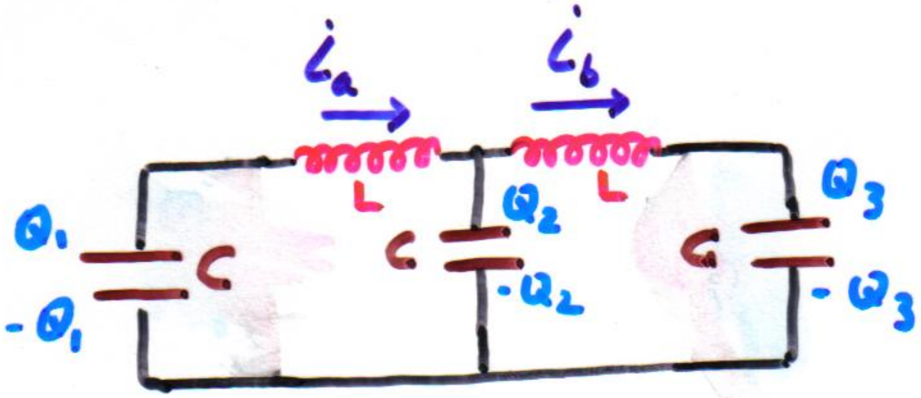
Example Two coupled LC circuits



$$i_a = -\frac{dQ_1}{dt} \qquad\qquad i_b = \frac{dQ_3}{dt}$$

$$\frac{dQ_2}{dt} = i_a - i_b$$

From the left loop:

$$\frac{Q_1}{C} - L\frac{di_a}{dt} - \frac{Q_2}{C} = 0$$

$\Rightarrow$

$$L\frac{d^2 i_a}{dt^2} = \frac{1}{C}\frac{d}{dt}(Q_1 - Q_2)$$

$$L\frac{d^2 i_a}{dt^2} = -\frac{1}{C} i_a + \frac{1}{C}(i_b - i_a) \qquad \text{Eq 1}$$

From the right loop

$$\frac{Q_2}{C} - L\frac{d\,i_b}{dt} - \frac{Q_3}{C} = 0$$

$$L\frac{d^2 i_b}{dt^2} = -\frac{1}{C}\frac{dQ_3}{dt} + \frac{1}{C}\frac{dQ_2}{dt}$$

$$L\frac{d^2 i_b}{dt^2} = -\frac{1}{C}i_b - \frac{1}{C}(i_b - i_a) \qquad \text{Eq. 2}$$

From Eq. 1 and Eq. 2 we obtain:

$$L\frac{d^2}{dt^2}(i_b + i_a) = -\frac{1}{C}(i_b + i_a)$$

and

$$L\frac{d^2}{dt^2}(i_b - i_a) = -3\frac{1}{C}(i_b - i_a)$$

} Eq. 3

1

If we arrange the experimental setup such that $i_b = i_a$, Eq. 3 becomes

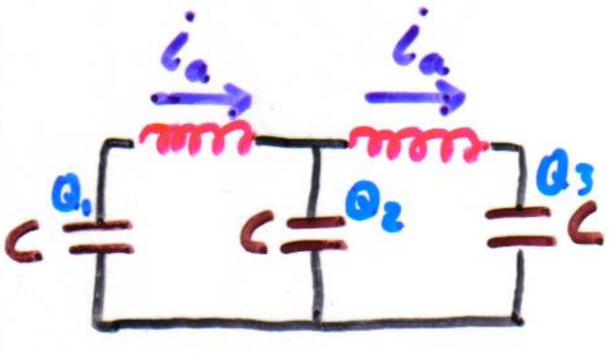
$$L \frac{d^2}{dt^2} i_a = -\frac{1}{C} i_a \Rightarrow \omega_1^2 = \frac{1/C}{L}$$

mode-1: $i_b = i_a$

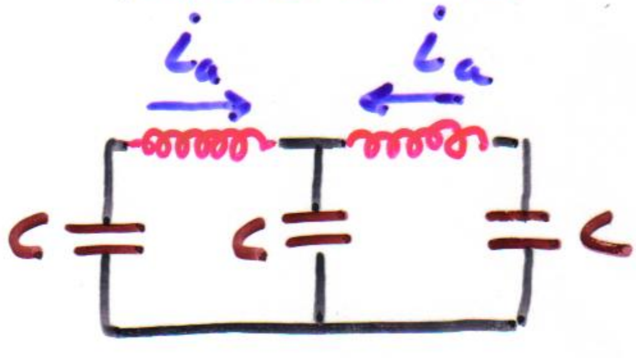
If we arrange the experiment such that $i_b = -i_a$, Eq. 3 becomes

$$L \frac{d^2 i_a}{dt^2} = -3\frac{1}{C} i_a \Rightarrow \omega_2^2 = 3\frac{1/C}{L}$$

MODE-1



MODE-2



Notice $\frac{d}{dt} Q_2 = 0$

So, the central capacitor does not participate

Free Oscillations

CASE: System with many degrees of freedom