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ENERGY CONSERVATION

The First Law of Thermodynamics and the Work/Kinetic-Energy Theorem

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ENERGY

TRANSFER of ENERGY

- **Heat-transfer *Q***
- **Macroscopic external Work *W'* done on a system**

ENERGY CONSERVATION LAW

- **The work/kinetic-energy theorem**
  - **Case: inelastic collision**
- **Generalization of the work/kinetic-energy theorem**
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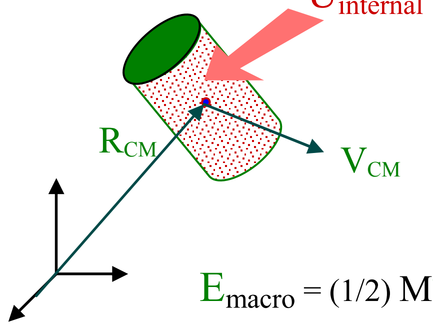
ENERGY

The total energy of a system has two distinct contributions:

$$U_{internal} = \sum_i \frac{1}{2} m_i u_i^2 + \sum_{i,j} \frac{1}{2} P_{ij}$$

"Disordered" energy
(gas molecules inside the container.)
u_i = velocities
P_{ij} = potential energies



$$E_{macro} = (1/2)\, M V_{CM}^2 + (1/2)\, I\, \omega^2 + M g z$$

Ordered energy
(of the can cylinder)

Fig. 1

A. MACROSCOPIC COMPONENT (“Ordered energy.”)

The ***total mechanical energy*** of the system, associated with the macroscopic position and motion of the system as a whole.

This mechanical energy comprises:

***i*)** Translational kinetic energy of the center of mass (CM) + + the rotational kinetic energy calculated with respect to the CM.

***ii*)** Potential energies associated to the position of the center of mass (gravitational potential energy, electrical potential energy, potential energy associated to the spring force, etc.)

B. MICROSCOPIC COMPONENT (“Disordered energy.”)

The other contribution to the energy is a vast collection of microscopic energies, known collectively as the **internal energy *U*** of the system.

U comprises:

The sum of individual kinetic and potential energies associated with the motion of, and interactions between, the individual particles (atoms and/or molecules) that constitute the system.

These interactions involve complicated potential energy functions on a microscopic distance scale. In principle, after an appropriate choice of the zeros of the potential energy functions, one can talk about a definite value of U of the system (when the latter is in a state of thermodynamic equilibrium.) But such calculation of U can be a complicated endeavor. It is relatively simpler to calculate the changes of U.

ΔU:

When a system changes its state of thermodynamic equilibrium, it is only the changes in the internal energy ΔU that are physically significantly.

TRANSFER of ENERGY

Different systems can transfer energy among themselves by two processes:

(1) Via **heat-transfer**, driven by temperature differences

(2) Via **work**, driven by external macroscopic forces

We will see that,

Heat-transfer to a system is fundamentally a **microscopic mechanism** for transferring energy to a system.

Work done on a system is a **macroscopic mechanism** for transferring energy to a system

Heat-transfer *Q*

It can occur via conduction, convection, and radiation

The mechanism is fundamentally microscopic (at the atomic and molecular level.) heat transfer is accomplished by **random molecular collisions** and other molecular interactions.

The direction is from the higher to lower temperature (an aspect better explained in the context of the second law of thermodynamics.)

Warning: Do not confuse heat-transfer Q with the internal energy U.

Heat transfer is not a property of the state of a system
(a system in thermal equilibrium does not have an amount of heat or heat-transfer.)
On the other hand, a system in thermal equilibrium does have (in principle) a specific internal energy.

That is,

Q is not a state variable
U is a state variable

Macroscopic external Work *W'* done on a system

The macroscopic external work *W'* done on a system can cause a change in either

the internal energy *U* of the system, or

the total mechanical energy *E* of the system

Example where the external work causes a change of purely internal energy

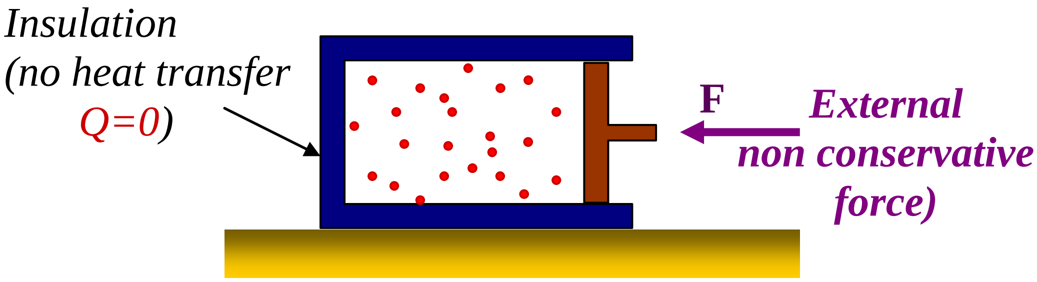


Fig. 2

Gas enclosed in an insulating container. The insulated walls ensure an absence of heat transfer from or toward the system (the gas.)

Movable piston allows an external agent to compress the gas (by pushing the piston), thus doing work on the system.

The work on the gas by the external agent results in an increase of the gas temperature (indicative of an increase in the internal energy *U*.)

On the other hand, simply lifting the gas container (described above) would be an example of increasing the mechanical energy of the system, without changing the internal energy.

ENERGY CONSERVATION LAW

The work/kinetic-energy theorem

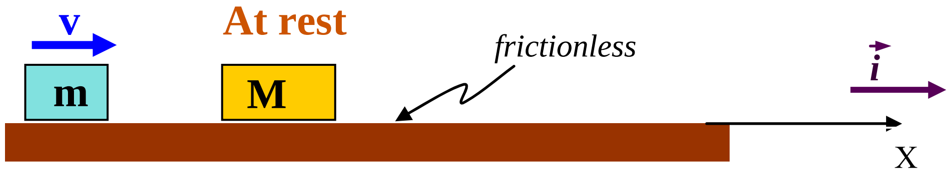
We are already familiar with the work/kinetic-energy theorem, which establishes the source (work) that causes a change in the kinetic energy of a system. We illustrated this theorem for the case of an individual particle, as well as for a system of particles constituting a rigid body. The later allowed us to solve, in a very straightforward manner, problems involving bodies rolling down an inclined plane, for example.[But cases involving work done by internal forces in non-rigid bodies were not considered. We will encounter such cases in this section.]

Case: Inelastic collision

In what follows, we illustrate the need for generalizing the work/kinetic-energy theorem, in order to include cases in which disordered (microscopic) energy is involved. To that effect, let's start consider an inelastic collision.

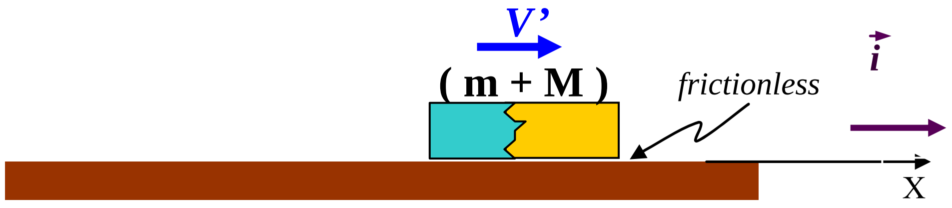
Before the collision

Both particles initially at the same temperature and in thermal equilibrium



Kinetic energy: $K_{\text{before}} = \frac{1}{2}\, m\, v^2$

After the collision



Kinetic energy $K_{\text{after}} = \frac{1}{2}\,(m + M)\, V'^2$

Since the linear momentum is conserved

$$mv = (m + M)\,V'$$

$$K_{after} = \tfrac{1}{2}(m + M)\,[\,(m/(m + M)\,v\,]^2$$
$$K_{after} = \tfrac{1}{2}\;[\,(m^2/(m + M)\,v^2\,]$$

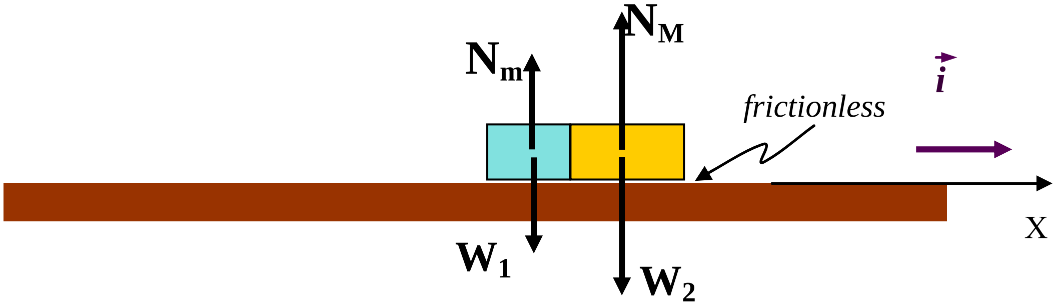
$$= \tfrac{1}{2}\;m\;v^2\,[\,(m/(m + M)\,]$$

The change in kinetic energy is given by,

$$\Delta K = K_{after} - K_{before} = [(m/(m + M) - 1\,]\,\tfrac{1}{2}\;m\;v^2$$
$$= -\,[(M/(m + M)\;]\,\tfrac{1}{2}\;m\;v^2$$

that is, the kinetic energy is less after the collision than before.

According to the work/kinetic-energy theorem this change should have resulted from the work done by the forces acting on the system. But notice, all the external forces acting on the system (normal forces and weight) are perpendicular to the displacement of the particles, hence, their work on the system is zero ($W_N = 0$, $W_W = 0$.)



Apparently, then, the work/kinetic-energy theorem $W_{total} = \Delta K$ appears not to be valid here.

The explanation lies in the fact that we are not including the work done by the internal friction forces. Such forces act during the inelastic collision. We say then,

$$W_{\text{internal-friction}} = \Delta K$$

Thus, in this particular example, we identify the decrease in the kinetic energy in the negative work done by the microscopic internal forces.

We would like to highlight that the change in kinetic energy ΔK may include not only the macroscopic kinetic energy (of the center of mass) but also (presumably) an increase also of the microscopic kinetic energy; that is,

$$W_{\text{internal-friction}} = \Delta K_{\text{macroscopic}} + \Delta K_{\text{microscopic}}$$

(*case for the inelastic collision depicted in the figure above*)

The work/kinetic-energy theorem

While we can in principle understand what is going on in the particular example of inelastic collision (where the system under study does not receive external heat-transfer), we would like to explore reformulating the work/kinetic-energy such that include also cases where thermal interaction (heat transfer) from the surrounding environment is allowed.

As a firs step , let’s express the work/kinetic energy theorem as follows,

$$W_{\text{internal}} + W'_{\text{external-non-conservative}} + W_{\text{external-conservative}} =$$

$$= \Delta K_{CM} + \Delta K_{\text{microscopic}}$$

(*generalization of the work/kinetic-energy theorem*)

Here $W'_{\text{external-non-conservative}}$ refers to the work done by forces like the one pushing the piston in Fig. 2 above. $W_{\text{external-conservative}}$ could be the work done, for example, the gravitational force.

That is, we are explicitly separating out the macroscopic work (done by external macroscopic forces, conservative and non-conservative) from the work done by microscopic forces. Similarly, we assume also that the kinetic energy changes in both macroscopic (the CM kinetic energy) and microscopic forms

i) For the conservative forces component, the work can be derived from a potential energy function E_p,

$$W_{\text{external-conservative}} = -\Delta E_p$$

which gives,

$$W_{\text{internal}} + W'_{\text{external-non-conservative}} + (-\Delta E_p) = $$
$$= \Delta K_{CM} + \Delta K_{\text{microscopic}}$$

$$W_{\text{internal}} + W'_{\text{external-non-conservative}} = \Delta\underbrace{(K_{CM} + E_p)} + \Delta K_{\text{microscopic}}$$

Calling $K_{CM} + E_p \equiv E_{\text{macro}}$ *the macroscopic mechanical energy*,

the work/kinetic-theorem can be written as,

$$W_{\text{internal}} + W'_{\text{external-non-conservative}} = \Delta E_{\text{macro}} + \Delta K_{\text{microscopic}}$$

ii) We can envision that, ultimately, W_{internal} causes a change in microscopic potential energies of the interacting microscopic particles that constitute the system. That is, $W_{\text{internal}} = \sum_{i\neq j} -\Delta P_{ij}$.

Hence,

$$\sum_{i\neq j} -\Delta P_{ij} + W'_{\text{external-non-conservative}} = \Delta E_{\text{macro}} + \Delta K_{\text{microscopic}}$$

$$W'_{\text{external-non-conservative}} = \underbrace{\Delta E_{\text{macro}}}_{\text{Change in macroscopic } \textbf{mechanical energy}} + \underbrace{\Delta K_{\text{microscopic}} + \sum_{i\neq j} \Delta P_{ij}}_{\text{Change in the } \textbf{internal energy U}}$$

The last two terms in the right side of the expression above constitute what we called, at the beginning of this section, the disordered Internal Energy *U* of the system.

Through the derivation process followed above, we notice that the work energy is deposited (transformed) into the system as either,

macroscopic mechanical energy, or

internal energy.

The work $W_{\text{external-conservative}}$ done by conservative macroscopic external forces has been assimilated into the mechanical energy, while the work W_{internal} done by microscopic forces ended up being grouped into the internal energy term.

The expression above also shows that the work energy $W'_{\text{external-non-conservative}}$ done by external non-conservative forces could end up either as macroscopic mechanical energy or internal energy (that the latter case can occur was illustrated in the example above where a gas was compressed by a piston; the force acting on the piston was the non-conservative force.)

Generalization of the work/kinetic-energy theorem

As illustrative as the expression above might be, it also reveals its limitations for dealing with cases in which the system is in thermal contact with a body at different temperature. Indeed, in such a case, the system can also receive energy via heat-transfer, a process driven by temperature differences.) Accordingly the expression above needs to be modified or generalized.

$$\underbrace{Q}_{\substack{\text{Heat-transfer}\\\text{into the}\\\text{system}}} + W'_{\text{external-non-conservative}} = \underbrace{\Delta E_{\text{macro}}}_{\substack{\text{Change in}\\\text{macroscopic}\\\textbf{mechanical}\\\textbf{energy}}} + \underbrace{\Delta K_{\text{microscopic}} + \sum_{i\neq j} \Delta P_{ij}}_{\substack{\text{Change in the}\\\textbf{internal}\\\textbf{energy U}}}$$

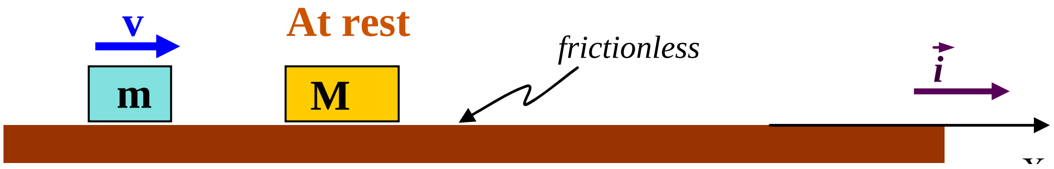
In a simplified form

$$\underbrace{Q}_{\substack{\text{Heat-transfer}\\\text{into the system}\\\text{caused by}\\\text{temperature}\\\text{difference}}} + \underbrace{W'_{\text{external-non-conservative}}}_{\substack{\text{Work done on the}\\\text{system by a non-}\\\text{conservative}\\\text{macroscopic}\\\text{external force}}} = \underbrace{\Delta E_{\text{macro}}}_{\substack{\text{Change in}\\\text{macroscopic}\\\textbf{mechanical}\\\textbf{energy}\text{ of the}\\\text{system}}} + \underbrace{\Delta U}_{\substack{\text{Change in the}\\\textbf{internal}\\\textbf{energy U}\\\text{of the system}}}$$

which constitutes our **Fundamental Energy Conservation Law.**

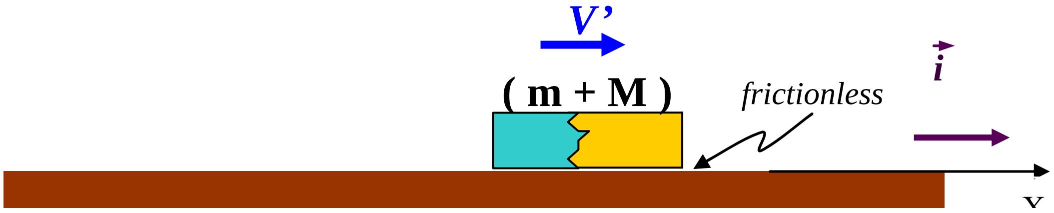
Inelastic collision (revisited)

Both particles initially at the same temperature and in thermal equilibrium



Kinetic energy $K_{\text{before}} = ½\, m\, v^2$

After the collision



- Here Q id the flow of energy by heat transfer, caused by temperature differences. In our case is zero.)
- W' is the work done by external forces. In our case it is zero.
- ΔU change in the internal energy
- ΔE change in the mechanical energy

  In our case $\Delta E = - [(M / (m + M)] \frac{1}{2} m v^2$

Accordingly,

$$0 + 0 = - [(M / (m + M)] \frac{1}{2} m v^2) + \Delta U$$

which gives,

$$\Delta U = [(M / (m + M)] \frac{1}{2} m v^2)$$

That is, the missing (ordered) kinetic energy appears as an increase in the internal (disordered)s energy ΔU of the system.

(The increase in the internal energy of the system typically manifest itself in an increase in the temperature of the system.

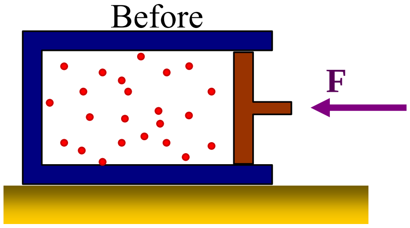
As the temperature of the system increases above the ambient environment because of the increase in the internal energy, heat-transfer subsequently occurs from the system to the environment until the system-ambient reach a common temperature.

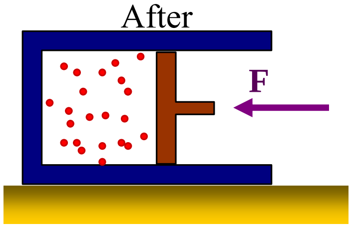
Case: Pure Thermodynamics

In pure thermodynamics, one typically considers only systems whose total mechanical energy does not change, $\Delta E_{macro} = 0$. The general statement of the energy conservation becomes,

$$\mathbf{Q} + W'_{external\text{-}non\text{-}conservative} = \Delta U$$

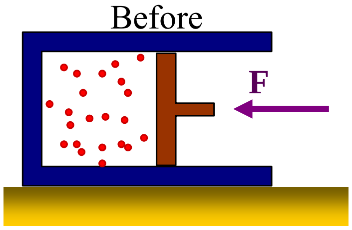
Notice

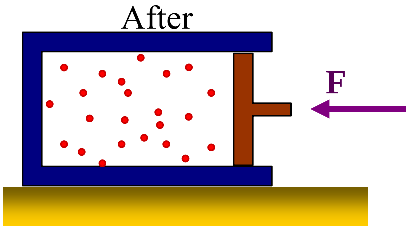




Work done by the external force F > 0

$$W'_{\text{external-non-conservative}} > 0$$





Work done by the external force F < 0

$$W'_{\text{external-non-conservative}} < 0$$

It is typical to consider the work done ***by*** the system (no the work done ***on*** the system by the external non-conservative forces.) Since, according to the Newton's third law, the force exerted by the system is equal in magnitude but opposite in direction, then

$$W'_{\text{external non-conservative}} = -W_{\text{done-by-the-system}}$$

Thus, for pure thermodynamic systems

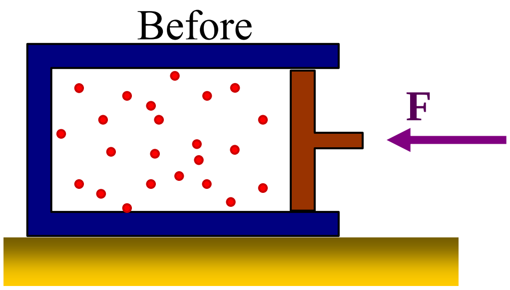
$$\mathbf{Q}_{\text{Heat-transfer-into-the-system}} - W_{\text{done-by-the-system}} = \Delta U$$

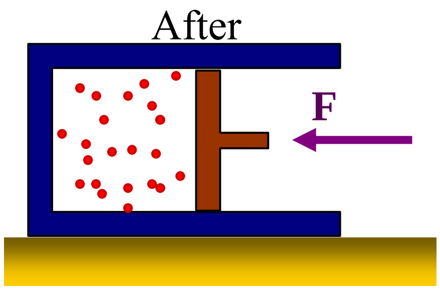
When all the terminology is understood, the subscripts are omitted and one simply writes

$$Q - W = \Delta U$$

The First Law of Thermodynamics

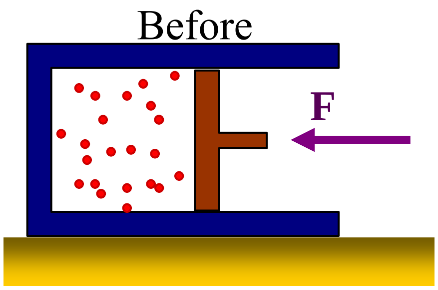
Notice

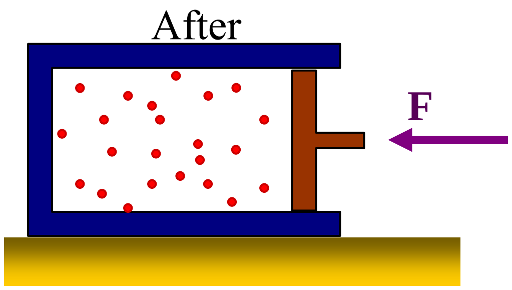




Work done by the gas < 0

W < 0

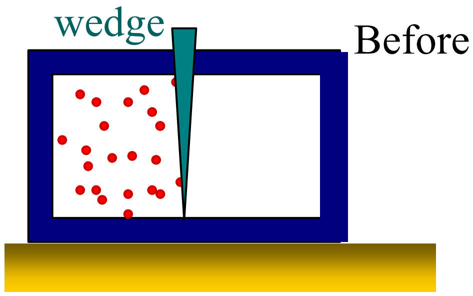


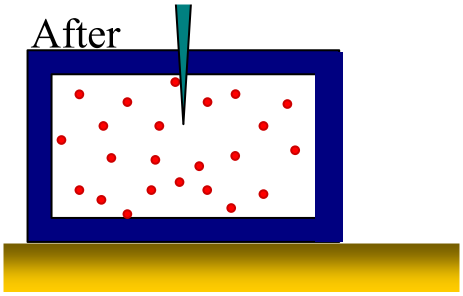


Work done by the gas > 0

W > 0

Question:





W = ?