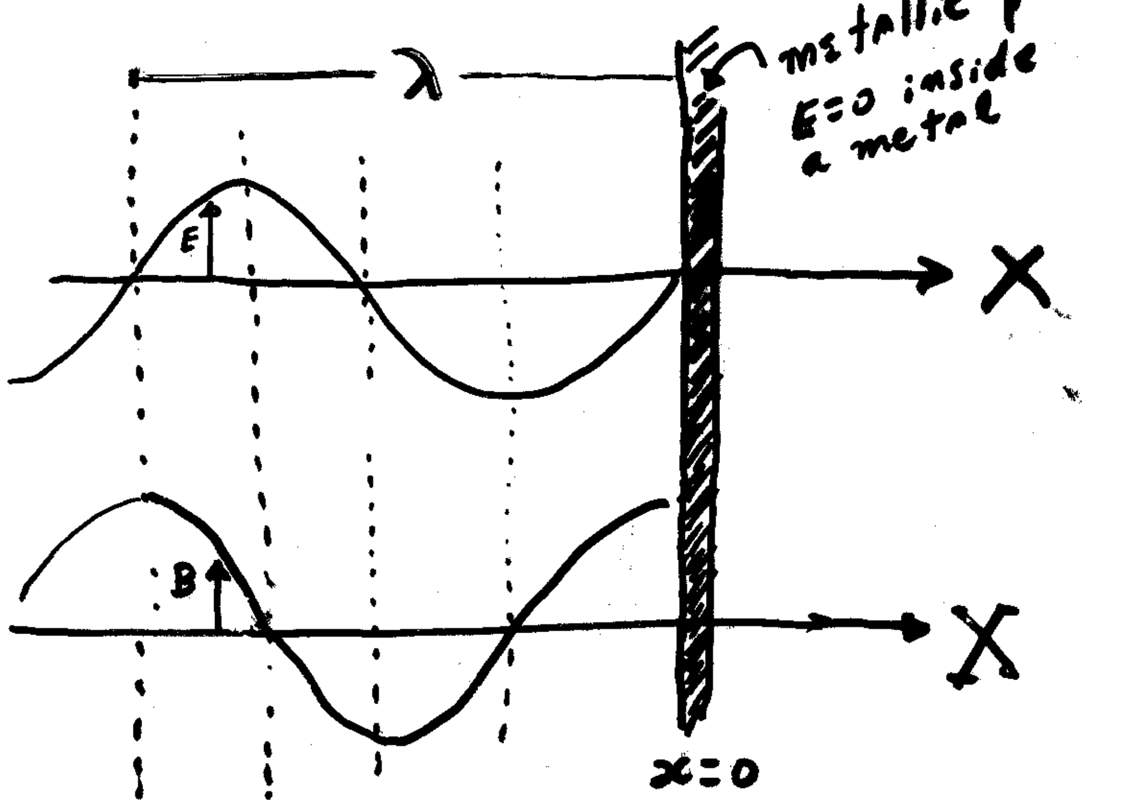
We found before that

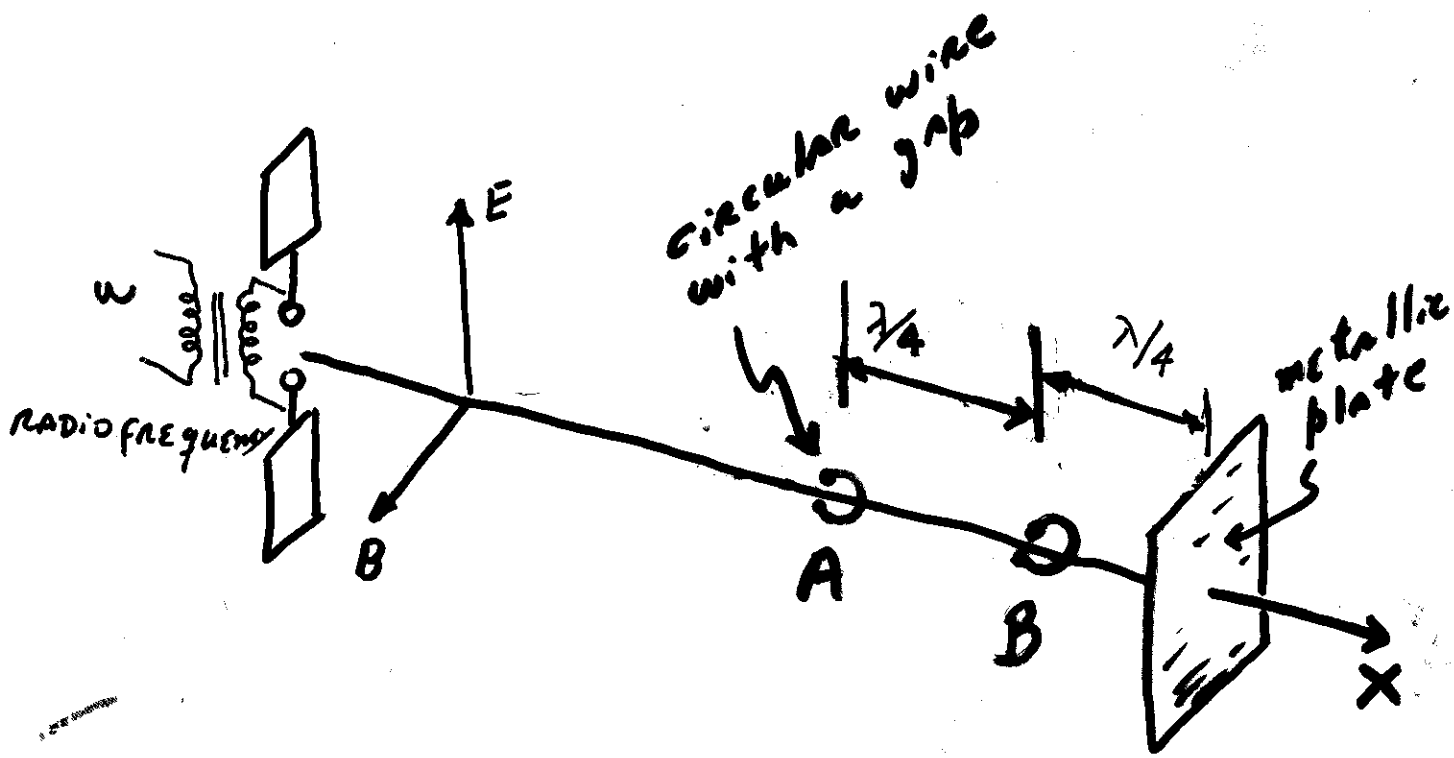
$$\frac{\partial}{\partial x} E_y = -\frac{\partial}{\partial t} B_z \,,$$

from which we obtain

B total $$B_z = -\left(2E_m \frac{k}{\omega}\right) \cos kx \sin \omega t$$

Notice E_{total} and B_{total} will have nodes and antinodes along the X axis





ANTINODE A

$\mathcal{E}$ electromotive force along the wire

$\mathcal{E}$ will produce a spark

NODE B

NO $\mathcal{E}$

NO spark.

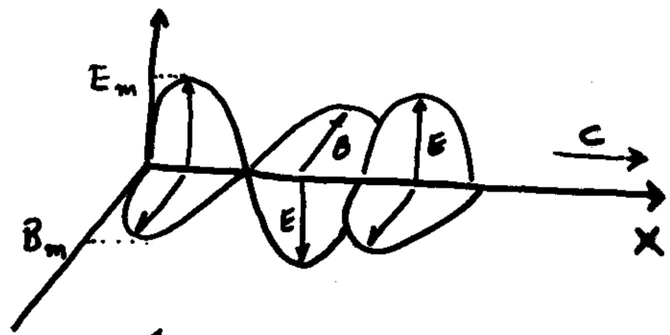
By measuring the distance between 2 antinodes, Hertz was able to figure out $\lambda/2$. Since he knew the frequency ω of the oscillator (transformer), he could calculate the velocity λf $\left(f = \frac{\omega}{2\pi}\right)$ of the electromagnetic waves. $V = 300{,}000$ Km/s

Relationship between the amplitudes E_m and B_m -

Let's consider a sinusoidal wave

$$E = E_m \cos(kx - \omega t)$$

$$B = B_m \cos(kx - \omega t)$$



$c = \frac{\omega}{k}$

EM propagating in the free space

and use equation ① (see page 5 in these notes)

$$\frac{\partial E}{\partial x} = -k E_m \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = +\omega B_m \sin(kx - \omega t)$$

$\underbrace{\text{eq ① implies}}_{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}} \longrightarrow k E_m = \omega B_m$ or $\boxed{B_m = \frac{E_m}{c}}$

$\vec{E}$ and $\vec{B}$ are perpendicular to one another

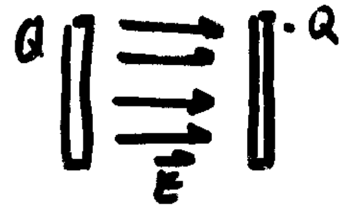
$\vec{E} \times \vec{B}$ points in the direction where the wave propagates toward to.

indicates vector product

ELECTROMAGNETIC WAVES TRANSPORT ENERGY POYNTING VECTOR

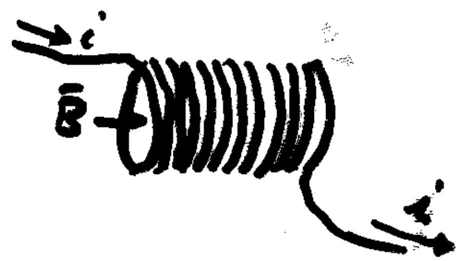
Wherever there is an electric field $\vec{E}$, there is an energy density u_E (energy per unit volume, Joule/m^3) associated to it:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (\text{Joule}/m^3)$$



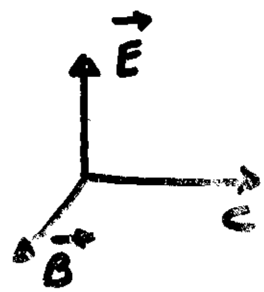
- Wherever there is a magnetic field $\vec{B}$, there is an energy density u_B (Joules/m^3) associated to it:

$$u_B = \frac{1}{2\mu_0} B^2$$



- In an electromagnetic wave, the electromagnetic energy density will be given by

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$



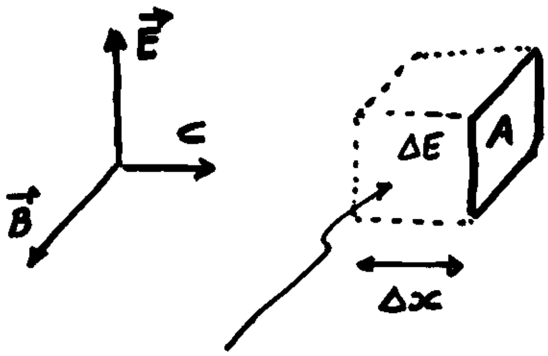
we can use $B = \frac{E}{c}$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$$ where we have used $c^2 = \frac{1}{\epsilon_0 \mu_0}$

electromagnetic energy density (Joule/m^3) $\boxed{u = \epsilon_0 E^2}$ or $\boxed{u = \frac{B^2}{\mu_0}}$ or $\boxed{u = \frac{EB}{\mu_0 c}}$

But this energy is propagating with velocity c

Let's consider a section of area A



the amount of energy in this cubic volume is $\Delta E = \mu A \Delta x$

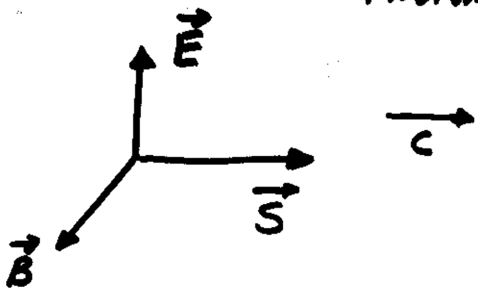
In an interval of time $\Delta t = \frac{\Delta x}{c}$ all the energy in that cubic volume will have passed through the section of area A

Energy per unit time crossing the area A $= \frac{\Delta E}{\Delta t} = \frac{\mu A \Delta x}{\Delta x / c} = \mu A c$

ENERGY per unit time and per unit AREA $\quad S = \mu c = \frac{E B}{\mu_0}$

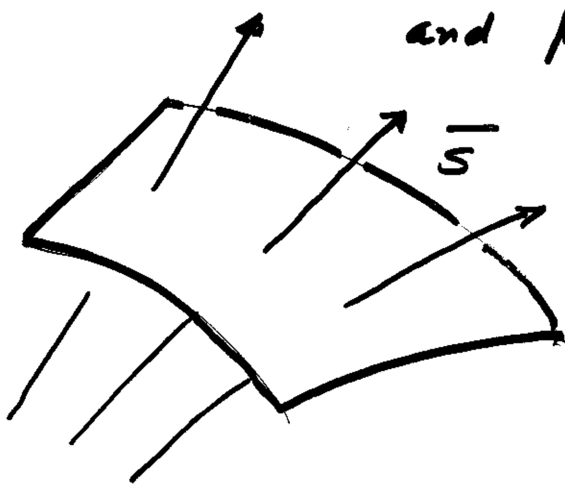
$$\boxed{\vec{S} = \frac{1}{\mu_0}\, \vec{E} \times \vec{B}}$$ Poynting vector

indicates vector product



S is the flow of electromagnetic energy traveling through the space, per unit area and per unit time.

$$\left[\frac{J}{m^2\,sec}\right]$$



$$S = u\,c$$

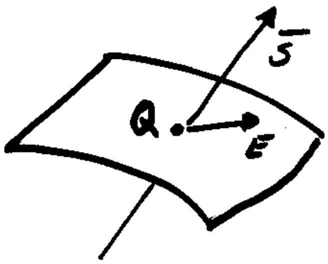
u: energy density

c: speed of light

$$\boxed{S = \frac{1}{\mu_0} E B} \qquad \boxed{S = \epsilon_0\, c\, E^2} \qquad \boxed{S = \frac{c}{\mu_0} B^2}$$

S: power per unit area $\left(\text{Watt}/m^2 \right)$

AVERAGE ENERGY $\langle S \rangle$: INTENSITY



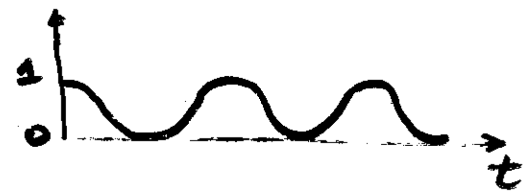
$$S = \epsilon_0 c E^2$$

Let's assume the electric field at the position "Q" is of the form

$$E = E_m \cos(\omega t)$$

The Poynting vector takes the form:

$$S = \epsilon_0 c E_m^2 \cos^2(\omega t)$$



If we take average over time:

$$\langle S \rangle = \epsilon_0 c E_m^2 \underbrace{\langle \cos^2(\omega t) \rangle}_{= \frac{1}{2}}$$

$$\boxed{I = \langle S \rangle = \frac{1}{2} \epsilon_0 c E_m^2}$$ Intensity — power per unit area (Watt/m²)

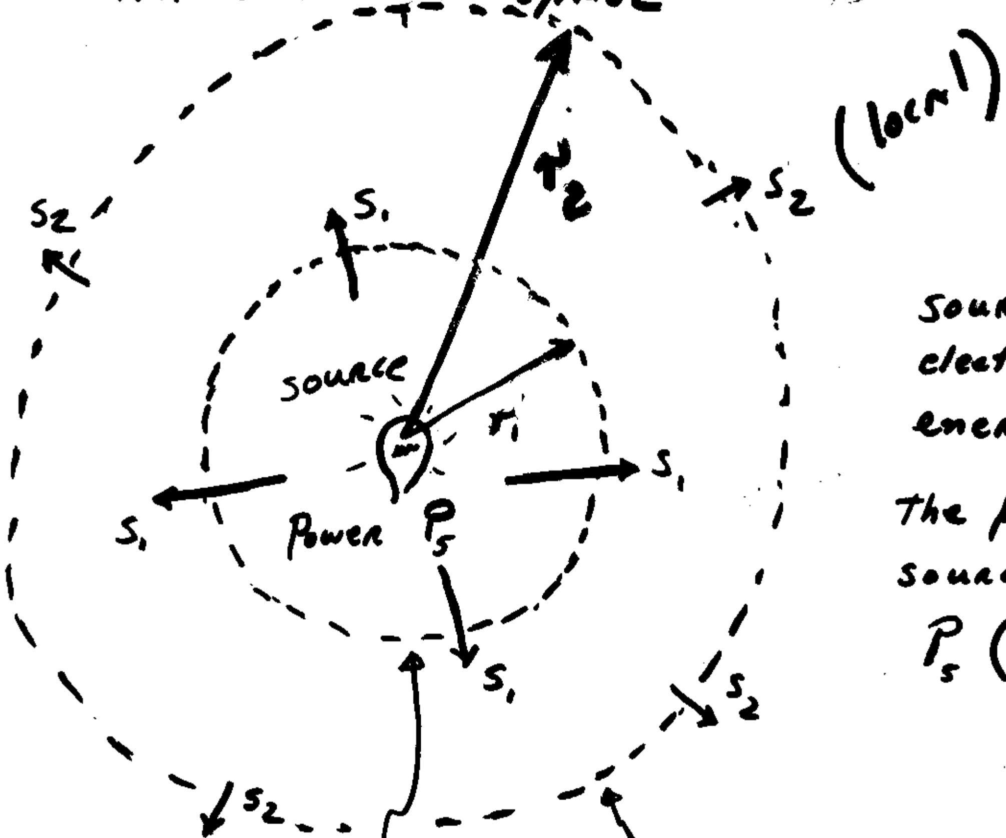
Sometimes, this result is given in terms of rms value of the electric field amplitude

$$E_{rms} \equiv \frac{E_m}{\sqrt{2}} \quad \text{(definition)}$$

$$\boxed{I = \epsilon_0 \cdot E_{rms}}$$ Intensity (Watt/m²)

Intensity decreases with distance from the source

CASE: PUNCTUAL SOURCE emiting energy All over the space



Source emits electromagnetic energy

The power of the source is:

P_s (Joules/sec)

Notice, conservation of energy implies:

$$P_s = \langle S_1 \rangle 4\pi r_1^2 = \langle S_2 \rangle 4\pi r_2^2$$

$$= I_1 4\pi r_1^2 = I_2 4\pi r_2^2$$

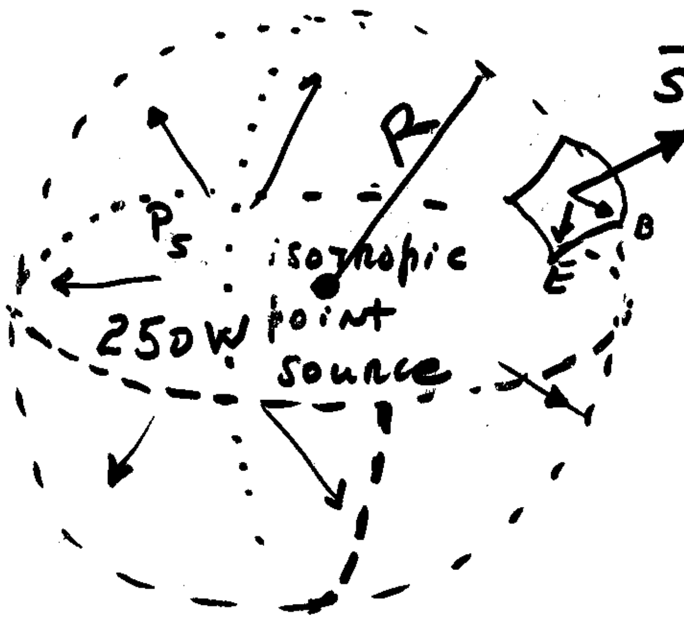
That is, for a constant value of P_s, the intensity varies with the distance r:

$$I(r) = \frac{P_s}{4\pi r^2}$$

Remember:

$$I = \langle S \rangle$$

Example: sample problem 34-1 (pg 811)



$\vec{S}$ $\left[\frac{\text{Joule}}{m^2\ sec}\right]$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$E = E_m \cos(\omega t)$$

$$\langle s\rangle = \frac{1}{2}\epsilon_0 c E_m^2$$

$$\frac{250\,W}{4\pi R^2} = \langle s\rangle = \frac{1}{2}\epsilon_0 c\ E_m^2 \quad *$$

$R = 1.8\ m$

$\epsilon_0 =$

$c =$

$$\Rightarrow \quad E_m^2 = \frac{2\times 250\ \frac{J}{s}}{4\pi(1.8\ m)^2\ \epsilon_0\ c}$$

Electric field $E = \frac{\text{volts}}{m}$ (Remember $E \cdot d = V$)

$$\text{volt} = \frac{\text{work}}{\text{unit charge}} = \frac{J}{C}$$

Remember

$$F = \frac{1}{4\pi\epsilon_0}\frac{q^2}{r^2}$$

so,

$$\epsilon_0 = \frac{C^2}{N\,m^2} = \frac{C^2}{J\,m}$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ \frac{F}{m}$$

$$= 8.85 \times 10^{-12}\ \frac{C^2}{J\,m}$$

Therefore

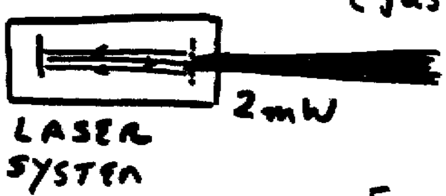
$$E_m^2 = \frac{2 \times 250\ \frac{J}{s}}{4\pi\,(1.8\ m)^2\ 8.85 \times 10^{-12}\ \frac{C^2}{J\,m} \times 3 \times 10^8\ \frac{m}{s}}$$

$$= 0.46 \times 10^4\ \frac{J^2}{m^2 C^2} = 0.46 \times 10^4\ \frac{V^2}{m^2}$$

$$E_m = 0.68 \times 10^2\ \frac{V}{m}$$

In team of rms: $E_{rms} = \frac{E_m}{\sqrt{2}} = 0.48\ \frac{V}{m} \times 10^2$

* Exercise: The beam diameter of a typical He-Ne laser is ~ 2 mm. (just at the output of the laser system case)



Find the amplitudes of the electric field and magnetic field of the laser beam.

$$I = \langle S \rangle = \frac{1}{2}\epsilon_0 c\, E_m^2 = \frac{\text{power}}{\text{unit area}} = \frac{2\ \text{mW}}{\pi\,(1\ \text{mm})^2}$$

$$= \frac{2\times 10^{-3}\ \text{J/s}}{\pi\,(10^{-3})^2\,\text{m}^2} = 0.64\times 10^{3}\ \frac{\text{J}}{\text{s m}^2}$$

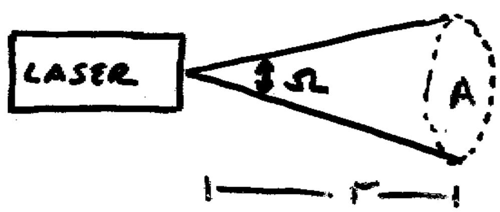
$$\Rightarrow E_m^2 = \frac{2\times 0.64\times 10^{3}\ \frac{\text{J}}{\text{s m}^2}}{\epsilon_0\, c}$$

$$E_m \approx 700\ \frac{\text{V}}{}$$

$$E_m^2 = 48.[illegible]\times 10^4$$

$$E_m = 7\times 10^{2}\ \frac{\text{V}}{\text{m}}$$

Note: Output laser beam is not perfectly collimated.



Solid angle Ω is measured in radians

$$\Omega = \frac{A}{r^2}$$

where

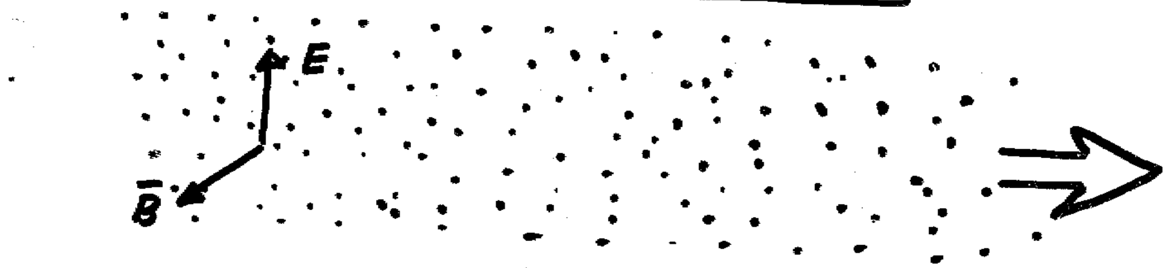
A : cross section area

r : distance

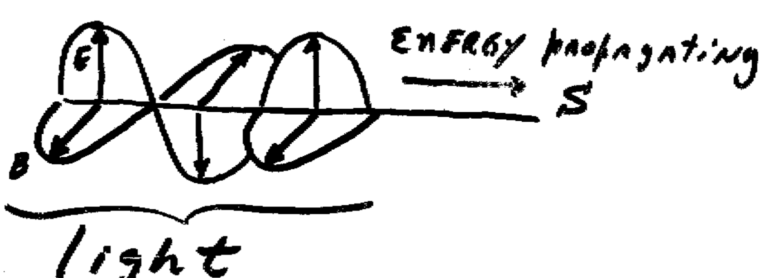
Excercise: 13 E (chapter-34)

Other practice problems: 15 E, 16 P, 17 P (chapter-34)

LIGHT as a WAVE



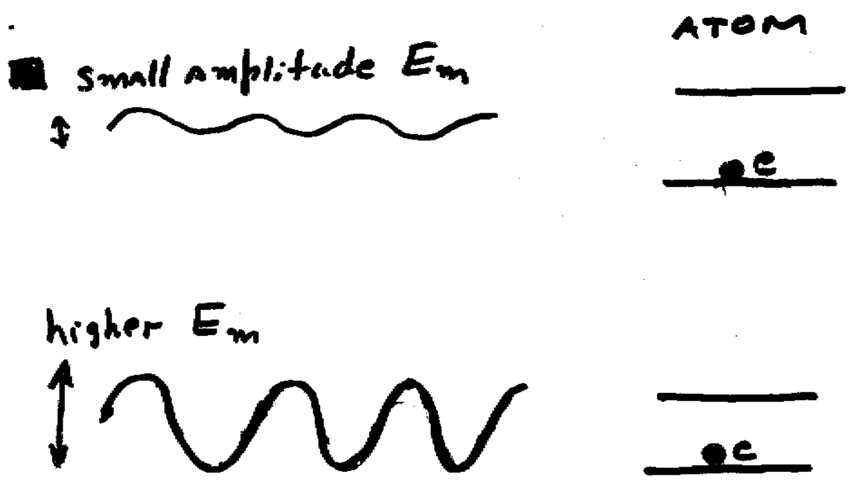
- Light is a wave of electric fields and magnetic fields
- the vector product $\vec{E} \times \vec{B}$ gives the direction of light propagation



$$\langle S \rangle = \tfrac{1}{2} \epsilon_0 c E_m^2$$

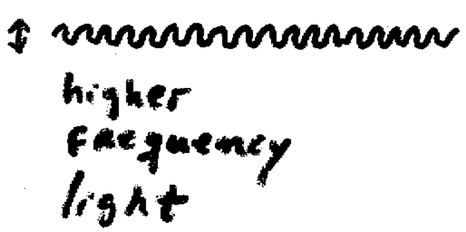
CLASSICAL VIEW → The higher E_m the higher the power propagating in the wave.

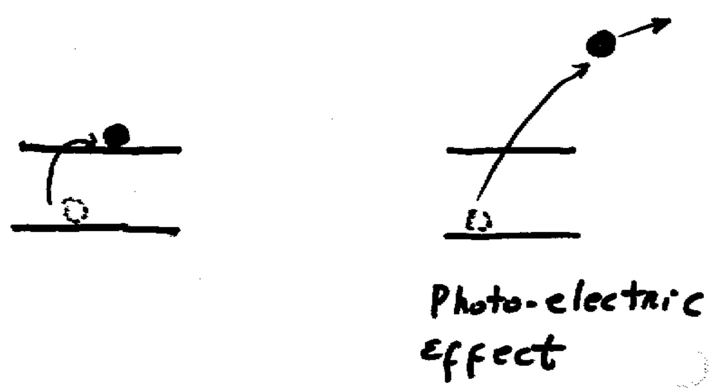
However, all this nice theory could not explain the following experimental result:



no matter how much you increase the incident light power (increase of E_m) the electron is <u>not</u> excited to the upper level of energy.

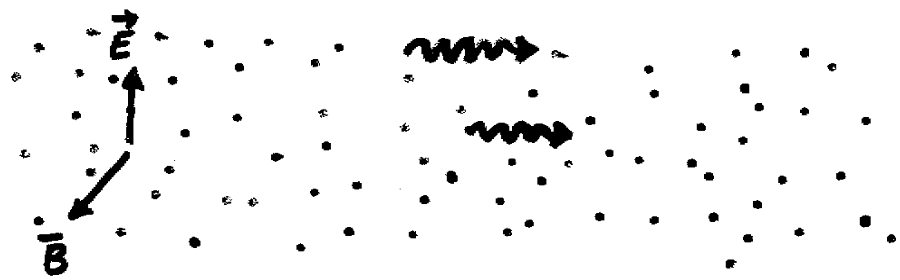
■ HOWEVER





how to explain it ?

LIGHT as a Radiation Field of Quantized energy packets



Light is made out of photons.

1 photon energy $= h\,f = \frac{hc}{\lambda}$

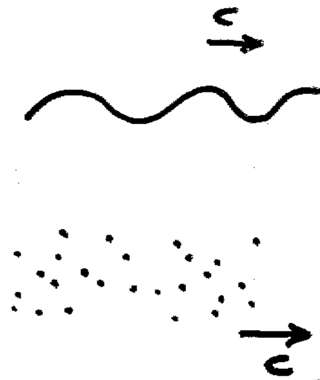
f: frequency of the radiation

h: constant #
"planck constant"
$= 6.6 \times 10^{-34}$ Joule . sec
$= 4.14 \times 10^{-15}$ eV . sec

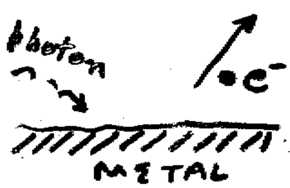
* Exercise: Find the energy of 1 photon whose wavelength is $\lambda = 500$ nm ("green light

LIGHT $\xrightarrow{\text{behaves as a}}$ WAVE

or any ELECTROMAGNETIC WAVE $\xrightarrow{\text{behaves as a}}$ particle



The particle nature of light was first proposed by Albert Einstein in 1905 in his explanation of the "photo electric effect."

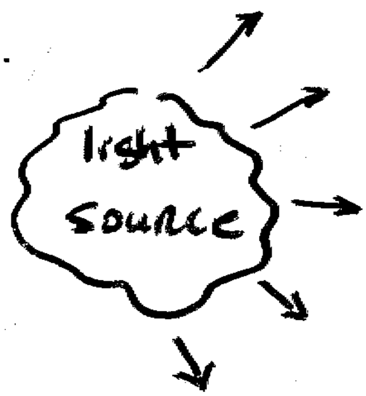


$$E_{photon} = hf = 6.6 \times 10^{-34} \text{ Joule}\cdot\text{sec} \cdot f$$

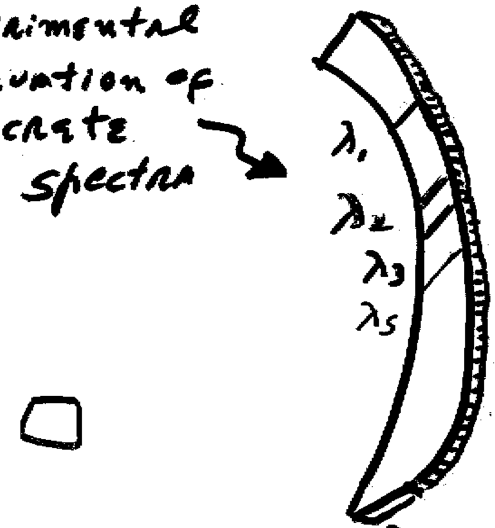
$$= 4.14 \times 10^{-15} \text{ eV}\cdot\text{sec} \cdot f$$

$$= h\frac{c}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{\lambda}$$

nm = nano-meter
$= 10^{-9}$ m

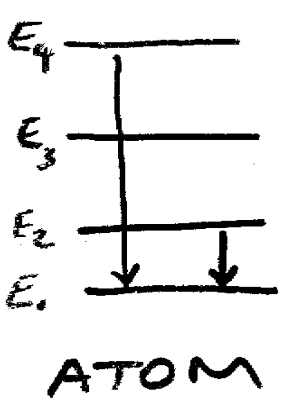


Experimental observation of a discrete lines spectra



Hypothesis

$$\Delta E = \frac{hc}{\lambda}$$



Measurements of discrete values of λ allowed to make models of the <u>ATOM</u>

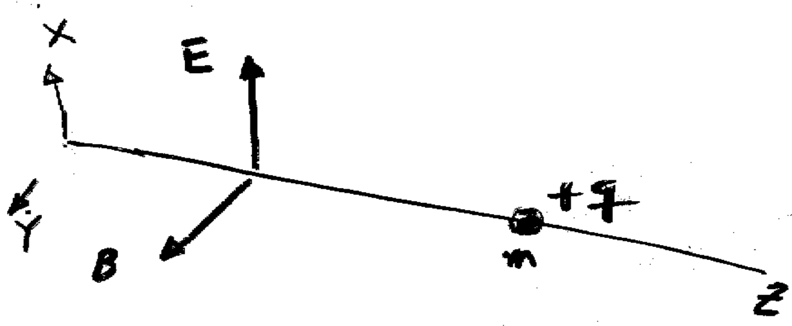
LIGHT WAVE-particle duality

the propagation of light is governed by its WAVE properties

the light-matter interaction is governed by its particle-like character.

If light behaves as a particle, does it have momentum?

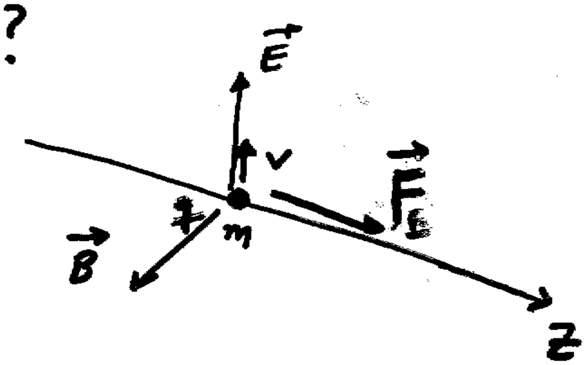
Angular momentum?



E $\longrightarrow$ will shake the charge up and down (along the X-axis)

What about B ?

$$\vec{F}_B = q\,\vec{v} \times \vec{B}$$



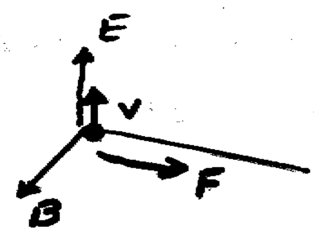
$\vec{F}_B$ acts in the direction of the propagation of light

So, the mass m will gain some linear momentum along the Z-axis

$$F = q v B$$

$$B = \frac{E}{c}$$

$$= \frac{v}{c}\, q E$$



velocity × force
= power
= work W done (by the electric field E) on the charge q, per unit time

$$F = \frac{1}{c}\frac{dW}{dt} = \frac{d}{dt}\Bigg(\underbrace{\frac{W}{c}}_{\text{momentum}}\Bigg)$$

W must come from the energy of the light

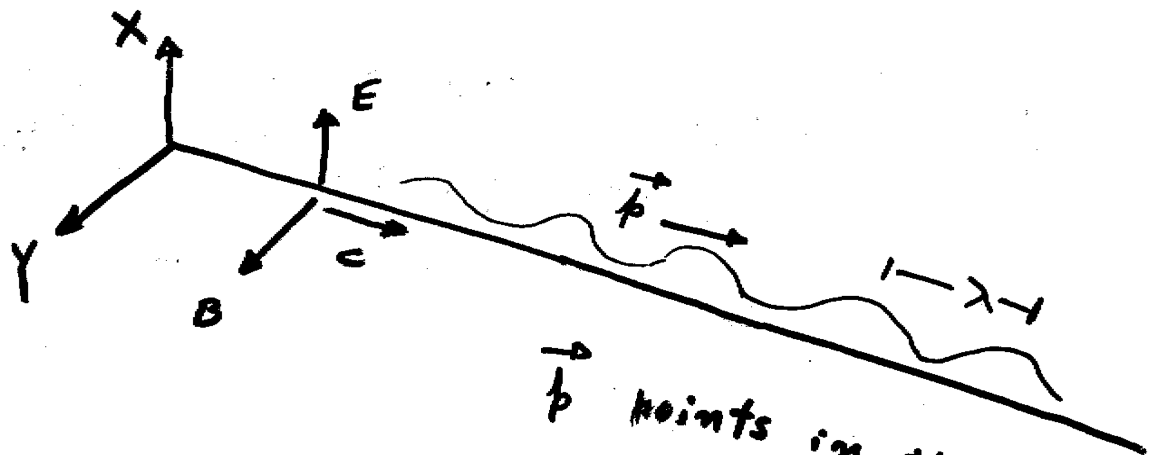
$$W = hF$$

h is the Planck's constant

$$p = \frac{W}{c} \xrightarrow{\text{if one photon is absorbed}} = \frac{hF}{c}$$

$$\boxed{p = \frac{h}{\lambda}}$$

Linear momentum carried by a photon



$\vec{p}$ points in the direction of the light propagation

$$p = \frac{h}{\lambda}$$

* Exercise: Problems 13E, 14E, 16P ; Chap 34, p. 828
6th Ed

MAXWELL's RAINBOW