

## SPECIAL RELATIVITY

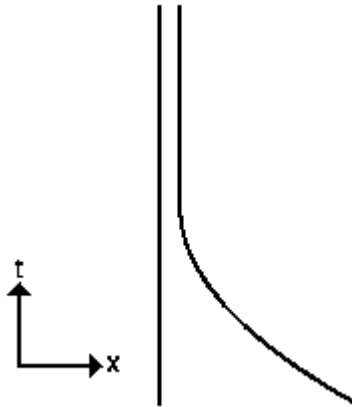
### Introduction:

When Mr. La Rosa started talking about relativity in class, I first was lost and didn't get the idea. At That time I was thinking of a subject for my project. I thought then that the subject of relativity would be interesting and so helpful to do some researches in. So here's a brief explanation to the idea relativity .

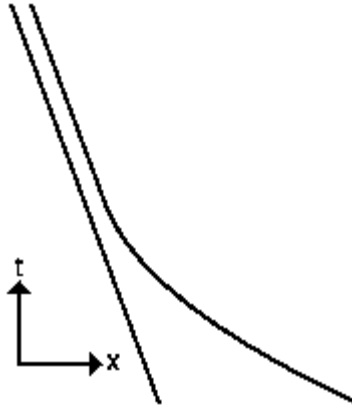
**Relativity**, involves considering how an event or series of events would look to one observer given that you know how it looks to another observer who may be moving with respect to the first. This is called "transforming" the observation from one frame to another, and relativity tells us how to do that. Thus, we are concerned with the way something seems to one observer **relative to** how it seems to another.

### Galilean Relativity

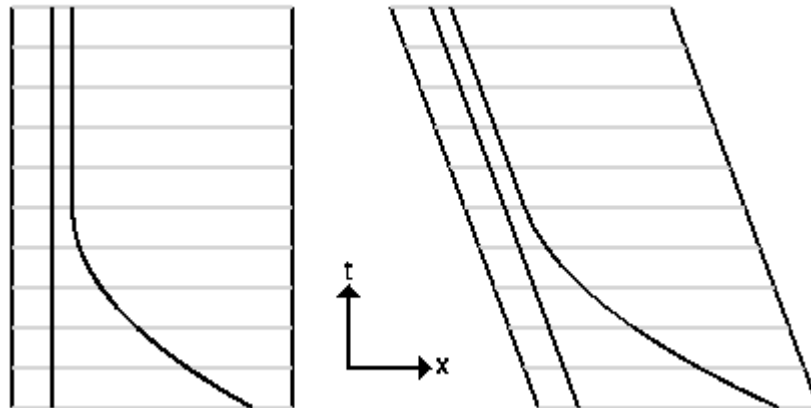
Contrary to popular opinion, *Einstein* did not invent relativity. *Galileo* preceded him. *Aristotle* had proposed that moving objects (on the Earth) had a natural tendency to slow down and stop. This is shown in the space-time diagram below.



Note the curved line above. This shows a variable velocity, or *acceleration*. Galileo objected to Aristotle's hypothesis, and asked what happened to an object moving on a moving ship.



Now it is still moving in its final state. *Galileo* proposed that it is only relative velocities that matter. Thus a space-time diagram can be transformed by painting it on the side of a deck of cards, and then skewing the deck to one side -- but keeping the edges along a straight line:



Straight lines (unaccelerated particles) remain straight in this process. Thus *Newton's* First Law is preserved, and non-accelerated lines are special. This Galilean transformation does not affect the time. Thus two observers moving with respect to each other can still agree on the time, and thus the distance between two objects, which is the difference in their positions measured at equal times, can be defined. This allowed Newton to describe an inverse square law for gravity.

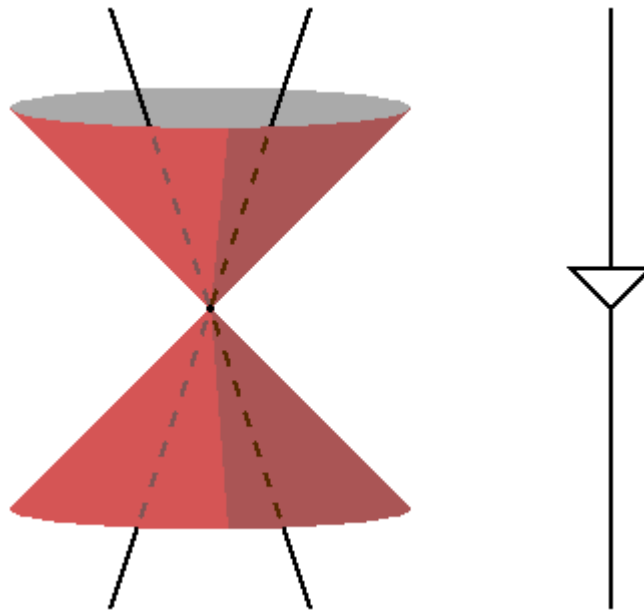
But Galilean transformations do not preserve velocity. Thus the statement "The speed limit is 70 mph" does not make sense. According to relativity, this must be re-expressed as "The magnitude of the relative velocity between your car and the pavement must be less than 70 mph". Relative velocities are OK.

## Special Relativity

But 200 years after *Newton* the theory of electromagnetism was developed into *Maxwell's* equations. These equations describe waves with a speed of

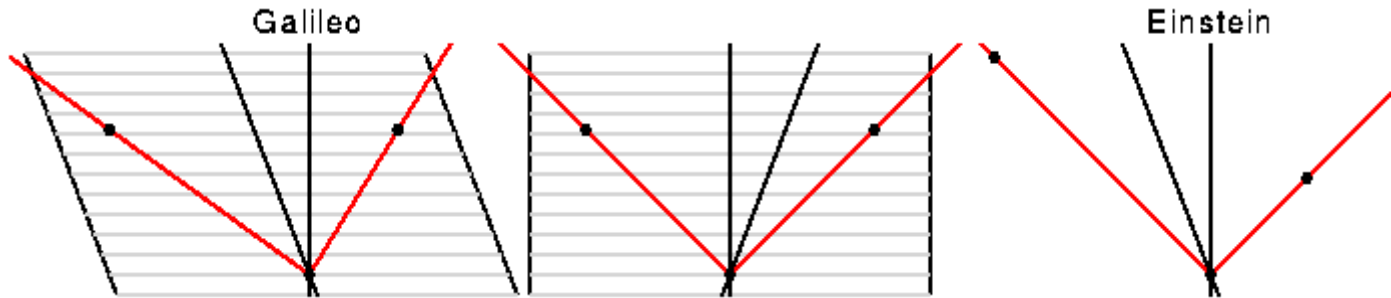
$$1/\sqrt{\epsilon_0 \cdot \mu_0},$$

where  $\epsilon_0$  is the constant describing the strength of the electrostatic force in a vacuum, and  $\mu_0$  is the constant describing the strength of the magnetic interaction in a vacuum. This is an absolute velocity -- it is not relative to anything. The value of the velocity was very close to the measured speed of light, and when Hertz generated electromagnetic waves (microwaves) in his laboratory and showed that they could be reflected and refracted just like light, it became clear that light was just an example of electromagnetic radiation. Einstein tried to fit the idea of an absolute speed of light into Newtonian mechanics. He found that the transformation from one reference frame to another had to affect the time -- the idea of sliding a deck of cards had to be abandoned. This led to the theory of *special relativity*. In special relativity, the velocity of light is special. Anything moving at the speed of light in one reference frame will move at the speed of light in all unaccelerated reference frames. Other velocities are not preserved.



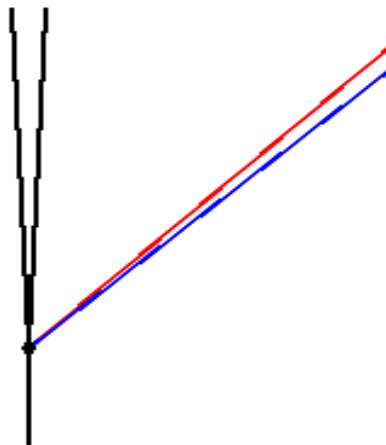
Because the speed of light is special, space-time diagrams are often drawn in units of seconds and light-seconds, or years and light-years, so a unit slope [45 degree angle] corresponds to the speed of light. The set of all light speed world lines going through an event defines the *light cones* of that event: the past light cone and the future light cone. An example of light cones is shown above. The fancy light picture on the left shows both the past and future light cones of the event where the two worldlines cross, while the schematic version on the right is easy to use in more complicated diagrams.

Thus in the situation shown in 3 space-time diagrams below, the central section shows the worldline of one stationary observer, one observer moving to the right, and two events on the future light cone on the event where the two observers' worldlines cross.



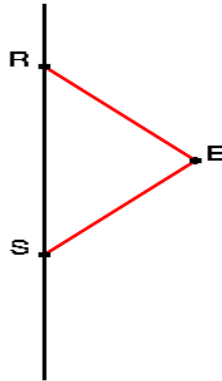
The left-hand section of the figure shows the Galilean transformation into the frame of reference of the moving observer. The events on the future light cone have shifted to the left, but they are still at the same time. Since the coordinates  $x$  and  $t$  just provide a way of describing space-time, and are not the space-time themselves, the two events are still on the future light cone. But now slopes of the light rays have changed, so the speed of light has changed. The Lorentz transformation appropriate for special relativity is shown on the right hand of the figure. The events on the future light cone have shifted to the left as before, but now their times have changed, so the slopes of the light rays do not change. The speed of light is invariant in Einstein's special relativity.

What is the evidence for the invariance of the speed of light? The hypothesis that the speed of light is  $c$  relative to its source can easily be disproved by the one-way transmission of light from distant supernovae. When a star explodes as a supernova, we see light coming from material with a large range of velocities  $dv$ , at least 10,000 km/sec. Because of this range of velocities, the spectral lines of a supernova are very broad due to the Doppler shift. After traveling a distance  $D$  in time  $D/c$ , the arrival time of the light would be spread out by  $dt = (dv/c)(D/c)$ .



However, this DOES NOT happen. For the Crab supernova, with  $D/c = 6000$  years,  $dv = 10,000$  km/sec would give a range of arrival times of 200 years. But the Crab was only bright for 1 year. For very distant supernovae with  $D/c = 5$  billion years, modern observations with spectrographs show that the redshifted and blueshifted light arrives at the same time: within 10 days. This limit on the spread is 5 billion times smaller than the prediction of the "bullet" model of light.

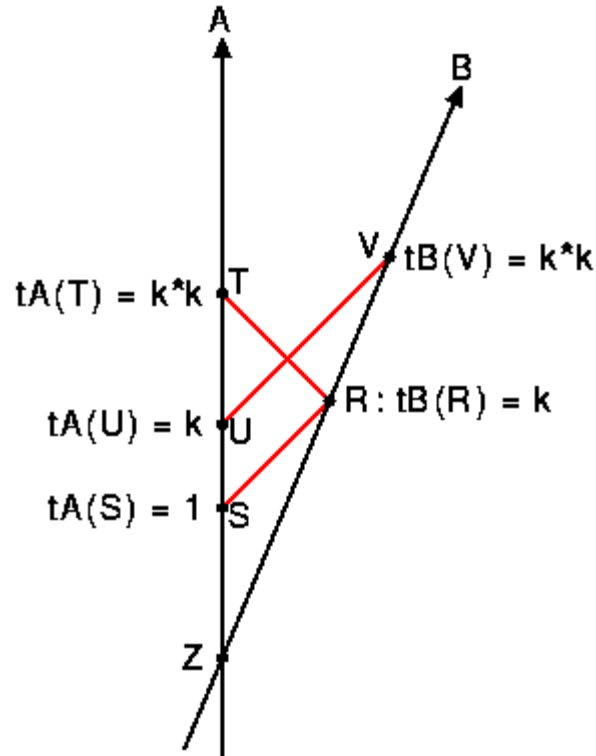
The constancy of the speed of light allows the use of *radar* (Radio Detection And Ranging) to measure the position and time of events not on an observer's worldline. All that we need are a clock and the ability to emit and detect radar pulses.



If we send out a radar pulse at time  $t_s$ , which is reflected at the event E, and the echo arrives back at time  $t_r$ , then we know that the light was traveling at  $c$  for the entire round-trip journey out to E and back, so the distance of E is  $D(E) = c \cdot (t_r - t_s) / 2$ . In our frame of reference we are stationary, and light travels at the same speed coming back from E as it did going out, so the time of the event E is obtained by averaging the send and receive times,  $t(E) = (t_r + t_s) / 2$ .

## Time Dilation

Armed with radar, we can determine the time of two events on the worldline of an observer moving with respect to us. We can then compare the time interval we measure to the time interval to the time interval measured by the moving observer. Consider the two observers A and B below.



They both set their clocks to zero at the event Z where their worldlines cross. A sends out a radar pulse at the event S at time  $t_A(S) = 1$ . This is received by observer B at event R with time  $t_B(R) = k$ . The factor  $k$  depends on the relative velocity of A and B, but since the light takes some time to travel between A and B, we know that  $k$  will be larger than 1. The notation  $t_A$  means times determined by A, while  $t_B$  are times determined by B. If A sends out a pulse at some other time,  $t_A = x$ , it will be received by B at time  $t_B = x \cdot k$  by the principle of similar triangles. In particular, if A sends out a pulse at time  $t_A = k$  it will be received by B at time  $t_B = k \cdot k$ . Now consider the radar pulse reflected by B at event R. It starts at time  $t_B = k$ . When will A receive it? Since the speed of A with respect to B is the same as the speed of B with respect to A, this time has to be  $t_A(T) = k \cdot k$ . We can now compute the distance of event R from A's worldline and time of event R according to A,  $t_A(R)$ . These are

$$D_A(R) = c(k \cdot k - 1)/2 \text{ and } t_A(R) = (k \cdot k + 1)/2.$$

Thus the speed of B according to A is

$$v = D/t = D_A(R)/t_A(R) = c(k \cdot k - 1)/(k \cdot k + 1).$$

We can solve for  $k$  giving

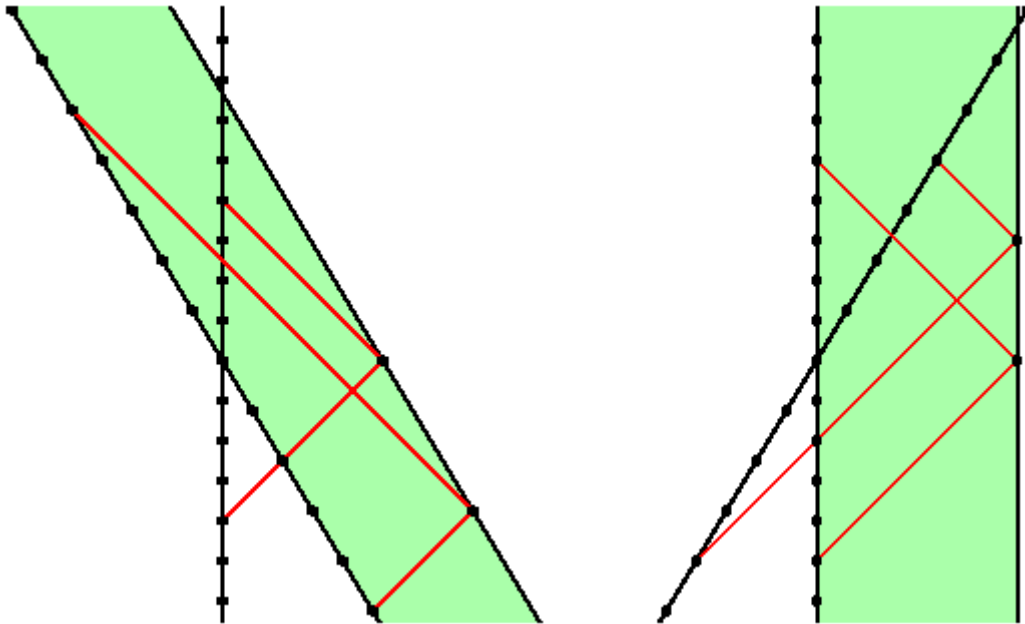
$$k = \sqrt{(1+v/c)/(1-v/c)}$$

which is the relativistic Doppler shift formula. But we also find that  $t_A(R) > t_B(R)$ , so A says that B's clock is running slow. The amount of this *time dilation* is

$$(1+k \cdot k)/(2 \cdot k) = 1/\sqrt{1-v^2/c^2}.$$

Thus moving clocks run slow. Note that B will also find that A's clock is running more slowly than his. There is a symmetric disagreement about clock rates.

This slow down factor is exactly the slow down calculated above in the ether model for a bouncing photon clock moving perpendicular to its bounce axis. The clock moving parallel to the axis slows down by the same amount under special relativity because of the *Lorentz-Fitzgerald contraction* of moving objects in the direction of motion.



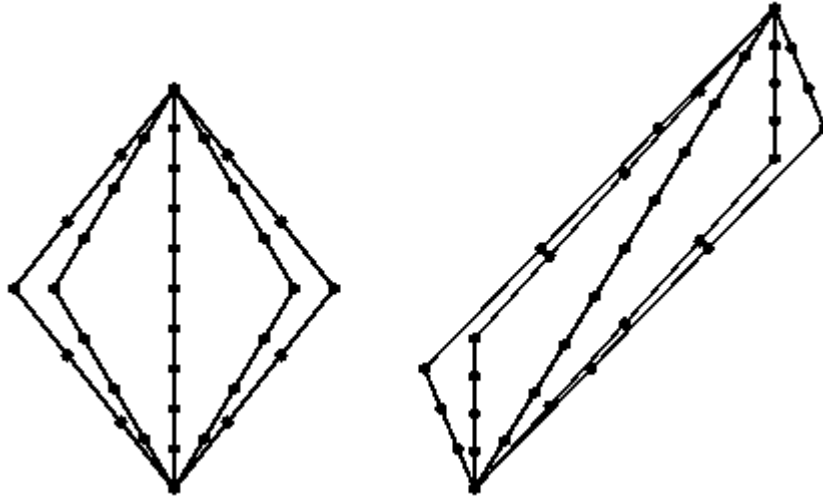
The space-time diagrams above both show a rod moving past an observer. On the left the rod is moving, while on the right the same situation is shown in the rod's frame of reference. The observer moving with respect to the rod makes a radar determination of its length, as does an observer moving along with the rod. The observer on the rod sees a length of 5 light-ticks because it takes 10 ticks for light to make the round trip to the end of the rod and back. The observer moving with respect to the rod at  $v = 0.6*c$  measures only 8 ticks for the round trip and thus gives the length of the rod as 4 light-ticks. Thus the length of a moving rod appears to be reduced by a factor of  $\sqrt{1-v^2/c^2}$ . Thus length contraction changes  $P(\text{par})$  for the bouncing photon clock to

$$\begin{aligned} P(\text{par}) &= [2L*\sqrt{1-v^2/c^2}/c]/(1-v^2/c^2) \\ &= [2L/c]/\sqrt{1-v^2/c^2} = P(\text{perp}) \end{aligned}$$

so the rate of a bouncing photon clock does not depend on the angle between its velocity and its bouncing axis.

Because the clocks of different observers run at different rates, depending on their velocities, the time for a given observer is a property of that observer and his worldline. This time is called the *proper time* because it is "owned" by a given particle, not because it is the "correct" time. Proper time is invariant when changing reference frames because it is the property of a particle, not of the reference frame or coordinate system. In general, given any two events A and B with B inside the future light cone of A, there is one unaccelerated worldline connecting A and B, just as there is one straight line connecting two points in space. In the frame of reference of the observer following this unaccelerated worldline, his clock is always stationary, while clocks following any other worldline from

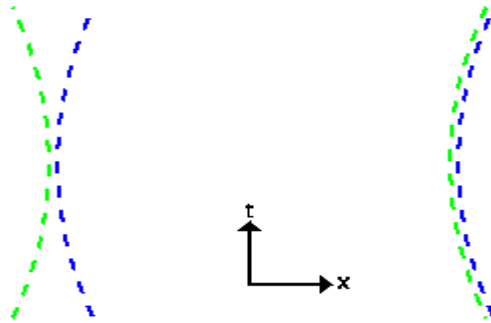
A to B will be moving at least some of the time. Because moving clocks run slow, these observers will measure a smaller proper time between events A and B than the unaccelerated observer. Thus the straight worldline between two events has the largest proper time, and all other curved worldlines connecting the two events have smaller proper times. This is exactly analogous to the fact that the straight line between two points has the smallest length of all possible curves between the points. Thus the "twin paradox" is no more paradoxical than the statement that a man who drives straight from LA to Las Vegas will cover fewer miles than a man who drives from LA to Las Vegas via Reno.



The pair of space-time diagrams above show quintuplets separated at birth. The middle worldline shows the quint who stays home. The space-time diagram on the left is done from the point of view of the middle quint. Each dot on a worldline is a birthday party, so the middle quint is 10 years old when they all rejoin each other, while the other quint is 6 and 8 years old. The space-time diagram on the right shows the same events from the point of view of an observer initially moving with one of the moving quints. When the quints come together their ages are still 6, 8, 10, 8, and 6 years. Thus the straight worldline between two events can be found by maximizing the proper time, just as the straight line between two points can be found by minimizing the length.

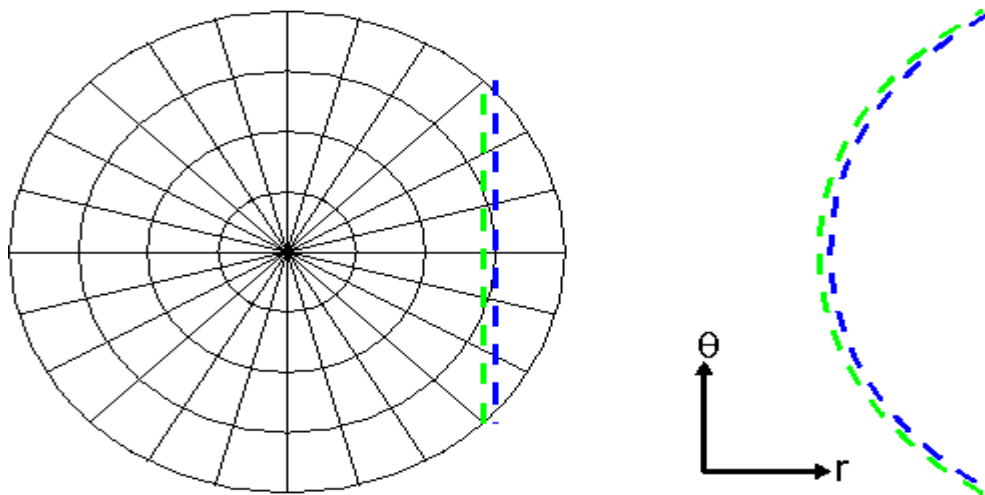
## General Relativity

Now we come to a matter of gravity: how can gravity be an inverse square law force, when the distance between two objects can not even be defined in Einstein's special relativity? Special relativity was constructed to satisfy Maxwell's equations, which replaced the inverse square law electrostatic force by a set of equations describing the electromagnetic field. So gravity was the only remaining action-at-a-distance inverse square law force. And gravity has a unique property; the acceleration due to gravity at a given place and time is independent of the nature of the body.



The space-time diagram above shows a proton and an antiproton moving under the influence of an electric field on the left and a gravitational field on the right. Gravity accelerates all objects equally. This fact was known to Newton, and tested to an accuracy of 1 part in 100 million .

Thus through any event in space-time, in any given direction, there is only one worldline corresponding to motion solely influenced by gravity. Compare this to the geometric fact that through any point, in any given direction, there is only one straight line. We are led to propose that worldlines influenced only by gravity are really straight worldlines. But how can an accelerating body have a straight worldline? It all depends on how you measure it. Suppose we plot a straight line on polar graph paper, and then make a plot of radius vs angle as shown below?

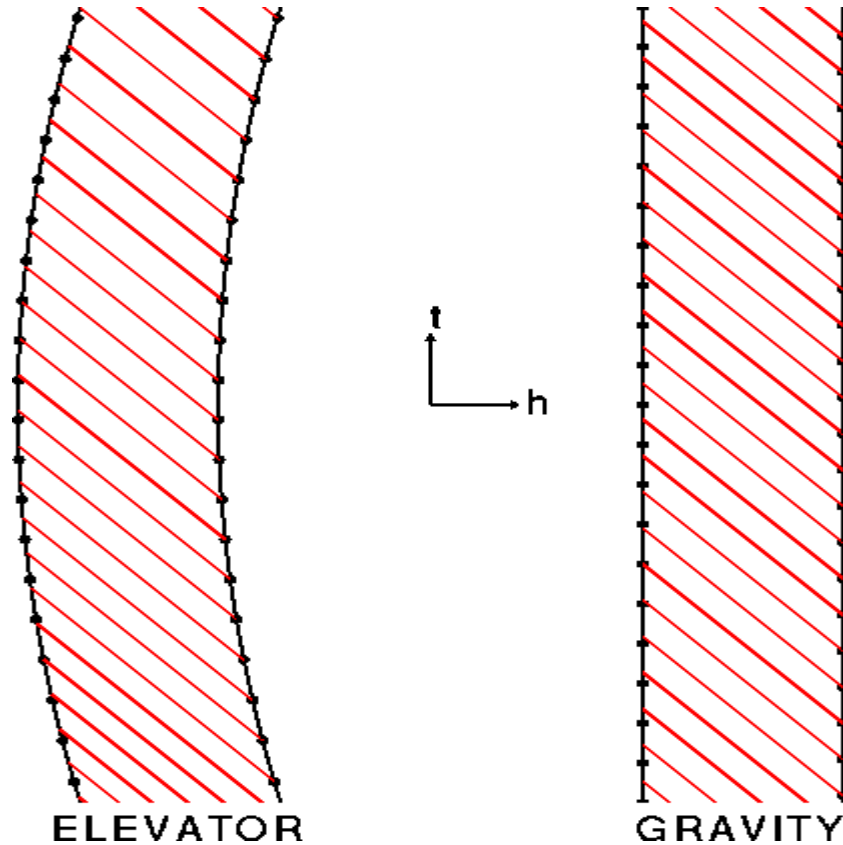


In the radius vs theta plot the straight line is curved. I have shown adjacent two lines dashed and labeled just like the proton and antiproton shown earlier to emphasize that while there are an infinite number of curved lines in the radius vs theta plot, there is only one straight line through the initial point with the initial direction.

### Principle of Equivalence

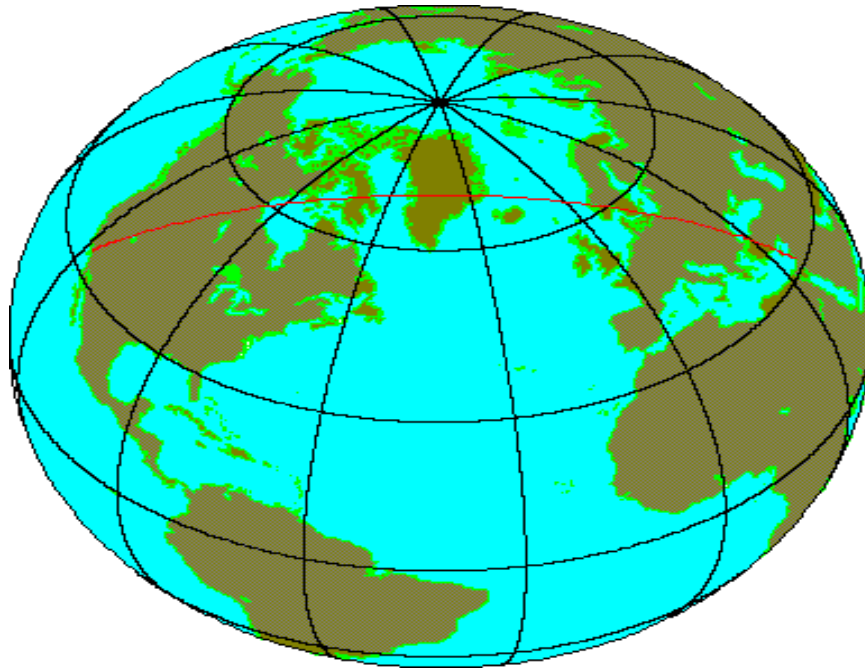
Einstein proposed that the effects of gravity (in a small region of spacetime) are equivalent to the effect of using an accelerated frame of reference without gravity. As example, consider the famous "Einstein elevator" thought experiment. If an elevator far

out in space accelerates upward at  $10 \text{ meters/second}^2$ , it will feel like a downward acceleration of gravity at  $1 \text{ g} = 10 \text{ m/s}^2$ . If a clock on the ceiling of the elevator emits flashes of light  $f$  times per second, an observer on the floor will see them arriving faster than  $f$  times per second because of the Doppler shift due to the acceleration of the elevator during the light transit time.

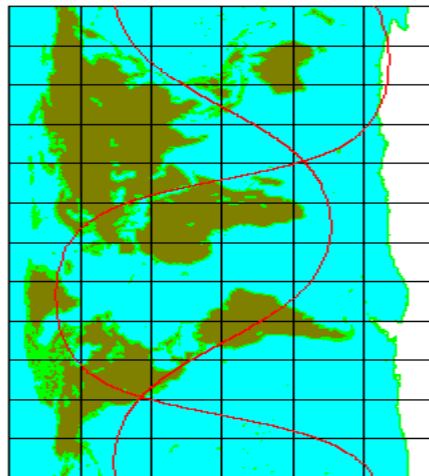


The space-time diagram on the left above shows the clock on the elevator ceiling emitting flashes of light. The light transit time is  $h/c$  where  $h$  is the height of the ceiling, and the velocity change is  $a \cdot h/c$  so the Doppler shift increases the rate of flash arrival by a factor of  $(1 + a \cdot h/c^2)$ , so the flash arrival rate is  $f' = (1 + a \cdot h/c^2) \cdot f$ . On the right is the same situation with stationary clocks in a gravitational field. In order to have the flash arrival rate faster by a factor of  $(1 + g \cdot h/c^2)$ , the clock on the ceiling must run faster by this factor. In other words, clocks run faster when they are high up in a gravitational field.

**Curved Spacetime** Curved coordinates alone, such as the polar graph, do not provide a satisfactory model for gravity. Two straight lines through the same point but with different directions will never cross again, while two worldlines influenced only by gravity which pass through the same event with different velocities can cross again. Consider the Galileo spacecraft, which made two Earth flybys. In between the flybys, Galileo was on an elliptical orbit with a 2-year period. In order to allow "straight" lines to cross multiple times, a curved space-time is needed. As a familiar example of a curved space, consider the surface of the Earth and the great circle arc connecting two cities. The great circle is the shortest distance between two points on the surface of the Earth, and it is the path followed by airliners.



Plotting latitude vs longitude, as if longitude were time and latitude position, gives the pseudo-spacetime diagram below.



The two great circles through Los Angeles, one to Tel Aviv and one to Singapore, are both "straight" lines, but they intersect in two places. This is impossible in plane geometry but it does occur in non-Euclidean geometry. The pseudo-spacetime diagram above is almost identical to a real spacetime diagram for objects moving in a tunnel drilled through the center of a massive sphere. Gravity produces oscillatory motions so worldlines for different objects, each influenced only by gravity, can cross at many events.

**Thus the fundamentals of relativity that are important for cosmology are:**

- The speed of light is a constant independent of the velocity of the source or the observer.
- Events that are simultaneous as seen by one observer are generally not simultaneous as seen by other observers, so there can be no absolute time.
- Each observer can define his own proper time -- the time measured by a good clock moving along his worldline.
- Observers can assign time and positions to events not on their worldlines using radar observations.
- Every observer will see his clock running faster than other clocks which are moving with respect to him, this is a mathematically consistent pattern required by the properties of radar observations.
- As a result, the unaccelerated worldline between two worldlines will have the longest proper time of all worldlines connecting these events.
- In the presence of gravity, the worldlines of objects accelerated only by gravity have the longest proper times.

**Reference:**

**Fundamentals of physics , Halliday / Resnek / Walker .**

[www.physics.purdue.edu](http://www.physics.purdue.edu)

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<http://casa.colorado.edu>

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