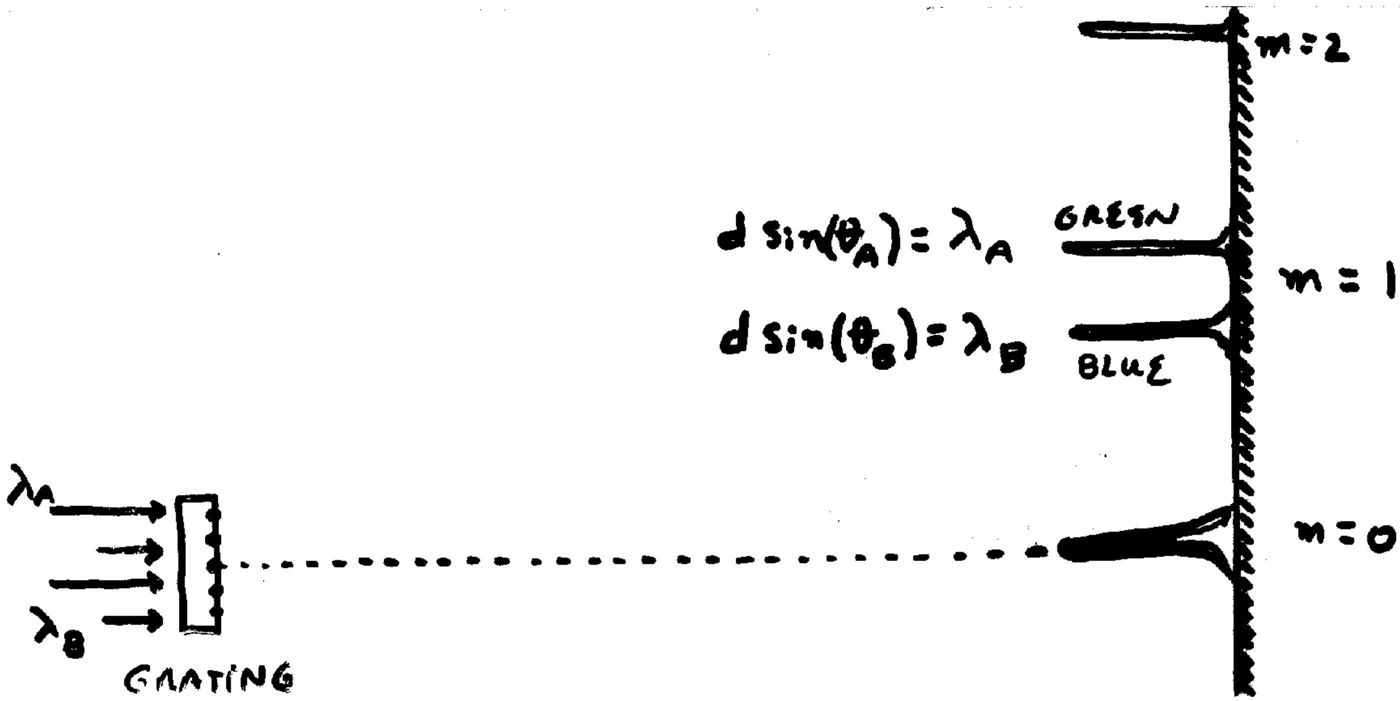


We realize: GRATINGS can be used for separating light into its different wavelegth components.

In this regard, gratings work like a prism

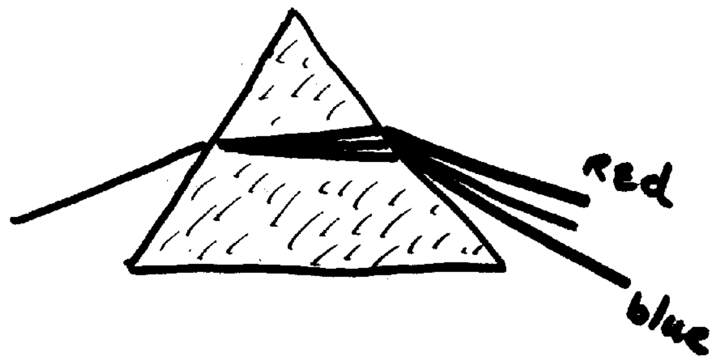
WARNING: there is a difference, however, in the way light is dispeased by a grating or dispersed by a prism.

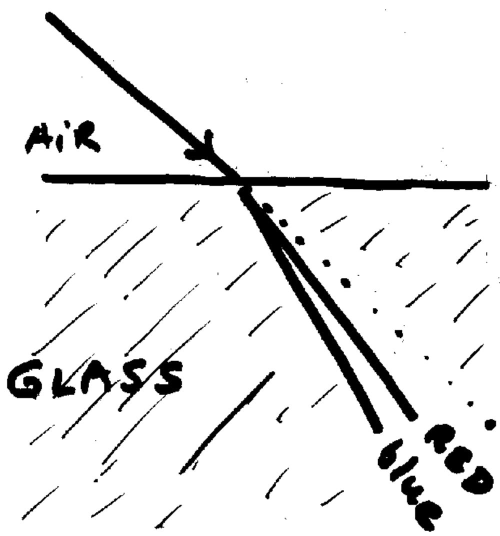
IN A GRATING:

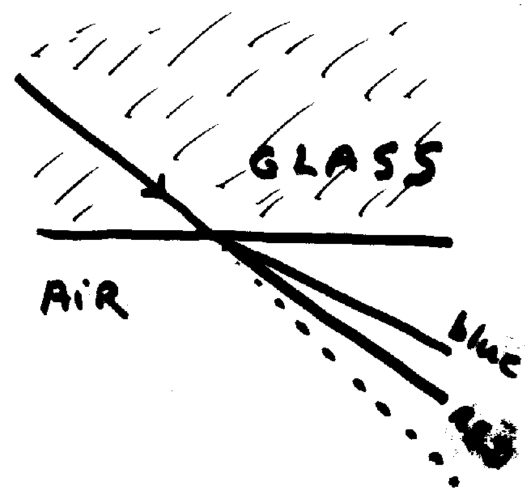
For intensity maxima of order $m \geq 1$, longer wavelengths deviate more from the incident direction

In a prism

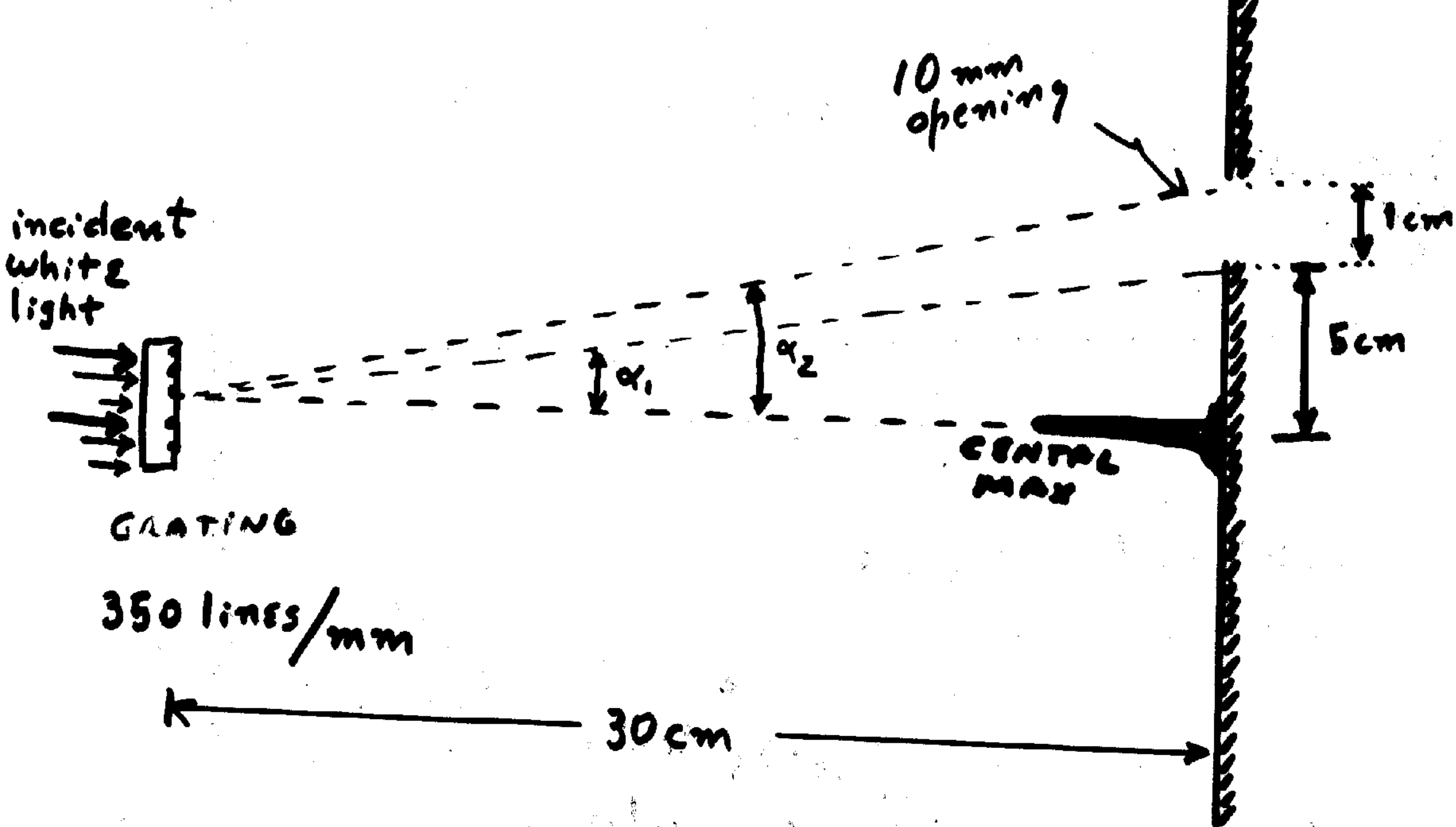
shorter wavelengths deviate more from the incident direction







Problem 44P



The question is; which wavelengths (colors) pass through the opening?

Solution:

- The opening window defines two angles:

$$\alpha_1 = \tan^{-1}\frac{5}{30} = 9.46^\circ \quad \text{and} \quad \alpha_2 = \tan^{-1}\frac{6}{30} = 11.31^\circ$$

- 350 lines/mm implies that the distance between line and line, d, is equal to $1000\,\mu m/350$

- Those wavelengths that produce maxima of intensity at angles θ (which are determined by the expression $d \sin\theta = m\lambda$) greater than 9.46° but lower than 11.31°, will pass through the opening.

- Let's find out which wavelength makes a big maximum of interference at $\alpha_1 = 9.46°$

$$d \sin(9.46°) = m\lambda$$

$$(2.86\,\mu m) \cdot \sin(9.46°) = m\lambda$$

$$0.47\,\mu m = m\lambda \qquad (1)$$

Since the white visible light includes wavelengths in the region from ~400 nm to 700 nm, that is λ in the expression above must satisfy $400\text{ nm} < \lambda < 700\text{ nm}$

therefore, expression (1) above can be satisfied with $m = 1$ and $\lambda = 470$ nm. ←

Choosing $m=2$ would have required λ to be 350 nm; but that wavelength is out of the white light range.

- Similarly, let's find out which wavelength makes a big max at $\alpha_2 = 11.31°$

$$d \sin(11.31°) = m\lambda$$

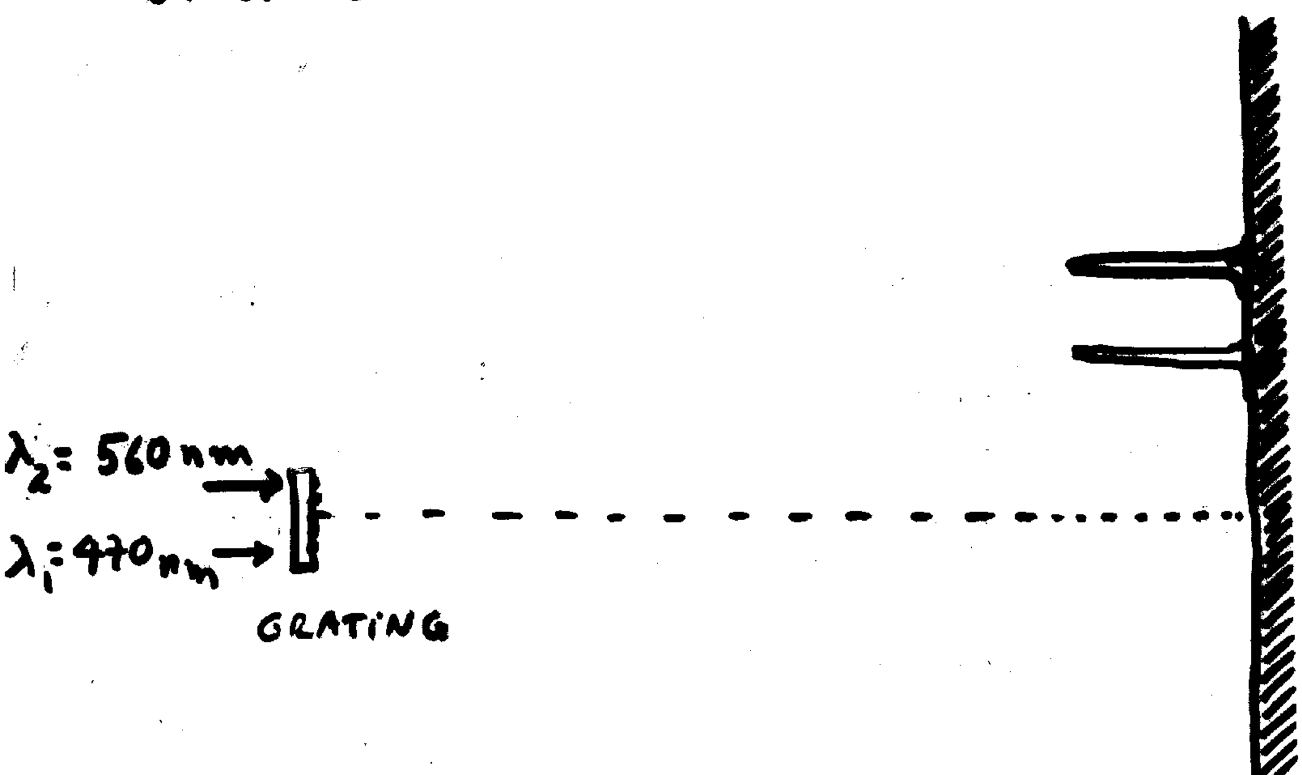
$$(2.86\,\mu m) \times \sin(11.31) = m\lambda$$

$$0.560\,\mu m = m\lambda \qquad (2)$$

For λ in the visible range this expression is satisfied with $m=1$, which gives $\lambda = 560\,nm.$ ←

Therefore, we conclude that only wavelength in the range $470\,nm < \lambda < 560\,nm$ will pass through the opening on the screen.

In the previous example we have found that radiation of wavelength $\lambda_1 = 470$ nm and $\lambda_2 = 560$ nm are easily separated by the grating. Their corresponding maxima of intensity are separated by an angle of $\Delta\theta \simeq 2^\circ$ or, equivalently, the maxima are located 10 mm apart from each other on a screen.



QUESTION: Given a grating, how much different the wavelengths λ_1 and λ_2 must be in order for the grating to be able to [illegible] in 2 distinguishable peaks ?

Obviously, if $\lambda_2 = \lambda_1$ only ONE peak will result.

If $\lambda_2 = \lambda_1 + \Delta\lambda$ the peak for λ_2 will move a litle bit up.

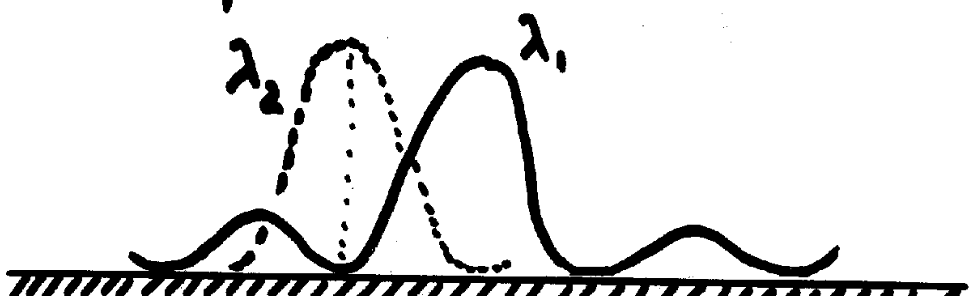
choosing $\Delta\lambda = 30\,nm$ will split the peak in 2 beams separated by $\sim 2^\circ$ (as we verified in the previous example, problem 44P)

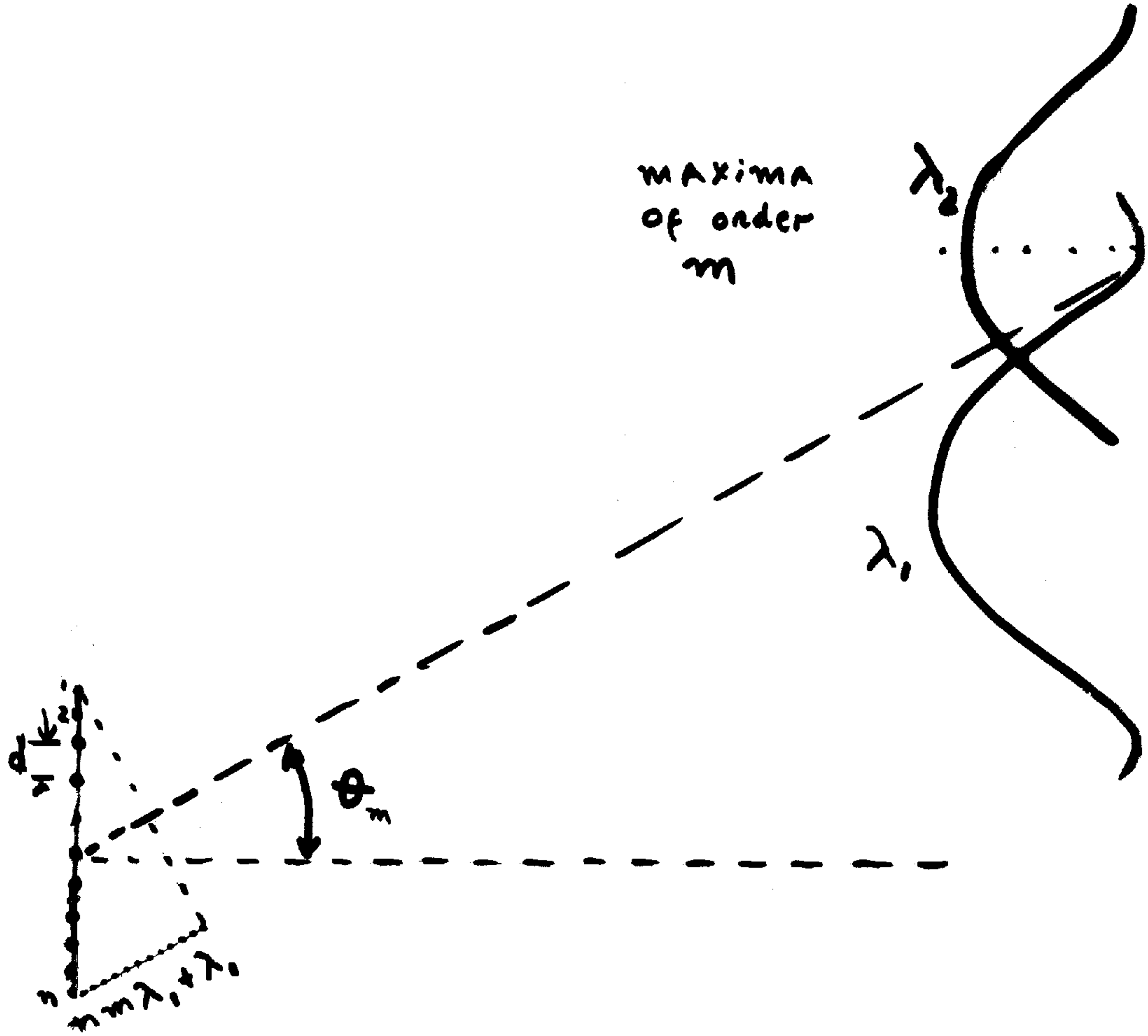
the question is, how small can we choose $\Delta\lambda$ and still be able to distinguish two maxima of intensity on the screen.

(can $\Delta\lambda$ be $10\,nm$? why not $\Delta\lambda = 1\,nm$? If we choose $\Delta\lambda = 0.01\,nm$, could we still distinguish 2 peaks on the screen?)

To answer this question, we will use the Raleigh's criterium:

The minimum of one of the peaks should sit at the maximun of the other peak





Since θ_m is the angle that correspond to a maximum of order m for the wavelength λ_2, the following relationship should hold

$$d \sin(\theta_m) = m \lambda_2 \qquad (3)$$

How much smaller than λ_2 can we choose λ_1 such that it produces a minimun at the same angular location θ_m?

Answer: λ_1 must be small enough such that

$$\underbrace{n}_{\substack{\text{\# of lines} \\ \text{in the} \\ \text{grating}}} d \sin(\theta_m) = n\, m\lambda_1 + \lambda_1 \qquad (4)$$

(remember how we calculated the line thickness of a maximum of intensity on page 22 of these notes)

From (3) and (4) we obtain

$$n m \lambda_2 = n m \lambda_1 + \lambda_1$$

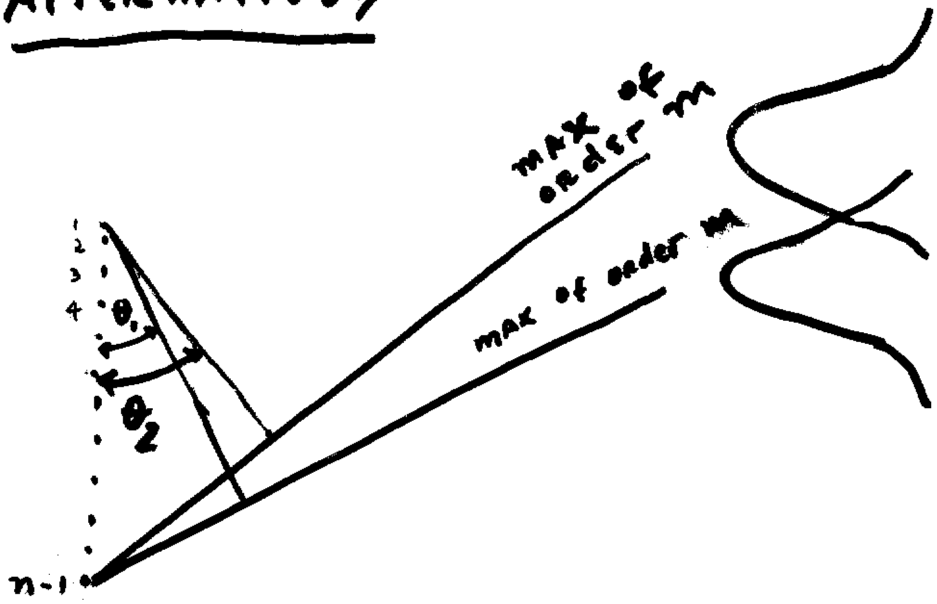
$$\Delta\lambda = \lambda_2 - \lambda_1$$

$$\boxed{\Delta\lambda = \frac{\lambda_1}{n m}}$$

This expression gives the resolution or RESOLVING POWER of the grating.

The higher n, the better resolution

Alternatively

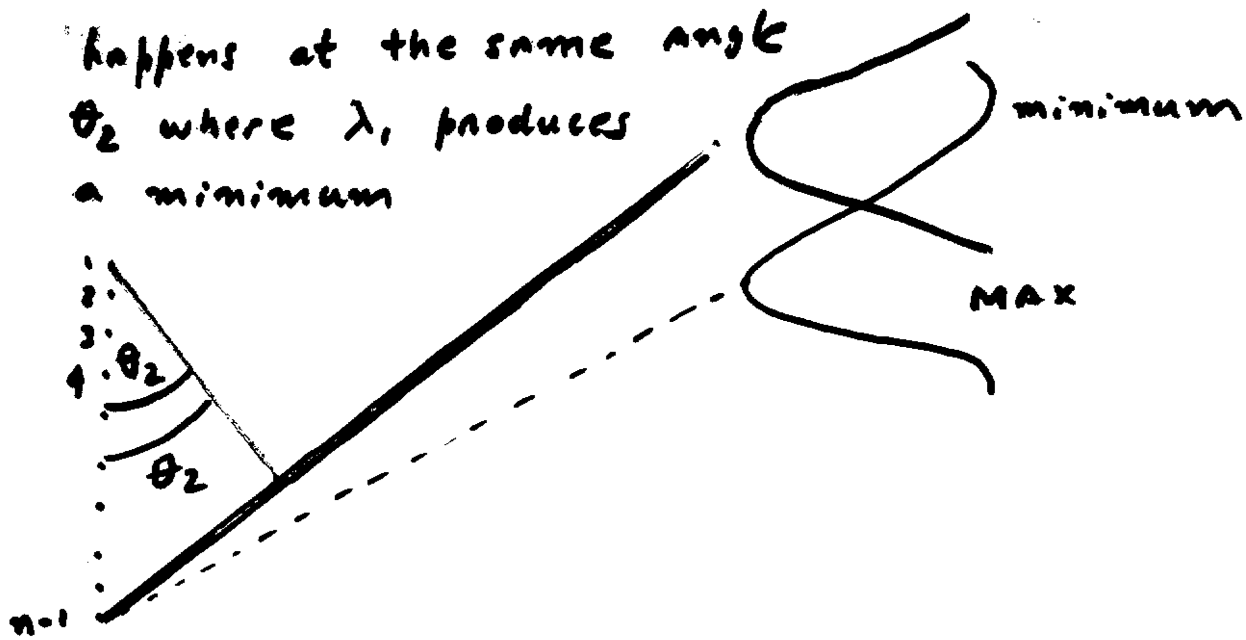


$$d \sin\theta_2 = m\,\lambda_2$$

$$d \sin\theta_1 = m\,\lambda_1$$

Let λ_1 be given

We want to find λ_2 such that its maximum happens at the same angle θ_2 where λ_1 produces a minimum



Condition for λ_1 to produce at minimum of intensity at θ_2: $n\,d \sin\theta_2 = n\,m\,\lambda_1 + \lambda_1$

$$n \underbrace{d \sin\theta_2} = n m \lambda_1 + \lambda_1$$

$$n \underbrace{m \lambda_2} = n m \lambda_1 + \lambda_1$$

$$\boxed{\lambda_2 = \lambda_1 + \frac{\lambda_1}{n m}}$$

- Given λ_1, the next higher wavelength that will have its max located at the minimum of λ_1 is $\lambda_1 + \frac{\lambda_1}{nm}$.

  These two wavelengths can be distinguished

- the bigger the n, the smaller the required increment in wavelength (and be able to distinguish the peaks of these two wavelengths)

$$\boxed{\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_1}{n m}}$$

The smaller $\Delta\lambda$ required to distinguish 2 wavelengths, the better the grating. It is conveniently then to define the resolving power of a grating as

$$\boxed{R \equiv \frac{\lambda}{\Delta\lambda}}$$

Using our previous result, a grating with composed of n lines has the following resolving power

$$R = n m$$

Example Determine whether a grating of 20,000 lines can resolve the two yellow lines of sodium, whose wavelengths are $\lambda_1 = 589.0$ nm and $\lambda_2 = 589.6$ nm

$$R = n\,m = 2 \times 10^4\, m$$

m is the order of the max intensity

The minimum $\Delta\lambda$ that this grating can distinguish is (let's take $m=1$)

$$\Delta\lambda = \frac{\lambda}{R} = \frac{\lambda}{2 \times 10^4}$$

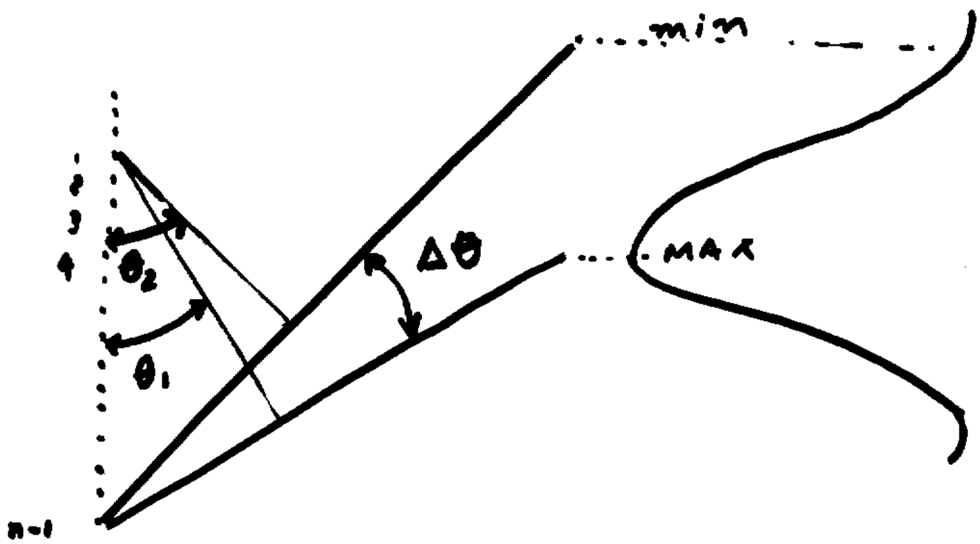
$\lambda_1 = 589.0$ nm

$$= \frac{589.0}{2 \times 10^4} = 294.5 \times 10^{-4}\ \text{nm} = \underline{0.02945\ \text{nm}}$$

Since the wavelengths λ_1 and λ_2, that we want to distinguish, are far apart by $\underline{0.6\ \text{nm}}$, the grating can indeed resolve them.

Complement

Angular width $\Delta\theta$ of a line of max intensity



Condition for θ_1: $\quad d \sin\theta_1 = m\lambda$

Condition for θ_2: $\quad n\, d \sin\theta_2 = n m \lambda + \lambda$

$$\Rightarrow \quad d \sin\theta_2 = m\lambda + \frac{\lambda}{n}$$

$$d \sin\theta_1 = m\lambda$$

$$\Rightarrow \quad d(\sin\theta_2 - \sin\theta_1) = \frac{\lambda}{n}$$

$$d \cos\theta_1 \times \Delta\theta = \frac{\lambda}{n} \quad \Rightarrow \quad \boxed{\Delta\theta = \frac{\lambda}{n d \cos\theta}}$$