Summary

MAXWELL EQUATIONS in VACUUM

$$\int_{surface} \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$

$$\int_{surface} \vec{B} \cdot d\vec{A} = 0$$

$$\int_{loop} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_{surf} \vec{B} \cdot d\vec{A}$$

$$\int_{loop} \vec{B} \cdot d\vec{l} = \mu_0 i + \mu_0 \epsilon_0 \frac{d}{dt} \int_{surf} \vec{E} \cdot d\vec{A}$$

E: Electric field

B: magnetic field

$E = E(t)$ $\xrightarrow{\text{produces}}$ B

$B = B(t)$ $\longrightarrow$ E

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 E}{\partial t^2} = 0 \qquad \frac{\partial^2}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} = 0$$

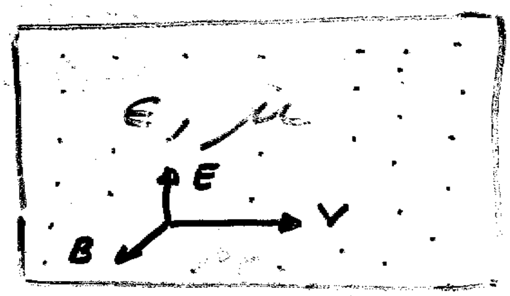
$$c = \sqrt{\frac{1}{\epsilon_0 \mu_0}} \qquad c = 300{,}000 \text{ km/s}$$

$E = E(x \pm ct)$ $\qquad$ $B = B(x \pm ct)$

INDEX of REFRACTION n

VACUUM → $\boxed{\mu_0, \epsilon_0}$ $c = \sqrt{\frac{1}{\epsilon_0 \mu_0}}$

- MATERIAL →
(gas, liquid, solid)



Electromagnetic waves travel through matter with speed v

$$v = \sqrt{\frac{1}{\epsilon \mu}}$$

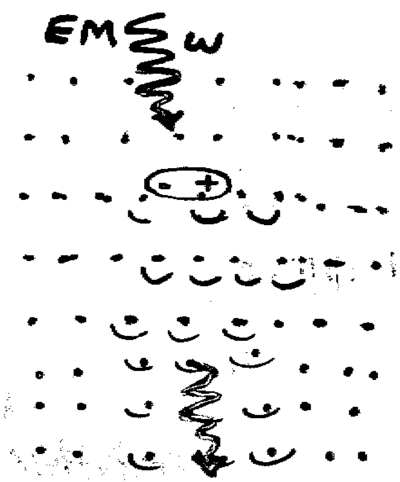
NOTICE

Since $\epsilon > \epsilon_0$, $\mu > \mu_0$ then $\boxed{v < c}$

Since $\epsilon = \epsilon(\omega)$, $\mu = \mu(\omega)$ then $v = v(\omega)$

- Definition index of REFRACTION $n = \frac{c}{v}$

Electromagnetic waves appear to propagate in matter with $V < c$



When EM waves falls on a piece of matter, it induces oscillations in the charged particles, which then emit secondary or scattered waves

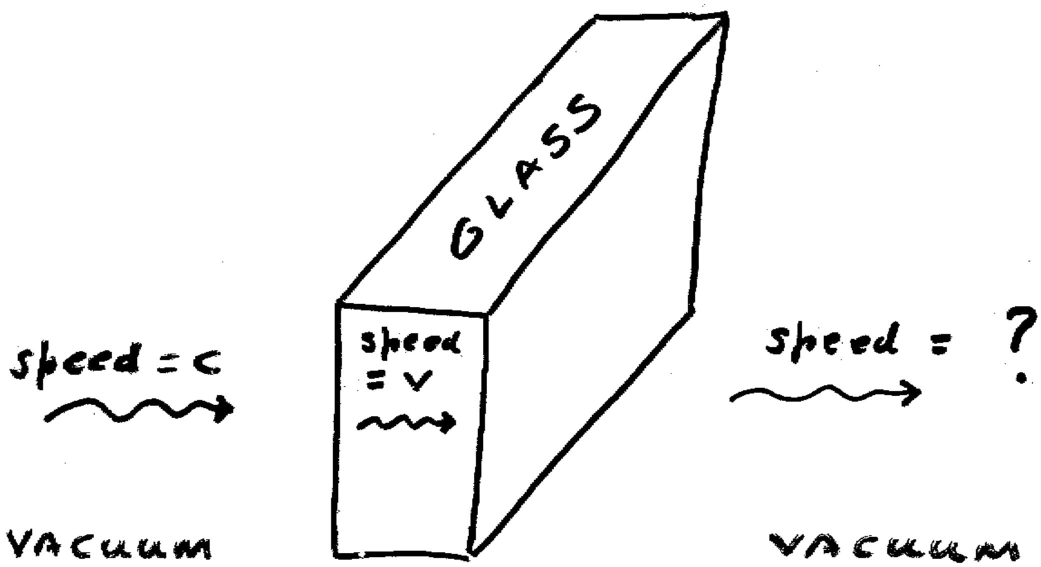
These scattered waves are superposed on the original EM wave, giving a resultant wave.

Incident EM wave (speed c)	+	scattered waves (speed c)	=	RESULTANT WAVE (speed v)

waves No necessarily in phase

This phase difference affects the resultant wave in such a way that this wave appears to have a velocity $< c$.

Does light slow down?



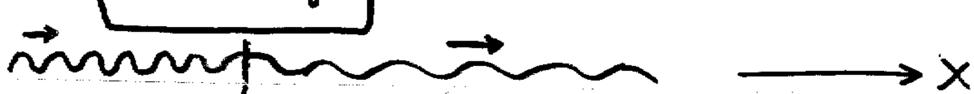
EM radiation propagating through different mediums

* frequency remains the same
  wavelength changes according to the index of refraction

GLASS	VACUUM
velocity → v	c
$n_g = \frac{c}{v}$	$n = \frac{c}{c} = 1$
$v = \lambda_g f$	$c = \lambda f$ → frequency remains the same

$$n_g = \frac{c}{v} = \frac{c}{\lambda_g f} = \frac{1}{\lambda_g}\frac{c}{f} = \frac{1}{\lambda_g}\lambda$$

$$\lambda_g = \frac{\lambda}{n_g}$$



λ_g wavelength inside glass

λ wavelength in vacuum

$$\lambda_g < \lambda$$

Index of refraction changes with frequency

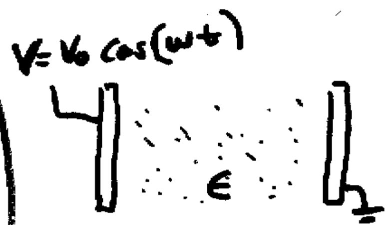
$$n = n(\omega) \quad \text{"DISPERSION"}$$

AIR | GLASS

$$n_g = \frac{c}{v} = \frac{\sqrt{\frac{1}{\epsilon_0 \mu_0}}}{\sqrt{\frac{1}{\epsilon \mu}}} = \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{K}$$

K ← dielectric constant

ϵ measures the polarizability of the medium

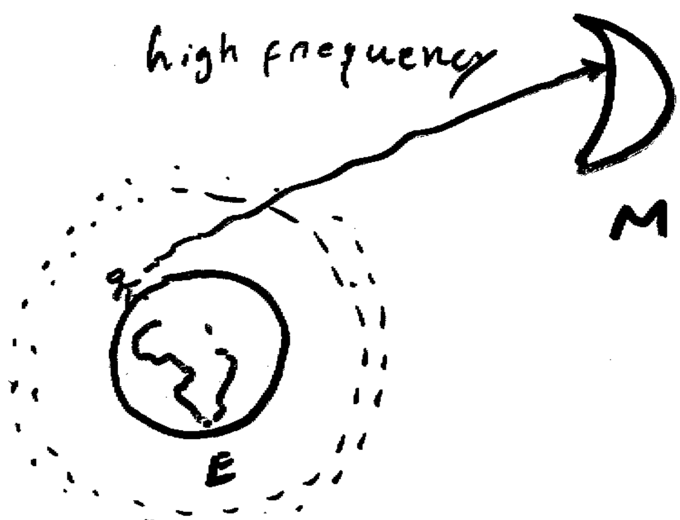


ϵ depends on the frequency ω

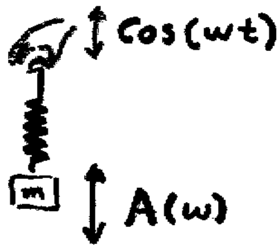
$$\epsilon = \epsilon(\omega)$$

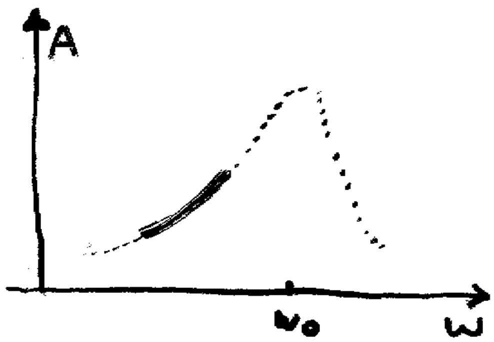
$n_a = n_a(\omega)$ | So, $n_g = n_g(\omega)$


low frequency signal
IONOSPHERE


high frequency
M
E

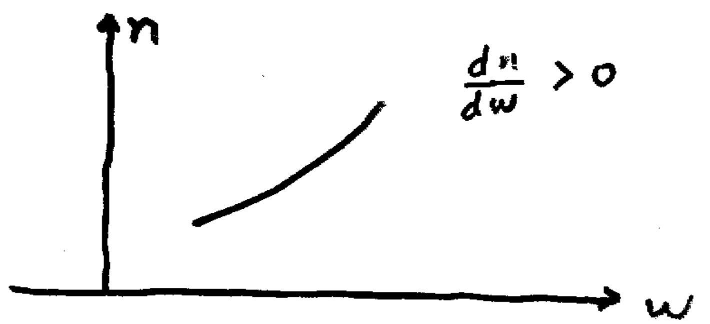
Mechanical analogy



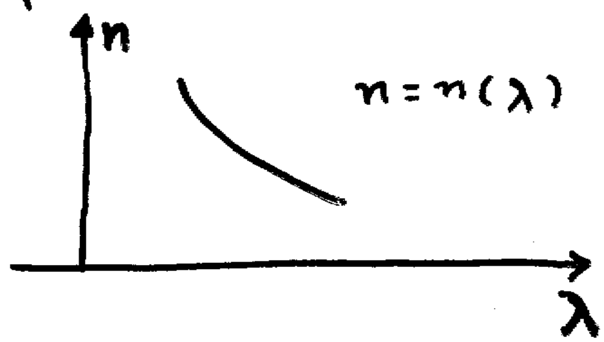


$$n_g = n_g(w)$$

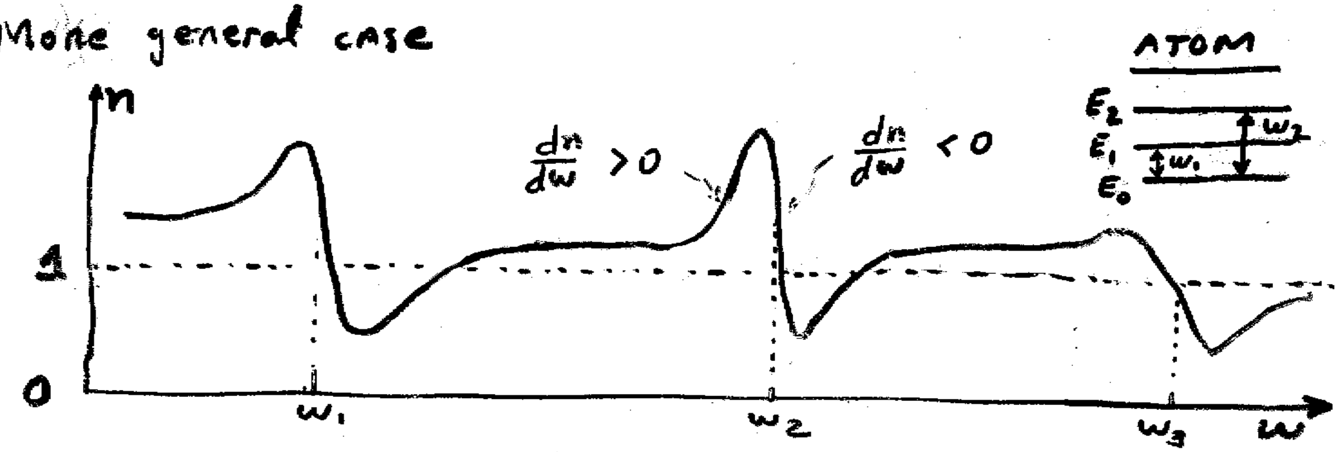
Typical variation of n vs w (called "normal" dispersion)

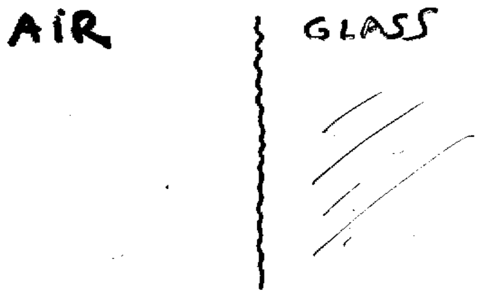


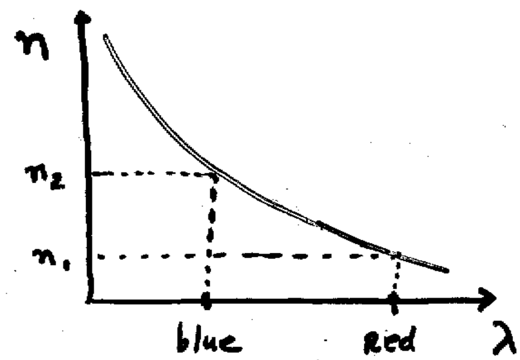
or, equivalent



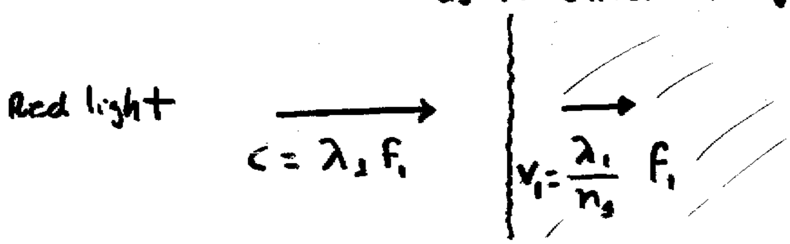
More general case





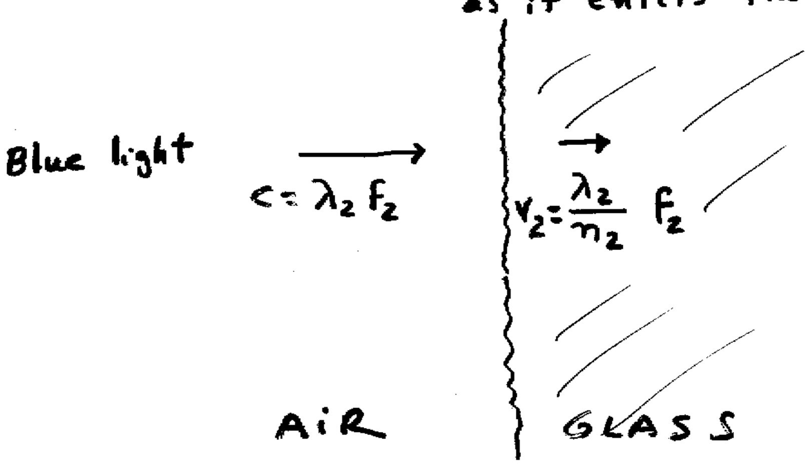


λ_1 "sees" an index of refraction n_1 as it enters the glass



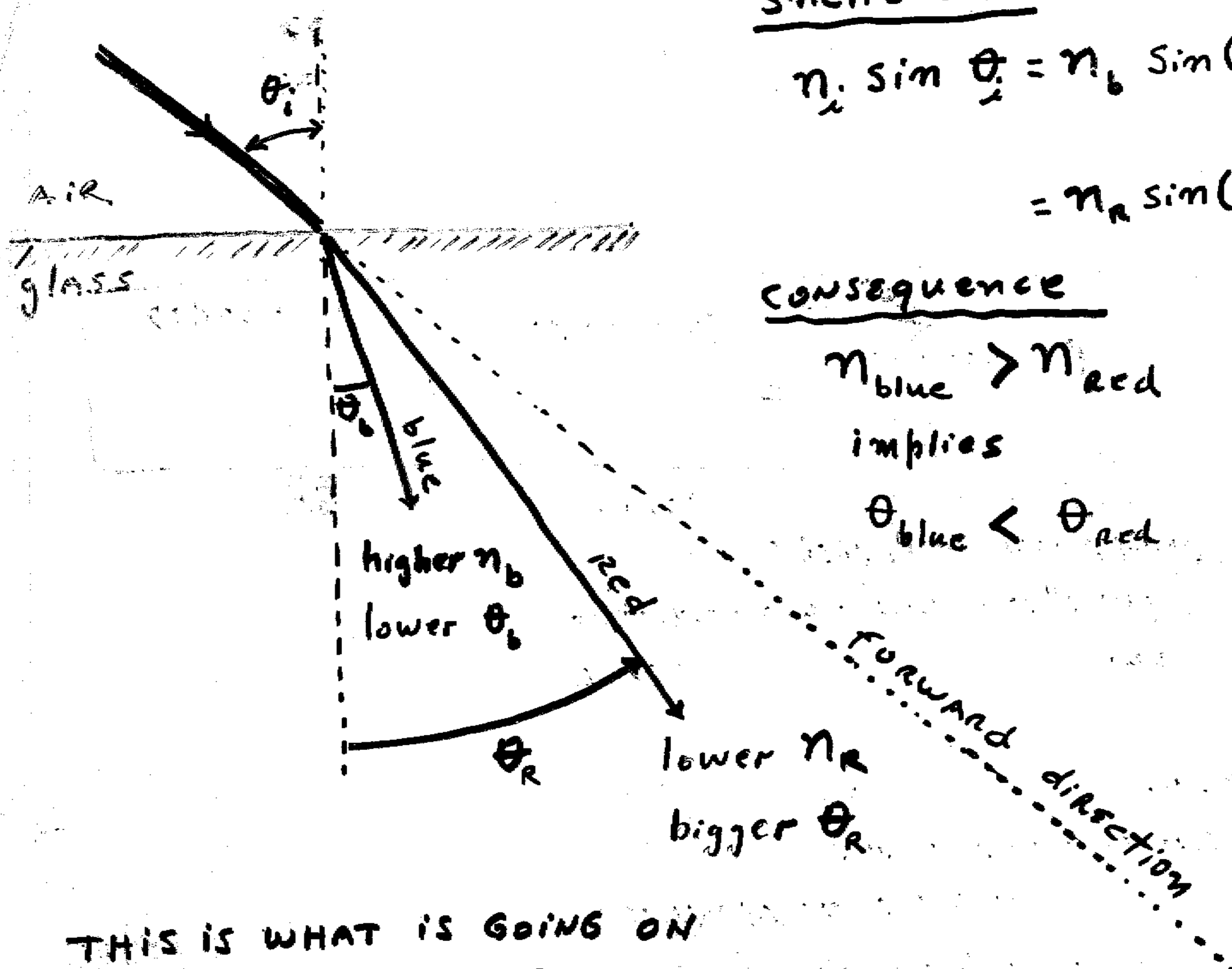
- Frequency f_1 is the same in air than in the glass
- wavelength decreases by a factor of n_1

λ_2 "sees" an index of refraction n_2 as it enters the glass



- Frequency f_2 is the same in air and in the glass
- wavelength decreases by a factor of n_2

Light dispersion at a air-glass interface.



Snell's law

$$n_i \sin \theta_i = n_b \sin(\theta_b)$$

$$= n_R \sin(\theta_R)$$

consequence

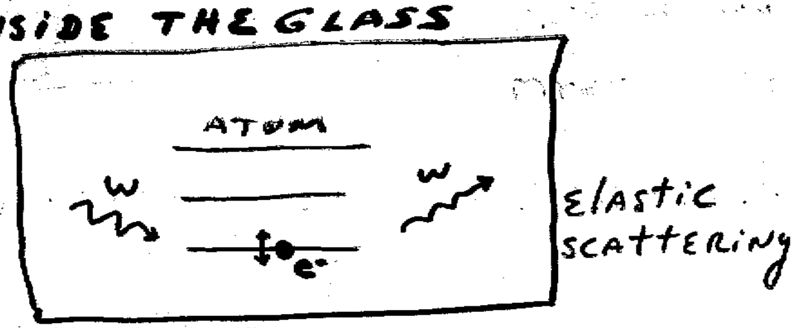
$n_{blue} > n_{red}$

implies

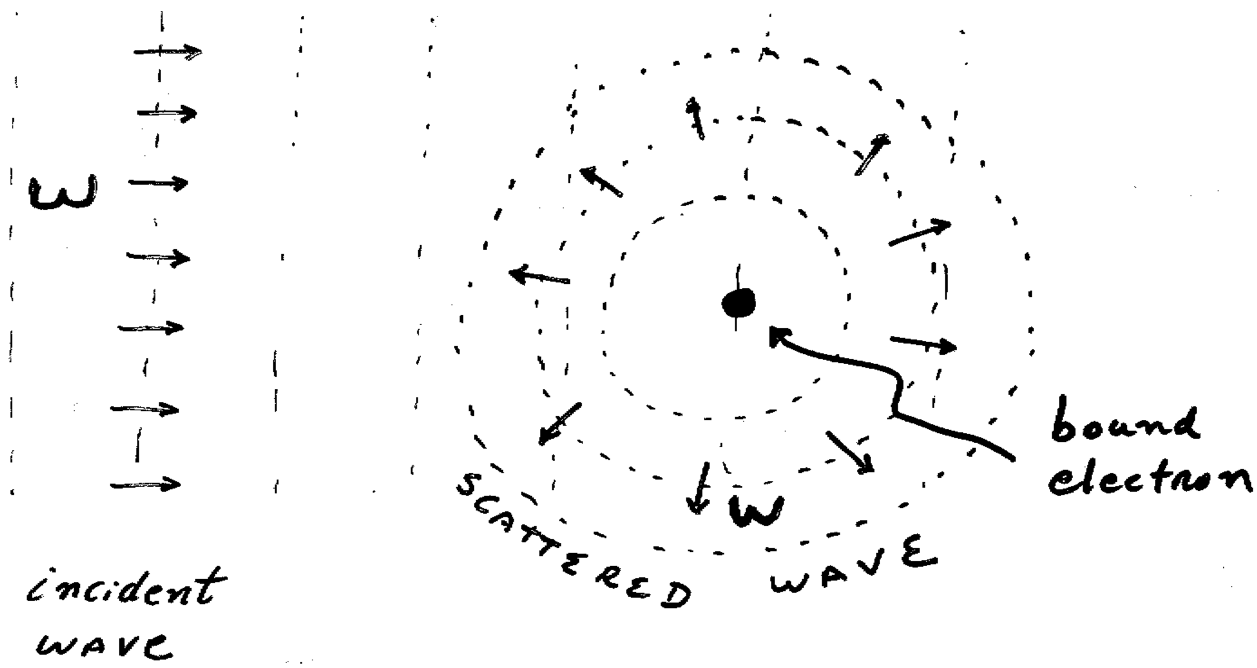
$\theta_{blue} < \theta_{red}$

blue light deviates more from this forward direction

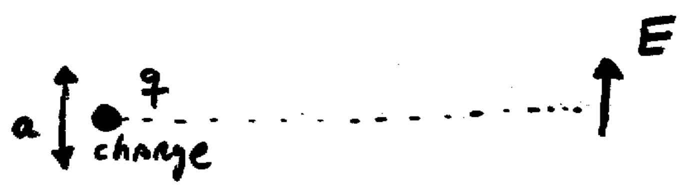
THIS IS WHAT IS GOING ON INSIDE THE GLASS



WHY IS THE SKY BLUE ?



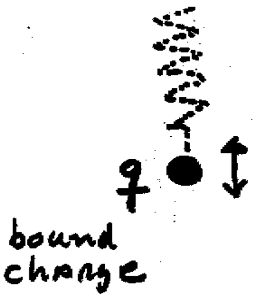
CASE $\lambda \gg \delta$ (δ = particle size)



An accelerated charge emits EM radiation

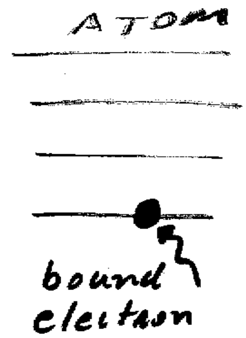
$E \sim a$

electric field proportional to the acceleration

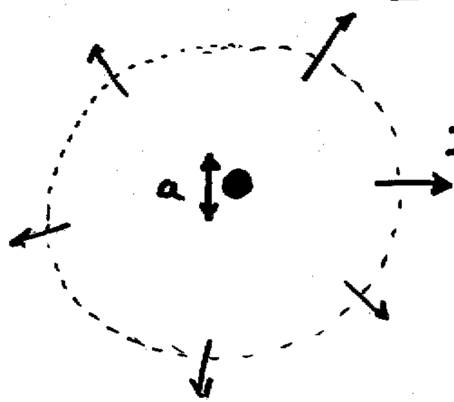


$x = A\cos(\omega t)$

$a \sim \omega^2$ (acceleration)



I: intensity (power)

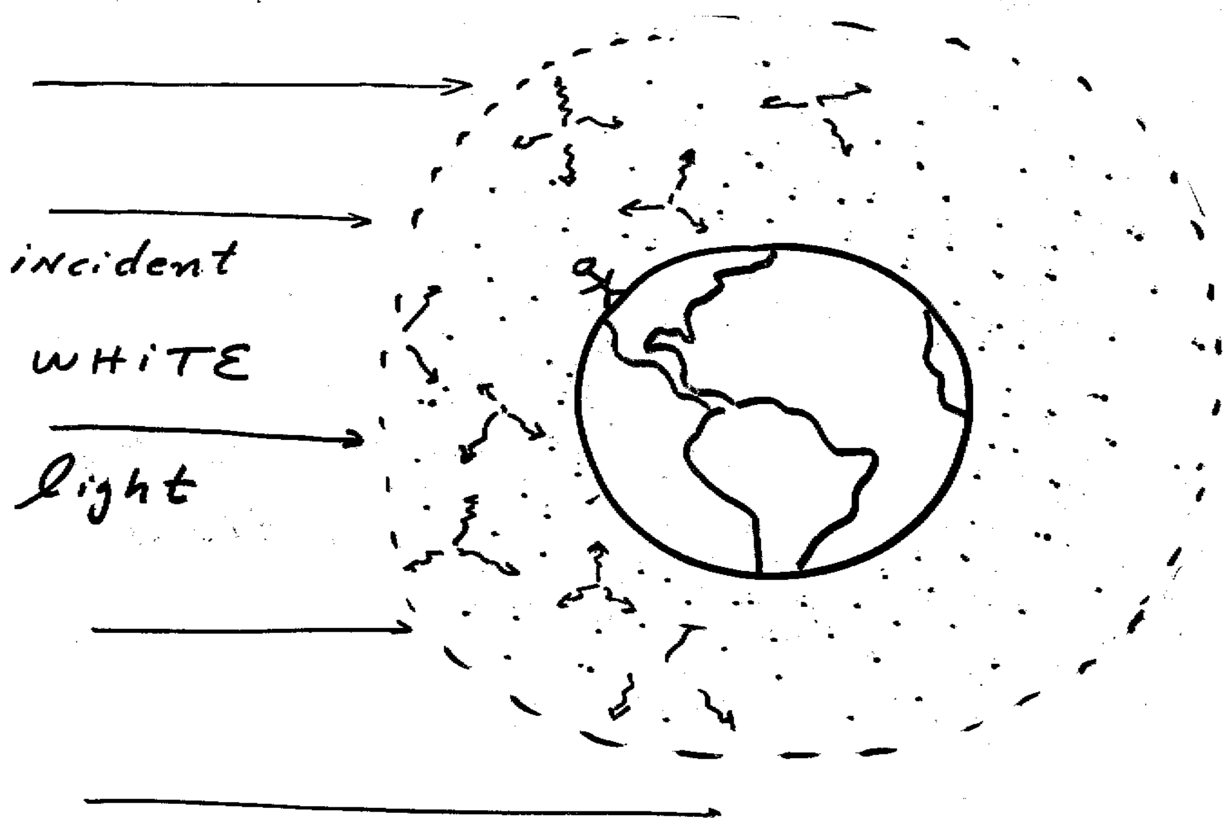


$I \sim E^2$

$$I \sim E^2 \sim (a)^2 \sim (\omega^2)^2$$

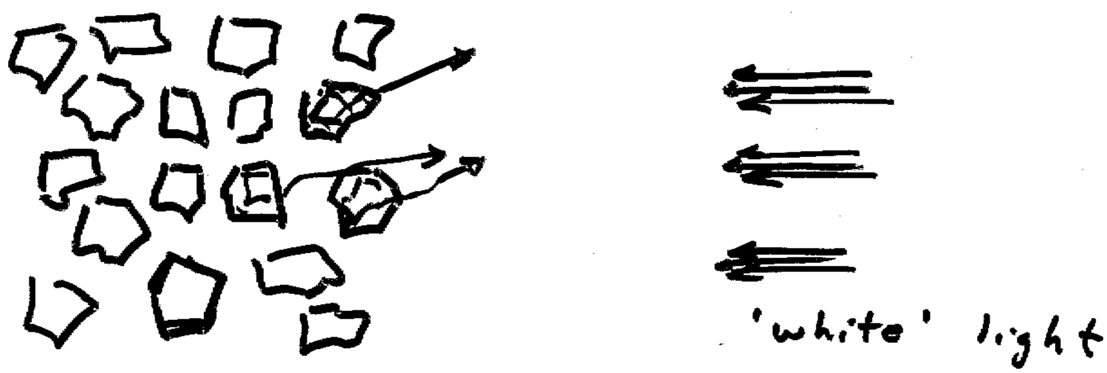
Thus $\boxed{I \sim \omega^4}$

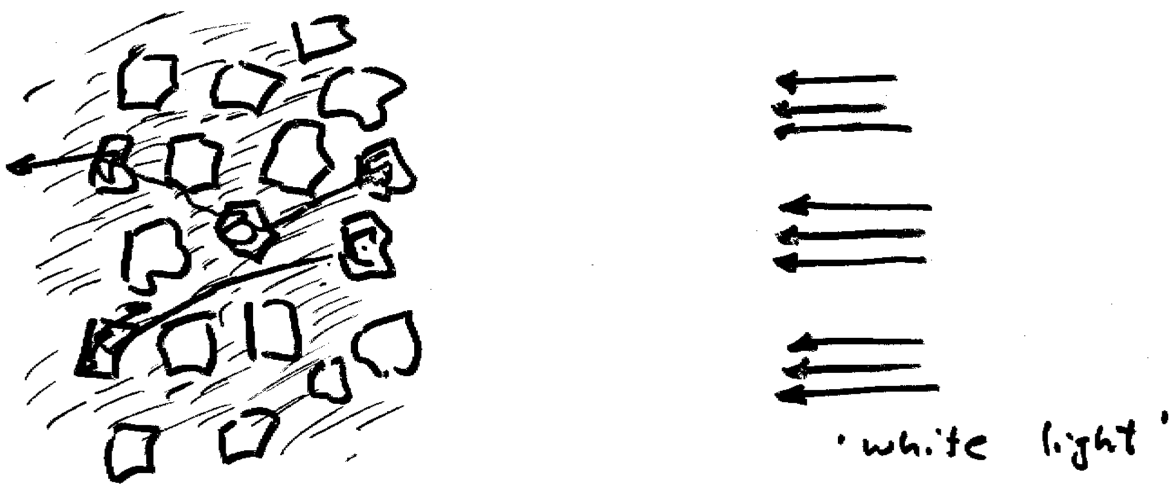
Light of higher frequency (i.e. blue) is scattered more efficiently than light of lower frequency (i.e. red)



In a bright clear day:
sunlight incident in ONE direction is scattered in all direction by the molecules in the atmosphere. You will receive light from every direction and the main color component will be blue (because of the ω^4 factor

CASE $\lambda < \delta$ (particle size)





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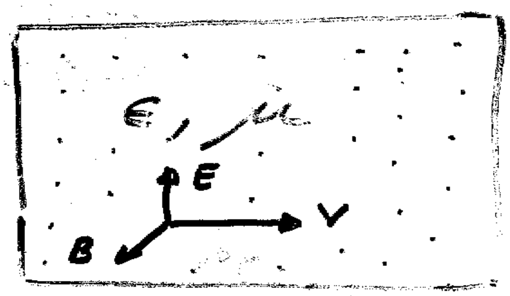

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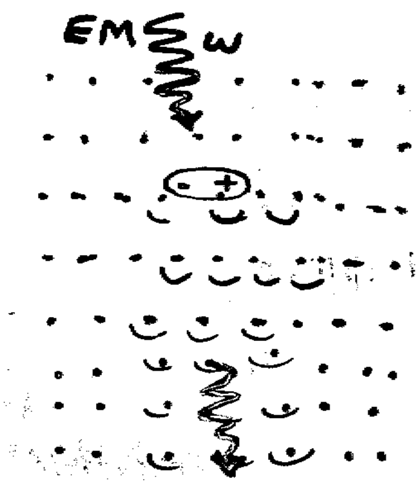
Electromagnetic waves appear to propagate in matter with V < c



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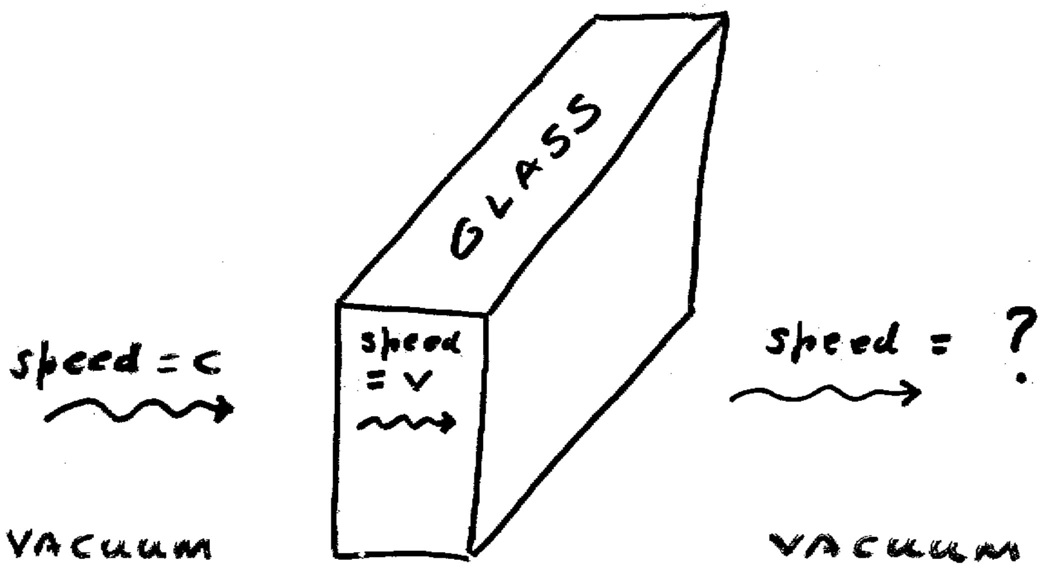

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$$\lambda_g = \frac{\lambda}{n_g}$$

λ_g wavelength inside glass

λ wavelength in vacuum

$$\lambda_g < \lambda$$