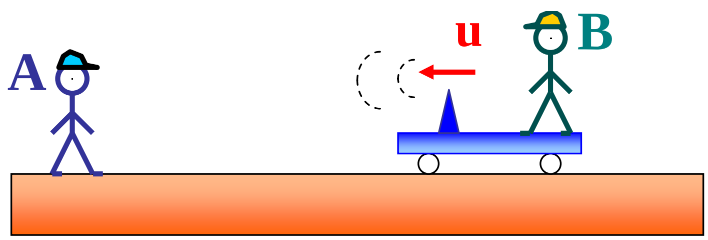
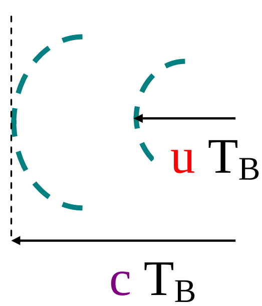
Including relativistic effects into the Doppler Effect

- **Observer: stationary**
  Source: in motion





T_B is the period of the wave (measured by the observer B)

After one pulse is emitted, the second pulse is not emitted but after T_B seconds. During that time the pulse has advanced a distance cT_B, while the source advances a distance uT_B

Observer A would expects to receive one pulse every

$$\frac{(c-u)}{c} T_B \quad \text{seconds}$$

However Observer-A knows relativity and suggests that, even though Observer-B sends one pulse every T_B seconds, in fact the pulses are spaced by γT_B.

NOTE:

EVENT: Sending a pulse from B

T_B: Delay T_B between two events that happen in the same place (according to observer B)

From relativity: According to observer A, the delay of the events that happen at the same place (according to observer B) is rather γT_B.

$$T_A = \gamma \underbrace{T_B}_{\text{"Proper time"}}$$

Thus, Observer-A should expect to receive one pulse every

$$T_A = \frac{(c-u)}{c} \gamma T_B \quad \text{seconds}$$

The frequency perceived by Observer A is then:

$$f_A = \frac{c}{(c-u)} \frac{1}{\gamma} f_B$$

$$= \frac{1}{(1-\frac{u}{c})} \quad \frac{1}{\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}} \; f_B$$

$$f_A = \underbrace{\sqrt{1-\left(\frac{u}{c}\right)^2}}_{\text{Relativistic correction}} \frac{1}{(1-\frac{u}{c})} f_B \qquad (1)$$

$$f_A = \frac{\sqrt{1+\frac{u}{c}}}{\sqrt{1-\frac{u}{c}}} f_B \qquad (2)$$

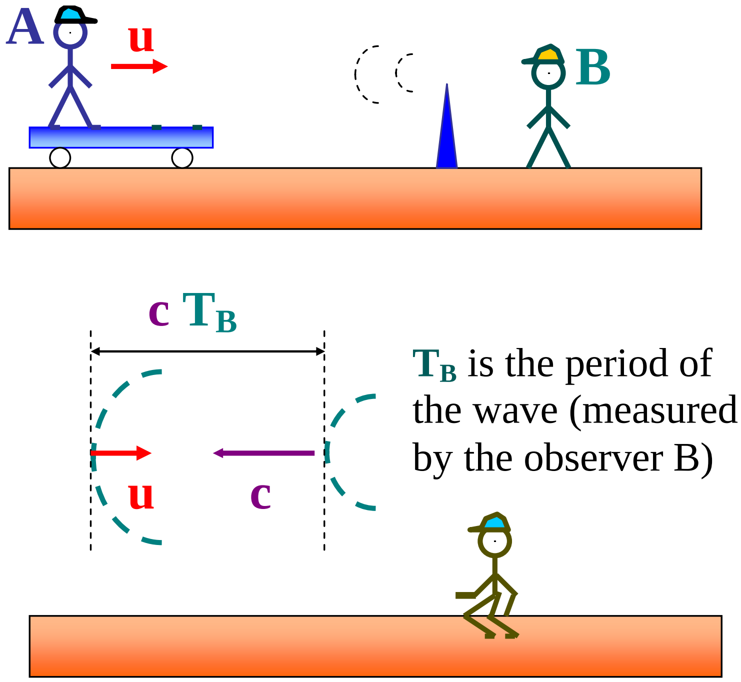
- **Observer: In motion**
  Source: stationary

The key aspect to analyze this case is to make predictions from a reference stationary with the platform. So we use a reference attached to the observer shown seated on the platform, or simply the observer B.

According to Observer B, the Observer A should receive one pulse every

$$\frac{cT_B}{c+u} \text{ seconds}$$

This estimation is according to an observer seated on the platform (stationary with respect to the source)



The pulses are spaced by a distance cT_B. Once the observer-A receives a pulse, he/she heads towards his/her encounter with the next pulse that is advancing with speed c.

NOTE:

EVENT: Receiving a pulse

T_A: Delay between two events that happen at the same place (according to observer A)

From relativity: According to observer B, the delay of events that happen at the same place (according to A) is rather γT_A. So,

$$\frac{cT_B}{c+u} = \gamma \underbrace{T_A}_{\text{“Proper time”}}$$

(i.e. for the traveler observer-A, the seconds in the platform are shorter by a factor of $\frac{1}{\gamma} = \sqrt{1-\left(\frac{u}{c}\right)^2}$). Thus, Observer A should receive one pulse every

$$T_A = \frac{cT_B}{c+u}\sqrt{1-\left(\frac{u}{c}\right)^2} \text{ seconds} \quad (T_A\text{'s seconds}).$$

The frequency perceived by Observer-A is then

$$f_A = \frac{c+u}{c}\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}} f_B =$$

$$f_A = \underbrace{\left(\frac{1}{\sqrt{1-\left(\frac{u}{c}\right)^2}}\right)}_{\text{Relativistic correction}} \left(1+\frac{u}{c}\right) f_B \qquad (3)$$

$$f_A = \frac{\sqrt{1+\frac{u}{c}}}{\sqrt{1-\frac{u}{c}}} f_B \qquad (4)$$

Notice, we have arrived to the same result (2). Observer A perceives the same frequency whether he/she approaches a stationary source, or the source approaches him/her.

That is, the Doppler Effect is compatible with Relativity.