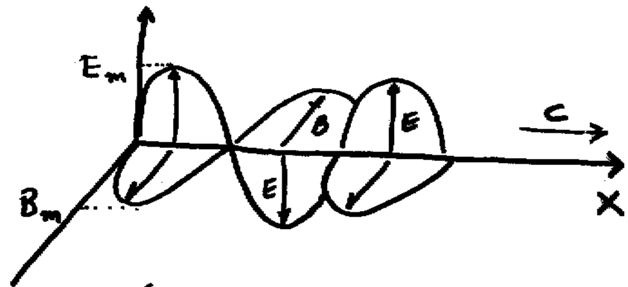
Relationship between the amplitudes E_m and B_m -

Let's consider a sinusoidal wave

$$E = E_m \cos(kx - \omega t)$$

$$B = B_m \cos(kx - \omega t)$$



$$c = \frac{\omega}{k}$$

EM propagating in the free space

and use equation ① (see page #8 in these notes)

$$\frac{\partial E}{\partial x} = -k E_m \sin(kx - \omega t)$$

$$\frac{\partial B}{\partial t} = +\omega B_m \sin(kx - \omega t)$$

$\underbrace{\text{Eq ① implies}}_{\frac{\partial E}{\partial x} = -\frac{\partial B}{\partial t}} \longrightarrow \quad k E_m = \omega B_m \quad$ or $\quad \boxed{B_m = \frac{E_m}{c}}$

$\vec{E}$ and $\vec{B}$ are perpendicular to one another

$\vec{E} \times \vec{B}$ points in the direction where the wave propagates toward to.

indicates vector product

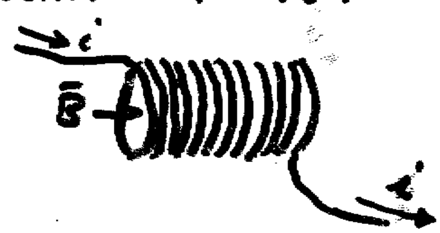
ELECTROMAGNETIC WAVES TRANSPORT ENERGY POYNTING VECTOR

- Wherever there is an electric field $\vec{E}$, there is an Energy density u_E (energy per unit volume, Joule/m^3) associated to it:

$$u_E = \frac{1}{2}\epsilon_0 E^2 \qquad (\text{Joule}/m^3)$$

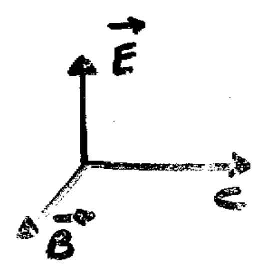
- Wherever there is a magnetic field $\vec{B}$, there is an Energy density u_B (Joules/m^3) associated to it:

$$u_B = \frac{1}{2\mu_0} B^2$$



- In an electromagnetic wave, the electromagnetic energy density will be given by

$$u = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2$$



we can use $B = \frac{E}{c}$

$$= \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2}\epsilon_0 E^2$$

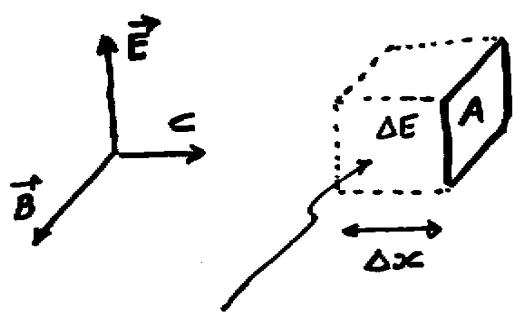
where we have used $c^2 = \frac{1}{\epsilon_0 \mu_0}$

Electromagnetic energy density (Joule/m^3)

$$\boxed{u = \epsilon_0 E^2} \quad \text{or} \quad \boxed{u = \frac{B^2}{\mu_0}} \quad \text{or} \quad \boxed{u = \frac{EB}{\mu_0 c}}$$

But this energy is propagating with velocity c

Let's consider a section of area A



the amount of energy in this cubic volume is $\Delta E = u \, A \, \Delta x$

In an interval of time $\Delta t = \dfrac{\Delta x}{c}$ all the energy in that cubic volume will have passed through the section of area A

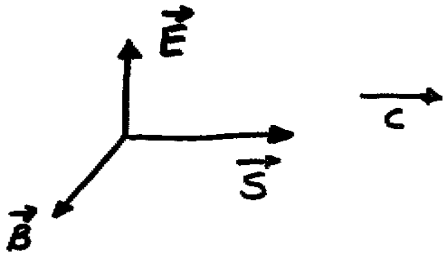
Energy per unit time crossing the area A $= \dfrac{\Delta E}{\Delta t} = \dfrac{u A \Delta x}{\Delta x / c} = u A c$

ENERGY per unit time and per unit area $\quad S = u\, c = \dfrac{E B}{\mu_0}$

$$\boxed{\vec{S} = \frac{1}{\mu_0}\,\vec{E}\times\vec{B}}$$

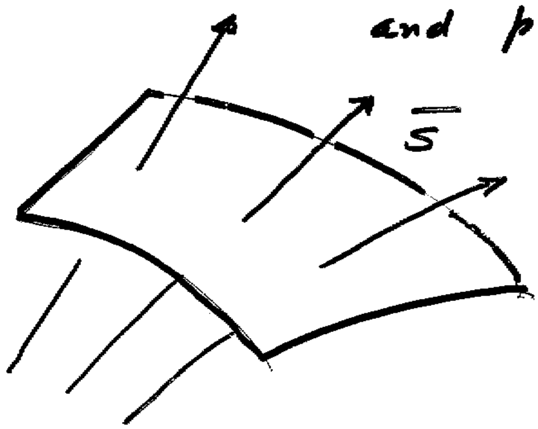
Poynting vector

indicates vector product



S is the flow of electromagnetic energy traveling through the space, per unit area and per unit time.

$$\left[\frac{J}{m^2\,sec}\right]$$



$$S = u\,c$$

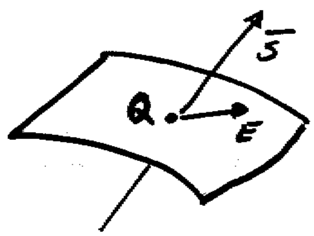
energy density

speed of light

$$\boxed{S = \frac{1}{\mu_0} E B} \qquad \boxed{S = \epsilon_0 c E^2} \qquad \boxed{S = \frac{c}{\mu_0} B^2}$$

S: power per unit area $\left(\text{Watt}/m^2\right)$

AVERAGE ENERGY $\langle S \rangle$: INTENSITY



$$S = \epsilon_0 c E^2$$

Let's assume the electric field at the position "Q" is of the form

$$E = E_m \cos(\omega t)$$

The Poynting vector takes the form:

$$S = \epsilon_0 c E_m^2 \cos^2(\omega t)$$

If we take average over time:

$$\langle S \rangle = \epsilon_0 c E_m^2 \underbrace{\langle \cos^2(\omega t) \rangle}_{= \frac{1}{2}}$$

$$\boxed{I = \langle S \rangle = \tfrac{1}{2} \epsilon_0 c E_m^2}$$ Intensity — power per unit area (Watt/m^2)

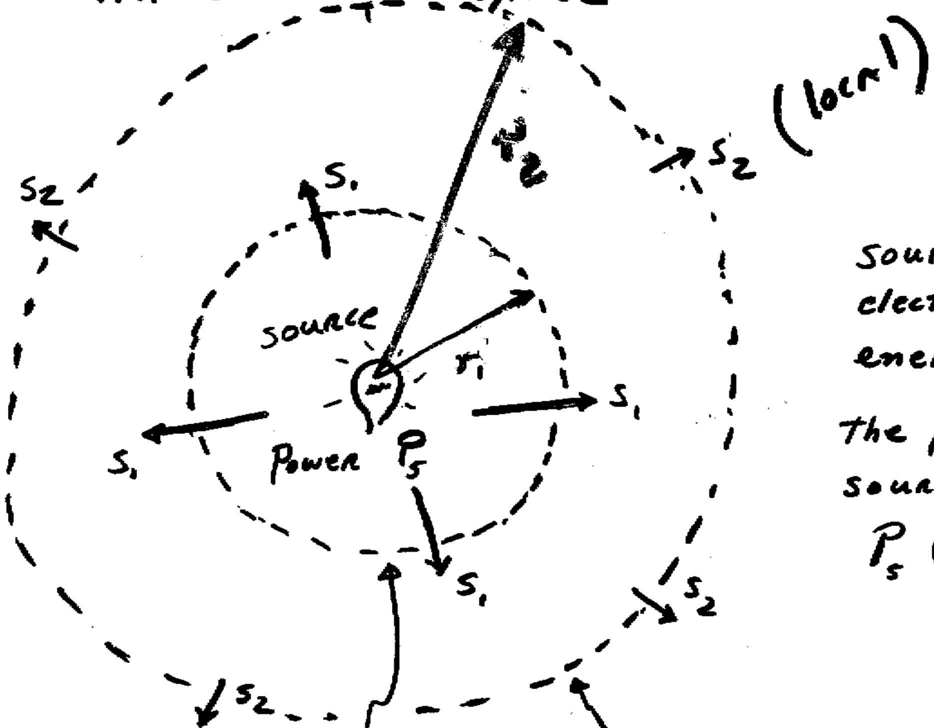
Sometimes, this result is given in terms of rms value of the electric field amplitude

$$E_{rms} \equiv \frac{E_m}{\sqrt{2}} \quad \text{(definition)}$$

$$\boxed{I = \epsilon_0 \cdot E_{rms}}$$ Intensity (Watt/m^2)

Intensity decreases with distance from the source

CASE: PUNCTUAL SOURCE emiting energy all over the space



Source emits electromagnetic energy

the power of the source is:

P_s (Joules/sec)

Notice, conservation of energy implies:

$$P_s = \langle S_1 \rangle 4\pi r_1^2 = \langle S_2 \rangle 4\pi r_2^2$$

$$= I_1 4\pi r_1^2 = I_2 4\pi r_2^2$$

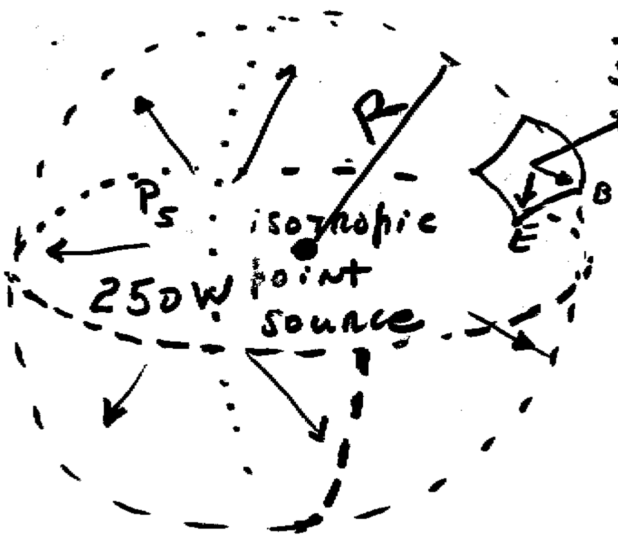
That is, for a constant value of P_s, the intensity varies with the distance r:

$$I(r) = \frac{P_s}{4\pi r^2}$$

Remember:

$$I = \langle S \rangle$$

Example: sample problem 34-1 (pg 811)



$\vec{S}$ $\left[\frac{\text{Joule}}{m^2\ \text{sec}}\right]$

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

$$E = E_m \cos(\omega t)$$

$$\langle s \rangle = \frac{1}{2}\epsilon_0 c E_m^2$$

$$\frac{250\,W}{4\pi R^2} = \langle s \rangle = \frac{1}{2}\epsilon_0 c\ E_m^2 \qquad *$$

$R = 1.8\ m$

$\epsilon_0 =$

$c =$

$$\Rightarrow \quad E_m^2 = \frac{2 \times 250\ \frac{J}{s}}{4\pi(1.8\,m)^2\ \epsilon_0 C}$$

Electric field $E = \frac{\text{Volts}}{m}$ $\left(\begin{array}{c}\text{Remember} \\ E \cdot d = V\end{array} \right)$

$$\text{volt} = \frac{\text{work}}{\text{unit charge}} = \frac{J}{C}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \frac{F}{m}$$

$$= 8.85 \times 10^{-12} \frac{C^2}{J\,m}$$

Remember

$$F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{r^2}$$

so,

$$\epsilon_0 = \frac{C^2}{N\,m^2} = \frac{C^2}{J\,m}$$

Therefore

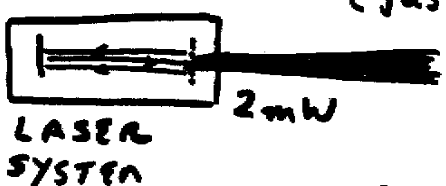
$$E_m^2 = \frac{2 \times 250 \frac{J}{s}}{4\pi\,(1.8\text{ m})^2\; 8.85 \times 10^{-12} \frac{C^2}{J\,m} \times 3 \times 10^8 \frac{m}{s}}$$

$$= 0.46 \times 10^4 \frac{J^2}{m^2 C^2} = 0.46 \times 10^4 \frac{V^2}{m^2}$$

$$E_m = 0.68 \times 10^2 \frac{V}{m}$$

In term of rms: $E_{rms} = \frac{E_m}{\sqrt{2}} = 0.48 \frac{V}{m} \times 10^2$

* Exercise: The beam diameter of a typical He-Ne laser is ~ 2 mm. (just at the output of the laser system case)



Find the amplitudes of the electric field and magnetic field of the laser beam.

$$I = \langle S \rangle = \frac{1}{2} \epsilon_0 c \, E_m^2 = \frac{\text{power}}{\text{unit area}} = \frac{2\,\text{mW}}{\pi (1\,\text{mm})^2}$$

$$= \frac{2 \times 10^{-3}\ \text{J/s}}{\pi (10^{-3})^2 \text{m}^2} = 0.64 \times 10^{3} \frac{\text{J}}{\text{s m}^2}$$

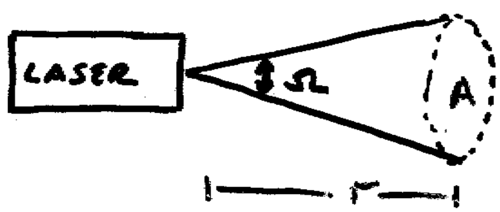
$$\Rightarrow E_m^2 = \frac{2 \times 0.64 \times 10^{3} \frac{\text{J}}{\text{s m}^2}}{\epsilon_0 \, c}$$

$$E_m \approx 700 \frac{\text{V}}{}$$

$$E_m^2 = 48.2 \times 10^{4}$$

$$E_m = 7 \times 10^{2} \frac{\text{V}}{\text{m}}$$

Note: Output laser beam is not perfectly collimated.



Solid angle Ω is measured in radians

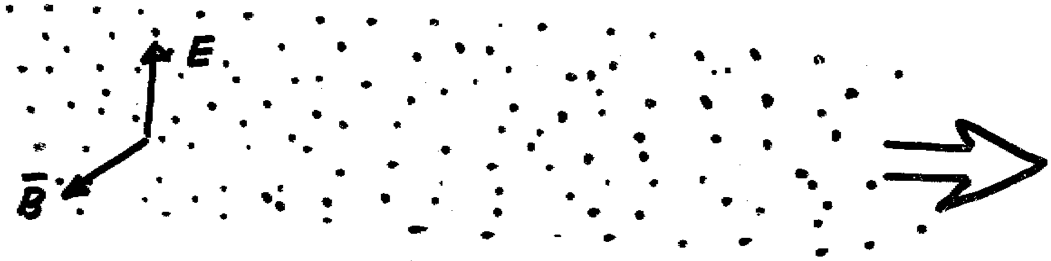
$$\Omega = \frac{A}{r^2}$$

where

A : cross section area

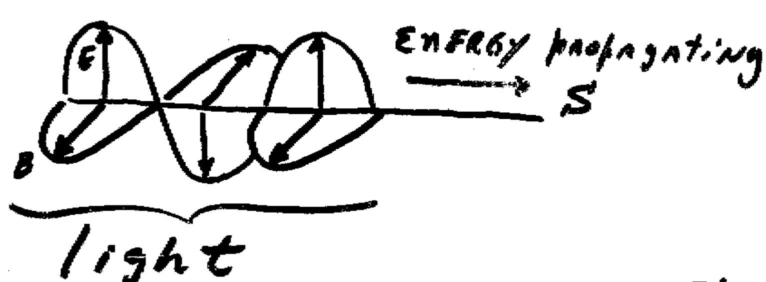
r : distance

LIGHT as a WAVE



- Light is a wave of electric fields and magnetic fields

- The vector product $\vec{E} \times \vec{B}$ gives the direction of light propagation



$$\langle S \rangle = \tfrac{1}{2} \epsilon_0 c E_m^2$$

CLASSICAL VIEW → The higher E_m the higher the power propagating in the wave.

However, all this nice theory could not explain the following experimental result:

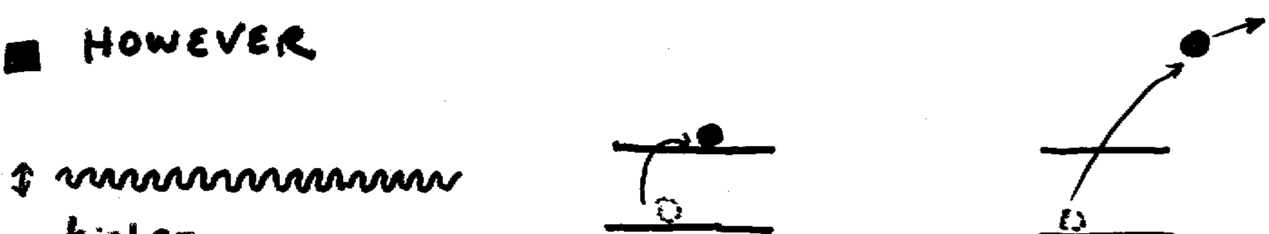
ATOM

■ Small amplitude E_m

higher E_m

no matter how much you increase the incident light power (increase of E_m) the electron is not excited to the upper level of energy.

■ HOWEVER

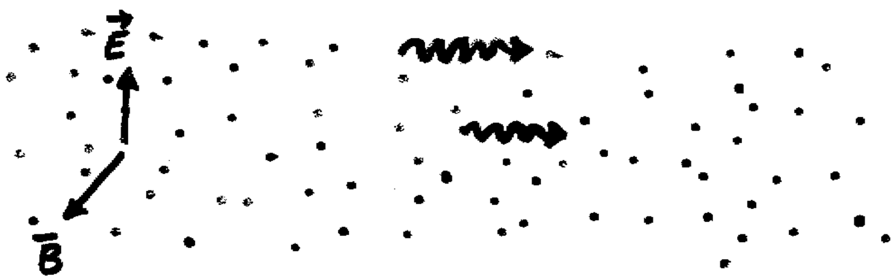


higher frequency light

Photo-electric effect

how to explain it ?

LIGHT as a Radiation Field of Quantized energy packets



Light is made out of photons.

$$1 \text{ photon energy} = h\,f = \frac{hc}{\lambda}$$

f: frequency of the radiation

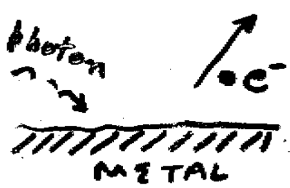
h: constant #
"planck constant"
$= 6.6 \times 10^{-34}$ Joule. sec
$= 4.14 \times 10^{-15}$ eV. sec

* Exercise: Find the energy of 1 photon whose wavelength is $\lambda = 500$ nm ("green light

LIGHT $\xrightarrow{\text{behaves as a}}$ WAVE

or any ELECTROMAGNETIC WAVE $\xrightarrow{\text{behaves as a}}$ particle

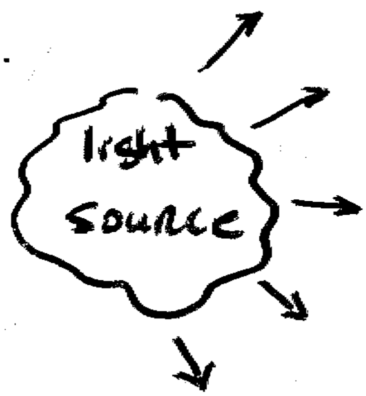
The particle nature of light was first proposed by Albert Einstein in 1905 in his explanation of the "photo electric effect."



$$E_{photon} = hf = 6.6\times 10^{-34}\ \text{Joule}\cdot\text{sec}\cdot f$$
$$= 4.14\times 10^{-15}\ \text{eV}\cdot\text{sec}\cdot f$$

$$= h\frac{c}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{\lambda}$$

nm = nano-meter
$= 10^{-9}$ m

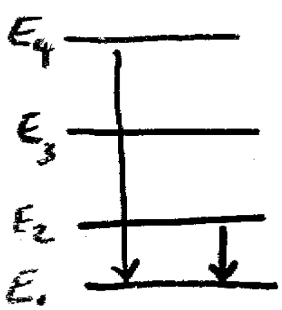


Experimental observation of a discrete lines spectra

photographic film

Hypothesis

$$\Delta E = \frac{hc}{\lambda}$$



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Measurements of discrete values of λ allowed to make models of the <u>ATOM</u>

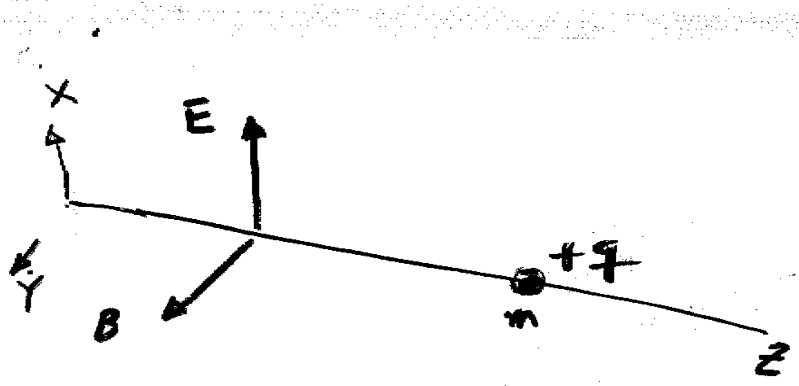
LIGHT WAVE-particle duality

the propagation of light is governed by its WAVE properties

the light-matter interaction is governed by its particle-like character.

If light behaves as a particle, does it have momentum ?

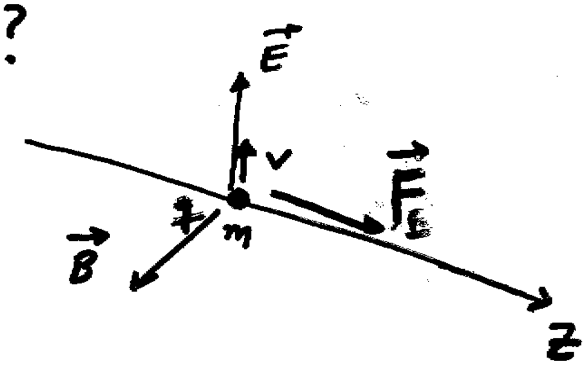
Angular momentum ?



E → will shake the charge up and down (along the X-axis)

What about B ?

$\vec{F}_B = q\,\vec{v} \times \vec{B}$



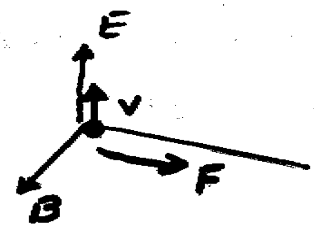
$\vec{F}_B$ acts in the direction of the propagation of light

So, the mass m will gain some linear momentum along the Z-axis

$$F = qvB$$

$$B = \frac{E}{c}$$

$$= \frac{v}{c}\, qE$$



velocity × force
= power
= work W done (by the electric field E) on the charge q, per unit time

$$F = \frac{1}{c}\frac{dW}{dt} = \frac{d}{dt}\underbrace{\left(\frac{W}{c}\right)}_{\text{momentum}}$$

W must come from the energy of the light

if one photon is absorbed

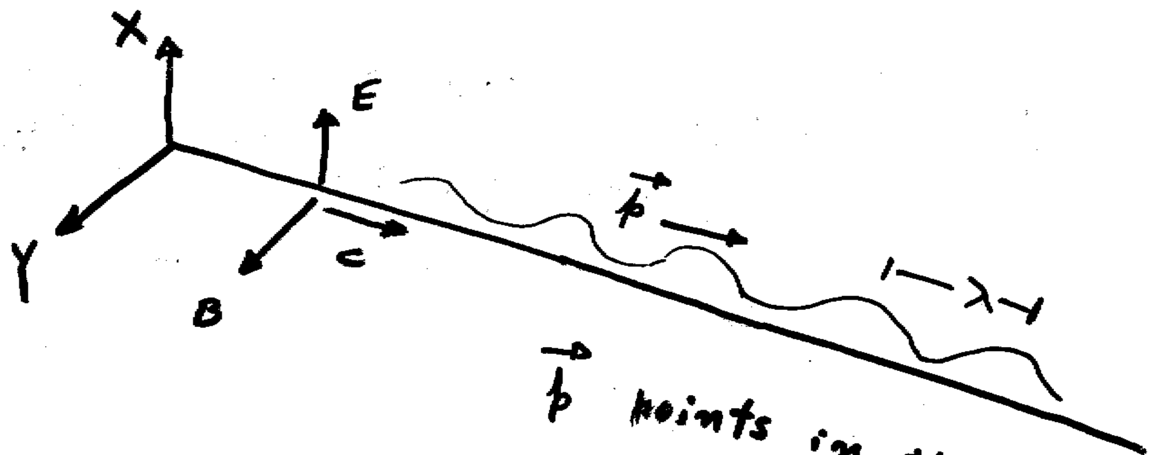
$$p = \frac{W}{c} = \frac{hF}{c}$$

$$W = hF$$

h is the Planck's constant

$$\boxed{p = \frac{h}{\lambda}}$$

Linear momentum carried by a photon



$\vec{p}$ points in the direction of the light propagation

$$p = \frac{h}{\lambda}$$

* Exercise: Problems 13E, 14E, 16P; Chap 34, p. 828
6th Ed

MAXWELL's RAINBOW