1. Introduction
   Phase shift

2. What is PHASE Φ ?
   - Definition.
   - Phase shift or phase difference between two wavefronts of the same wave

3. Effects of a medium1-medium2 interface ~~on~~ the PHASE of a propagating wave
   - mathematical description to obtain a relationship between the amplitudes of the incident, reflected and transmitted electromagnetic waves
   - Similarities between mechanical and electromagnetic waves

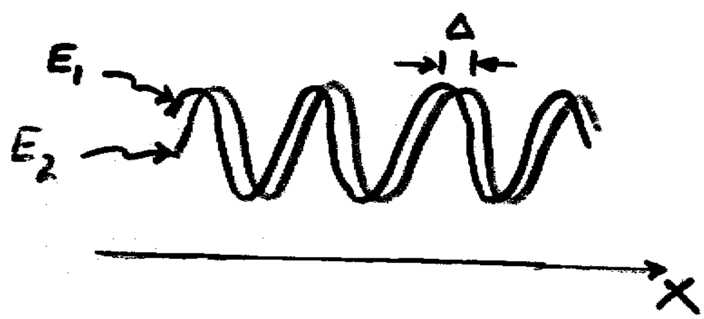
4. More about the phase difference between two wavefront of the same wave

5. Phase shift between waves that travel through different materials

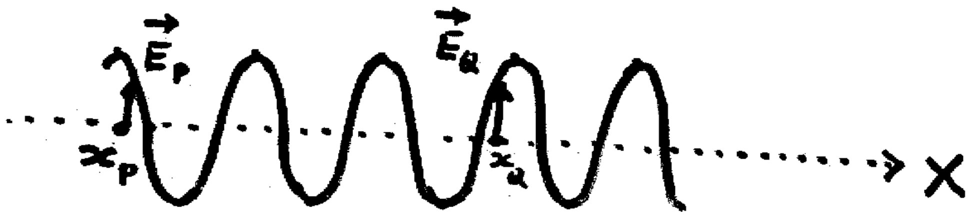
Phase shift

We already have an intuitive idea about what "phase shift" means.

For example, the figure below shows a 'snapshot' of two waves E_1 and E_2. After measuring the distance Δ we say "the waves are phase shifted by Δ"



We are going to introduce the concept of PHASE, in such a way that it will allow us to talk not only about the phase difference or phase shift between two waves (as in the figure above, but also about the phase difference between two wavefronts of the same wave (as in the next figure)



"Electric field $\vec{E}_1$ is out of phase with the electric field $\vec{E}_2$"

Definition of PHASE

Given the wave

$$E = E_m \cos(\underbrace{kx - \omega t + \delta}_{\text{this is called the PHASE of the wave}})$$

$$= E_m \cos(\phi)$$

Let's apply this definition to the case illustrated in the figure above, where a snapshot of the wave has been taken at $t = t_0$, and the electric field $\vec{E}$ at two locations P and Q are also shown.

the electric field at the location x_P is

$$E_P = \cos(\underbrace{k\,x_P - \omega t_0 + \delta}_{\phi_P})$$

the electric field at the location x_Q is

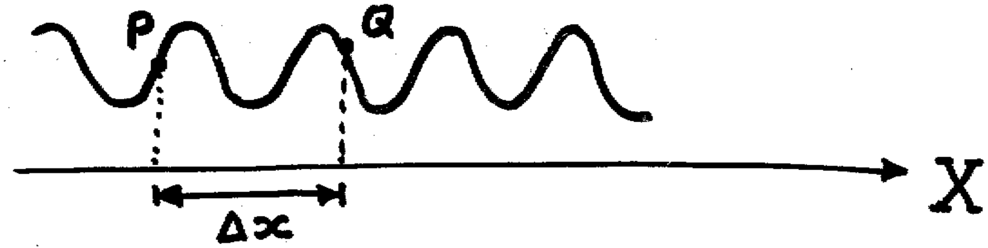
$$E_Q = \cos(\underbrace{k\,x_Q - \omega t_0 + \delta}_{\phi_Q})$$

We say, the electric fields at the locations x_P and x_Q are out of phase by $\Delta\phi$

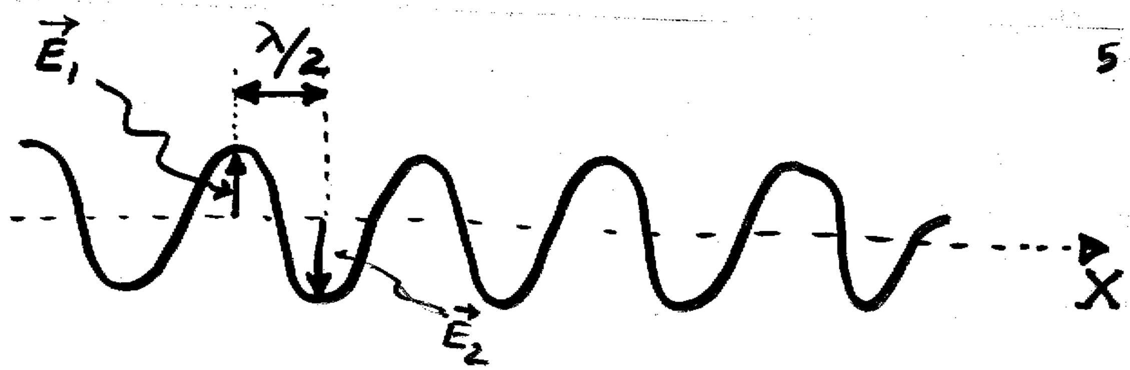
$$\Delta\phi = \phi_Q - \phi_P$$

$$= k \cdot (x_Q - x_P)$$

$$= k \cdot \Delta x$$

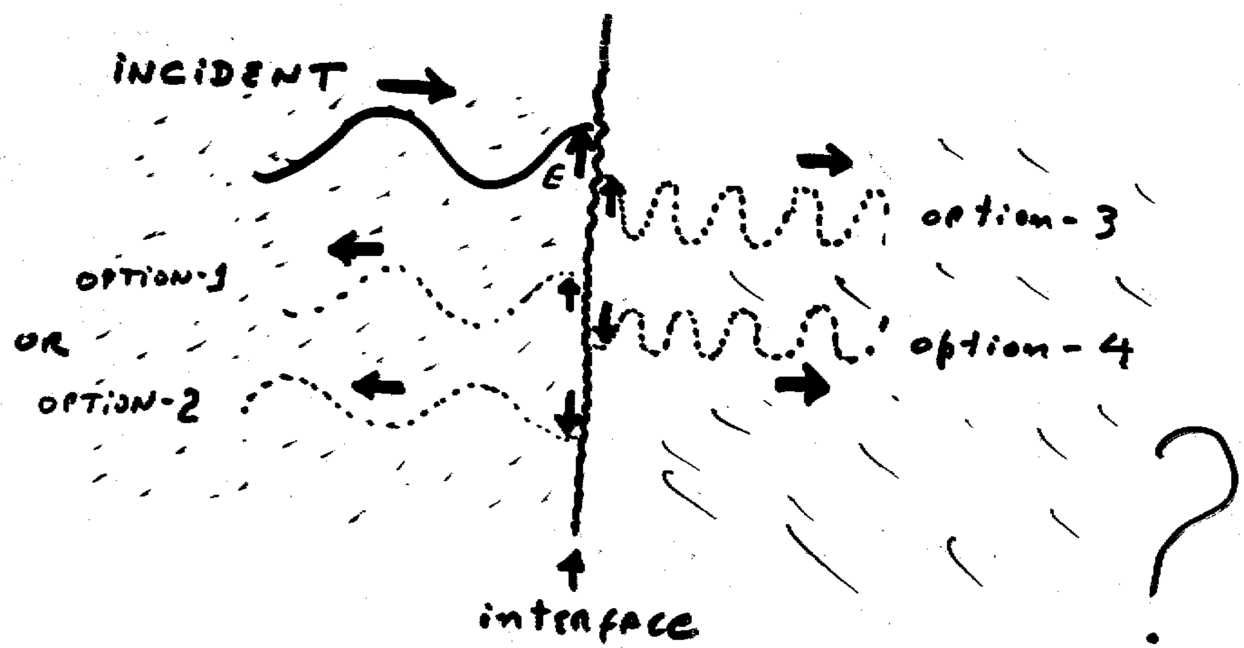


• Notice: If $\Delta x = \frac{\lambda}{2}$ THEN $\Delta\phi = k\frac{\lambda}{2} = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{2} = \pi\,\text{rad} = 180°$



that is, two electric field vectors $\vec{E}_1$ and $\vec{E}_2$ that are out of phase by $\Delta\phi = \pi \text{ rad} = 180°$ have opposite vector orientation.

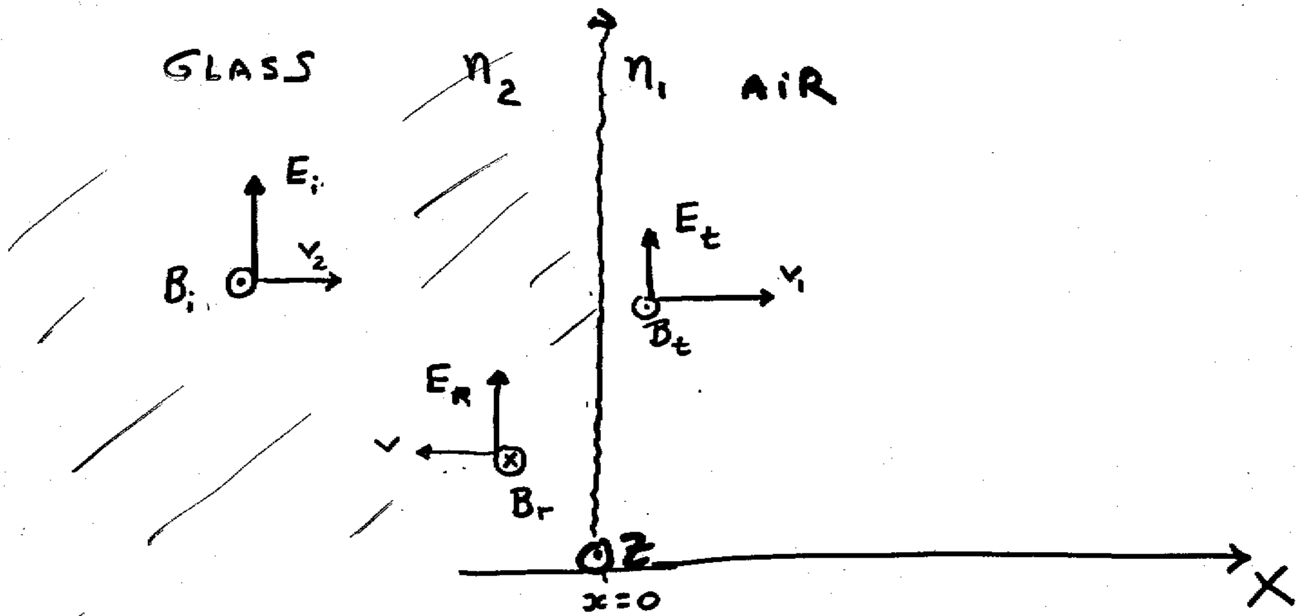
Effects of a medium1-medium2 interface on the phase of a propagating wave :-



Notice:

In option-1: The electric field $\vec{E}$ of the incident wave does not change direction upon reflection at the interface

In option-2: The incident electric field $\vec{E}$ changes its orientation upon reflecting at the interface. That is, $\vec{E}$ experiences a phase shift of π radians



$$E_i = E_{io} \cos(k_2 x - wt)$$

$$B_i = \frac{E_{io}}{V_2} \cos(k_2 x - wt)$$

$$E_R = E_{Ro} \cos(k_2 x + wt)$$

$$B_R = -\frac{E_{ro}}{V_2} \cos(k_2 x + wt)$$

$$E_t = E_{to} \cos(k_1 x - wt)$$

$$B_t = \frac{E_{to}}{V_1} \cos(k_1 x - wt)$$

Notice: - All the waves have the same ω
- waves in the same medium have the same k
- waves in different media have different k

At the interface, $x=0$, we should have

Total electric field at the left = Total electric field at the right

$$E_{io} + E_{Ro} = E_{to} \qquad (1)$$

Total magnetic field at the left = total magnetic field at the right (assuming the magnetic permeability of the media are the same $\mu_1 = \mu_2$)

$$\frac{E_{io}}{V_2} - \frac{E_{Ro}}{V_2} = \frac{E_{to}}{V_1} \qquad (2)$$

* Exercise: Using expressions (1) and (2) show that for the REFLECTED light:

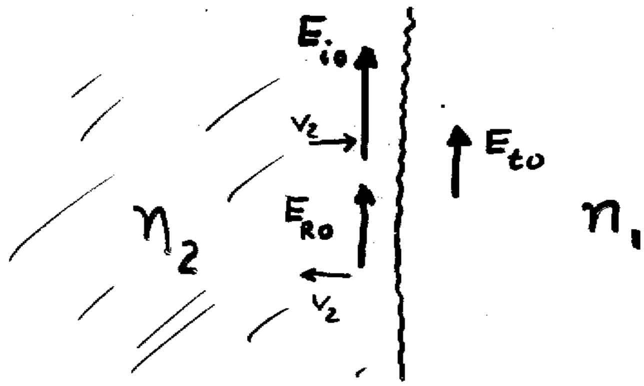
$$E_{Ro} = E_{io}\frac{V_1 - V_2}{V_1 + V_2} = E_{io}\frac{n_2 - n_1}{n_2 + n_1} \qquad (3a)$$

For the TRANSMITTED light

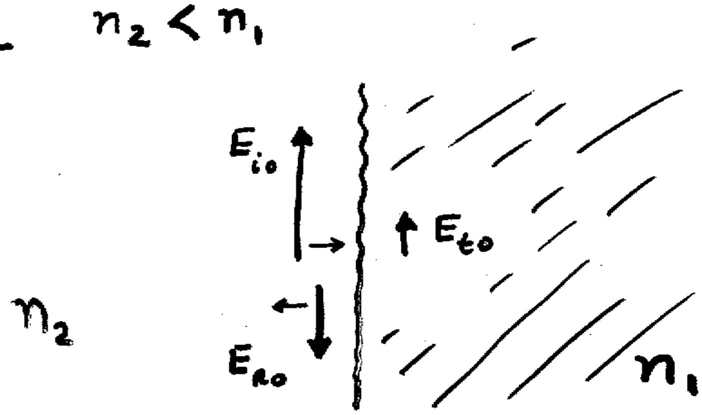
$$E_{to} = E_{io}\frac{2V_1}{V_1 + V_2} = E_{io}\frac{2n_2}{n_2 + n_1} \qquad (3b)$$

CASE $\eta_2 > \eta_1$

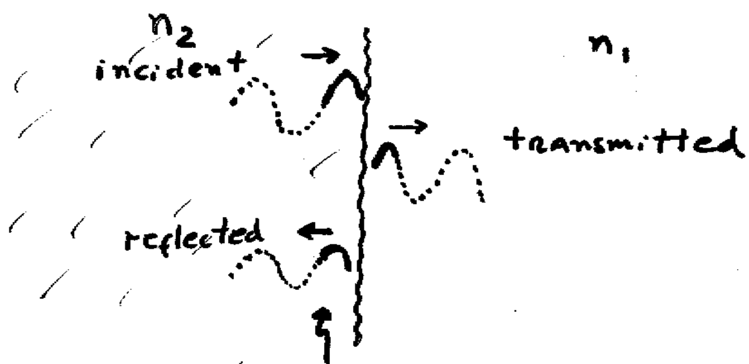
$$E_{R0} = E_{i0}\,\frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$



CASE $\eta_2 < \eta_1$



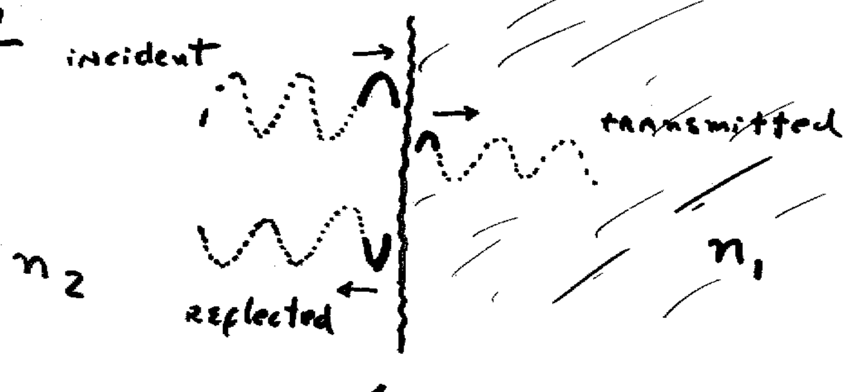
$n_2 > n_1$



Electromagnetic waves

No phase shift in the reflected wave

$n_2 < n_1$

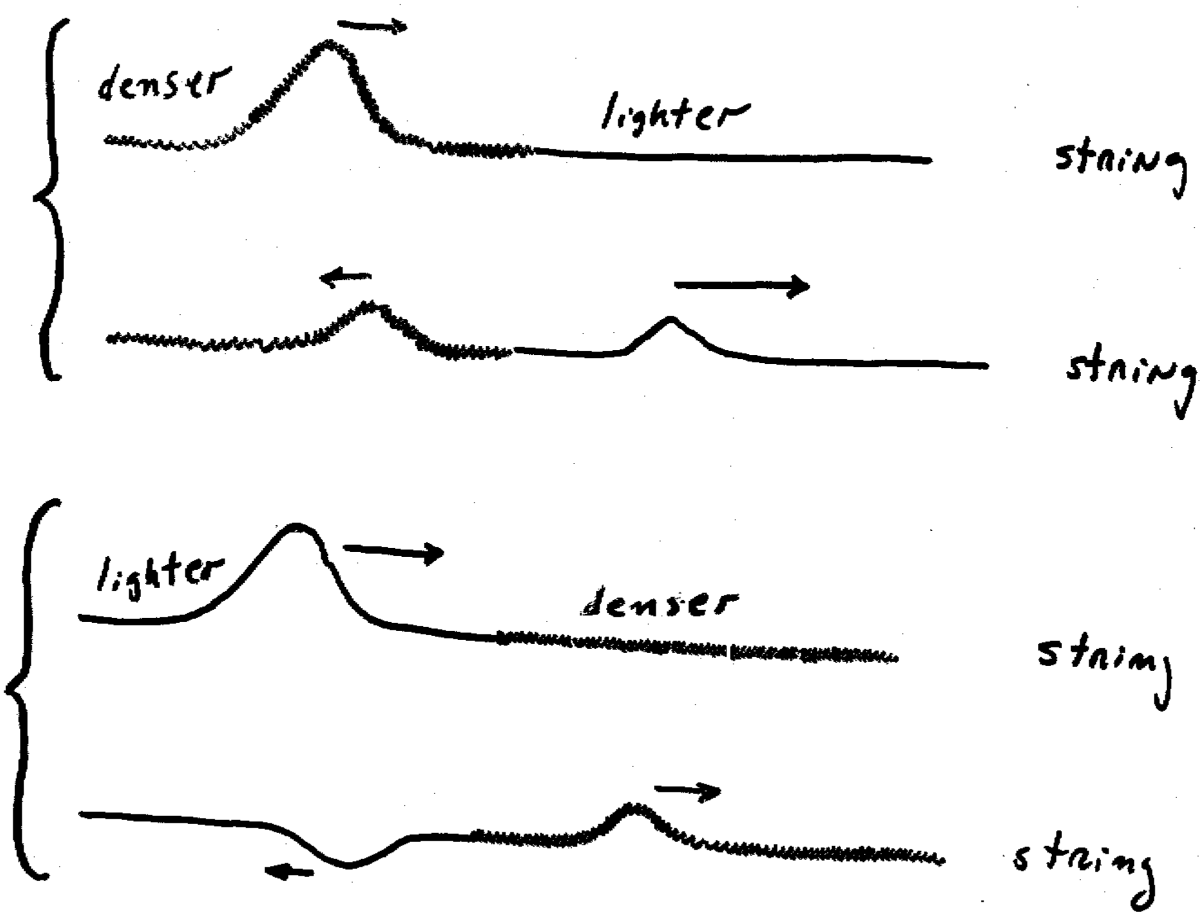


ELECTROMAGNETIC WAVES

"Reflected wave undergoes a phase shift of $\lambda/2$ or 180° relative to the incident wave at the interface"

EXERCISE: question #10, page 883 (assume normal incidence) (6th edition)

Notice the similarity of
phase shift at a interface
when dealing with electromagnetic waves
and
phase shift at a interface when we
studied mechanical waves in a string



More about the phase difference between two wavefronts of the same wave

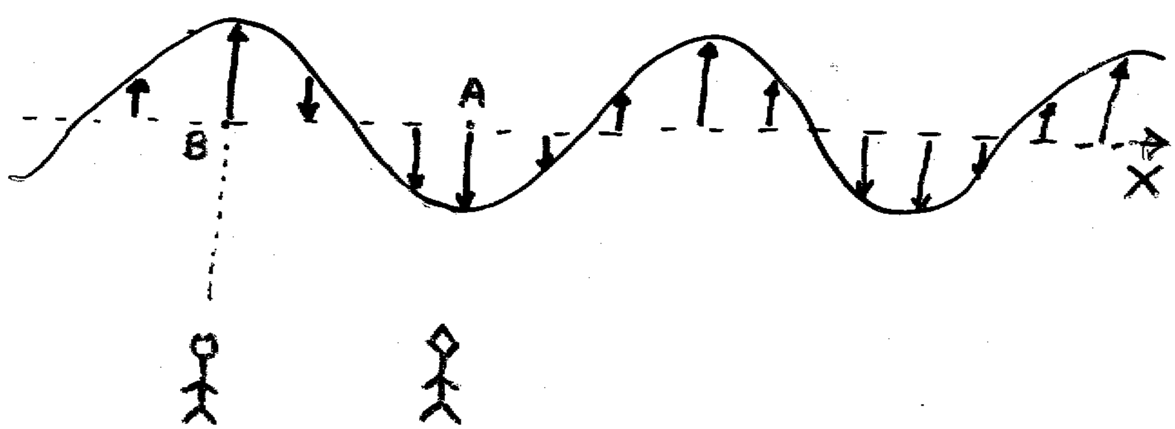
Let's assume an electromagnetic wave traveling in the +X direction is described by the expression

$$E(x,t) = E_0 \cos(\underbrace{kx - \omega t + \phi})$$

At a given time $t = t_0$, the wave is described by:

$$E(x) = E_0 \cos(kx - \omega t_0 + \phi) \quad \Big\} \leftarrow \text{A function of } x \text{ only}$$

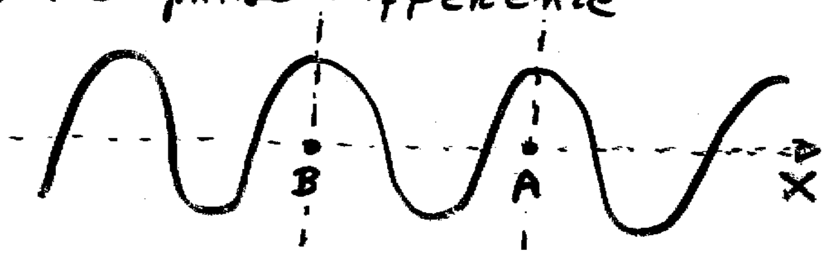
which looks like this:



We say:

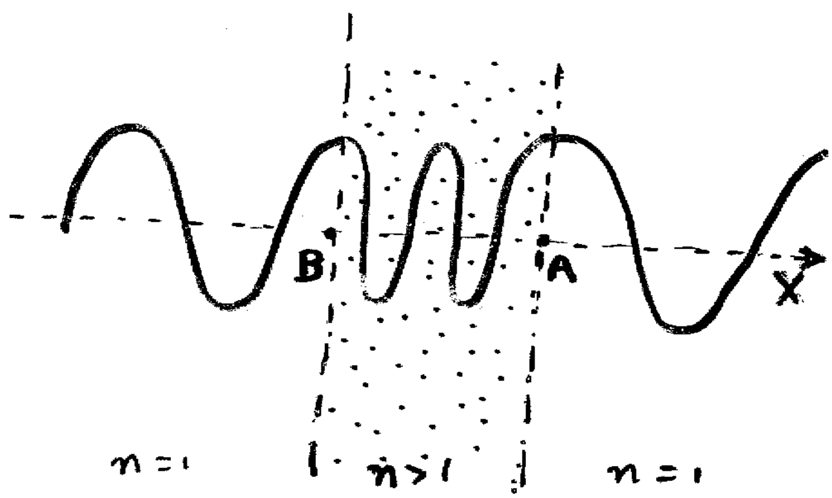
"The electric field at A is out of phase 180° with respect to the electric field at B." 13

- Another example where we can intuitively guess the phase difference



$\phi_A - \phi_B = ?$

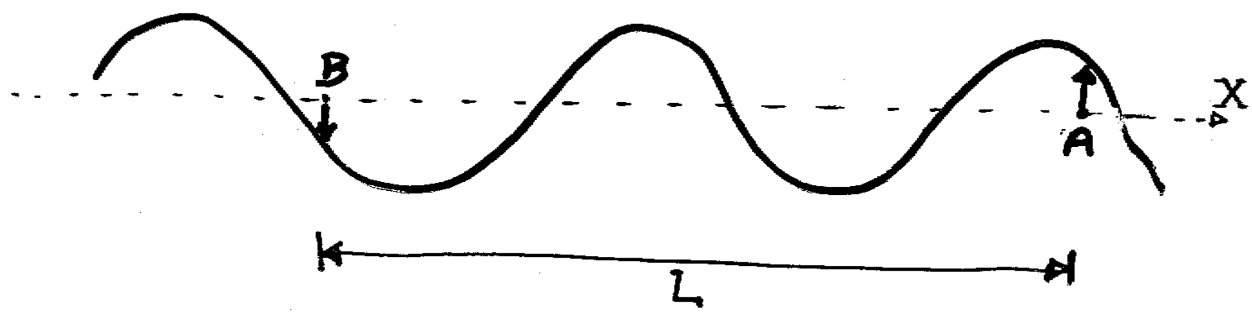
360°



$\phi_A - \phi_B = ?$

4π rad.

• In general, it is not obvious to guess what is the phase difference between 2 point A and B,



all we know is the distance L between A and B.

In such cases, we resort to our formulas.

We say:

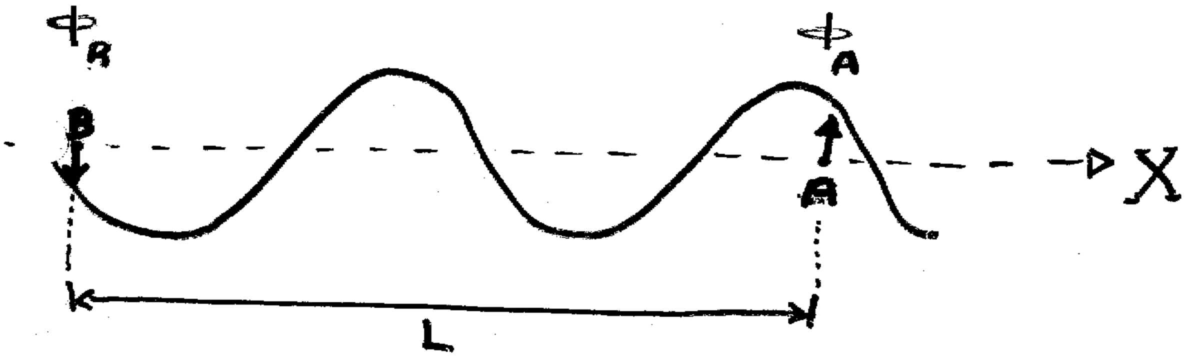
At a given time $t = t_0$, the electric field

at $x = x_A$ is $E_A = E_0 \cos(\underbrace{K x_A - \omega t_0 + \phi}_{\phi_A})$

at $x = x_B$ is $E_B = E_0 \cos(\underbrace{K x_B - \omega t_0 + \phi}_{\phi_B})$

so, $\phi_A - \phi_B = K(x_A - x_B)$

$$\boxed{\Delta\phi = K L = \frac{2\pi}{\lambda} L}$$



WAVE traveling in vacuum

λ: wavelength of the wave in vacuum

$$\boxed{\phi_A = \phi_B + 2\pi \frac{L}{\lambda}}$$

Of course, if the EM wave is traveling in a medium of index of refraction n, then we change our formulas accordingly:

↳ The wavelength is $\lambda_n = \dfrac{\lambda}{n}$

(where λ is the wavelength in vacuum)

↳ $\phi_B = \phi_A + 2\pi \dfrac{L}{\lambda_n}$ *

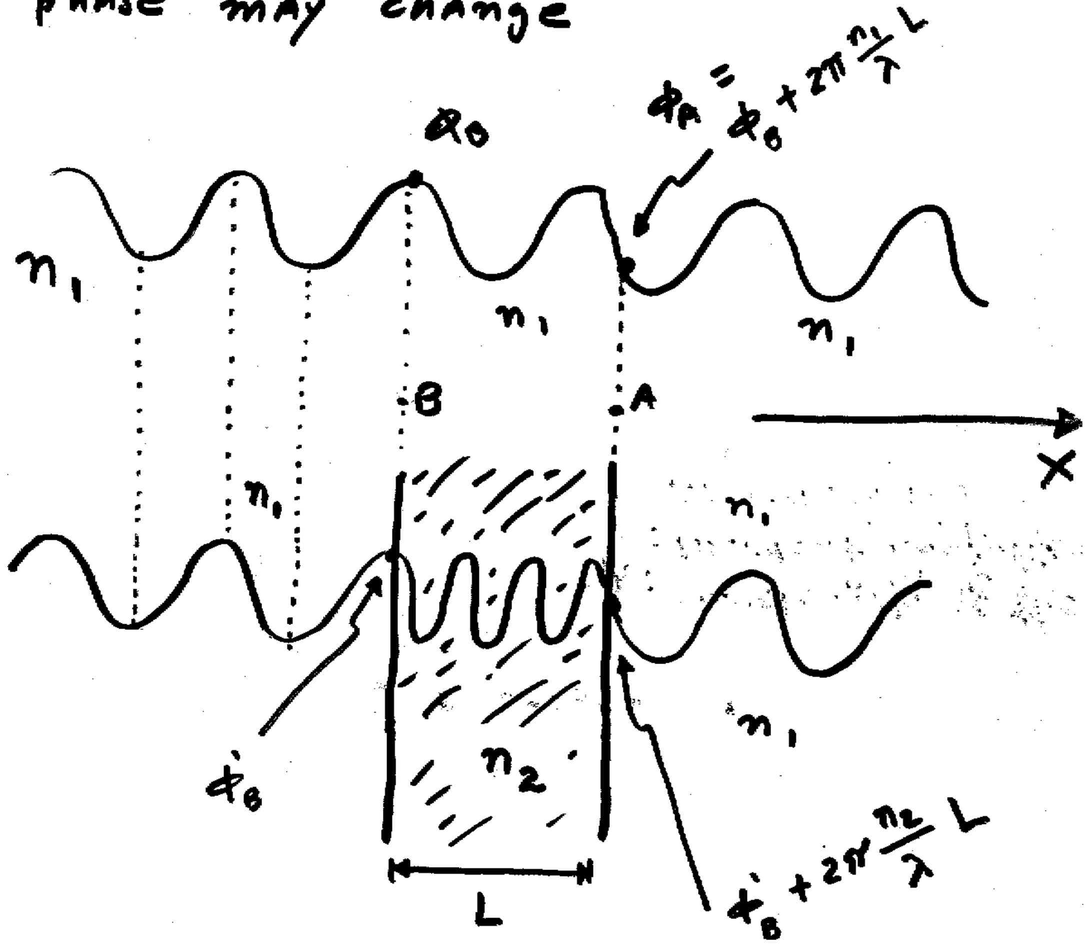
$$\boxed{\phi_B = \phi_A + 2\pi n \frac{L}{\lambda}}$$

WAVE traveling in a medium of index of refraction n

PHASE SHIFT between waves that travel through different materials

, Let's consider now 2 waves, initially in phase with respect to each other.

If at some point they travel through different materials that have different index of refraction, their relative phase may change



At point B: Both waves are in phase

$$\phi_B = \phi_B'$$

At point A: the phase of one wave is

$$\phi_A = \phi_B + \underbrace{2\pi \frac{n_1}{\lambda}}L$$

The phase of the other wave is

$$\phi_A' = \phi_B' + 2\pi \frac{n_2}{\lambda} L$$

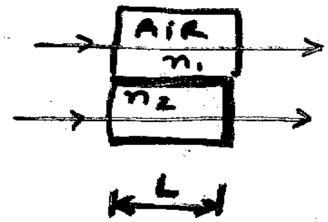
Far away, at the right side, the relative phase between the waves is now

$$\Delta = \phi_A' - \phi_A$$

$$\boxed{\Delta = 2\pi \frac{n_2 - n_1}{\lambda} L}$$

SAMPLE problem 36-1 page 865

$\lambda = 550\ nm$ $\quad n_1 = 1$ air $\quad n_2 = 1.6$ plastic $\quad L = 2.6\ \mu m$



solution:

$$\Delta\phi = 2\pi \frac{1.6-1}{550\ nm}\ 2600\ nm$$

$1\ \mu m = 10^3\ nm$

$1\ nm = 10^{-9}\ m$

$$\Delta\phi = 2\pi \times 2.84 \quad \checkmark$$

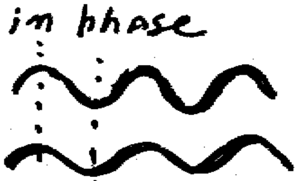
$$= 2\pi(2+0.84) = \underbrace{2\pi \times 2}_{\text{we would not distinguish this phase difference}} + 2\pi \times 0.84$$

$\Delta \longrightarrow 2\pi \times 0.84$ radians $= 5.3$ rad

or

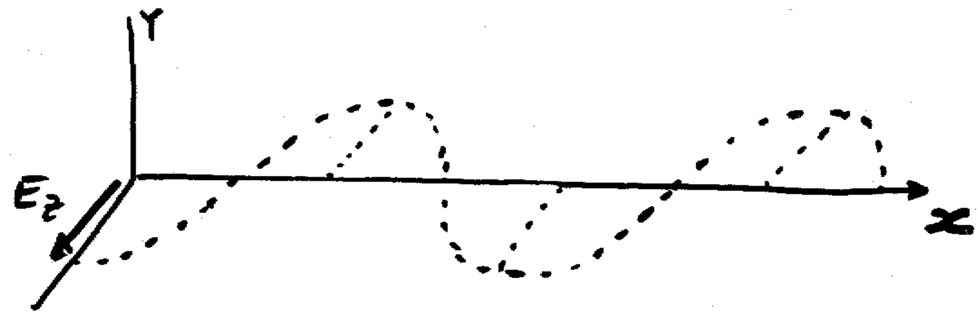
$\longrightarrow 360 \times 0.84$ degrees $= 302°$

in phase

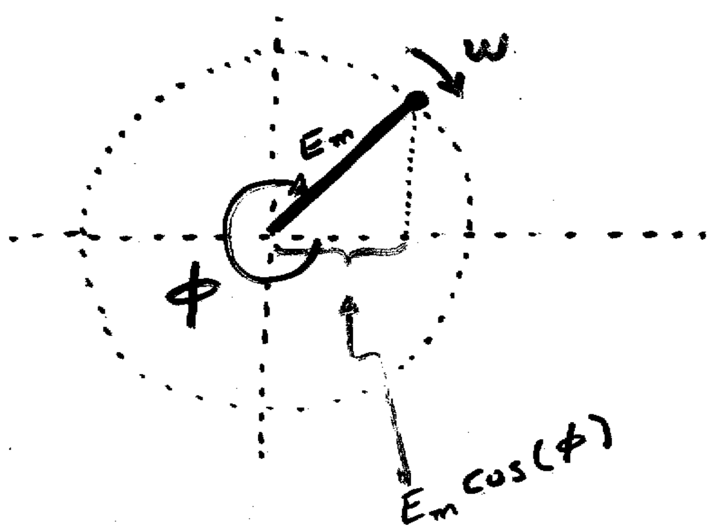


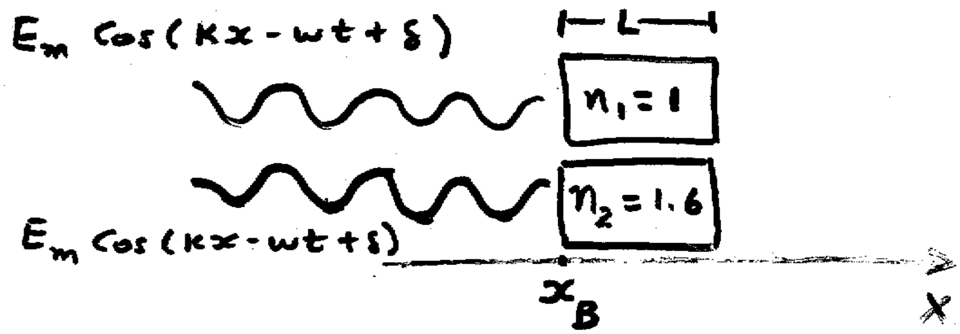
PHASORS METHOD

$$E_z = E_m \cos(kx - wt + \delta) = E_m \cos(\phi)$$



We visualize this component of the electric field as the horizontal projection of a rotating segment of length E_m





Consider 2 waves, initially on phase and incident separately on two media of index of refraction n_1 and n_2

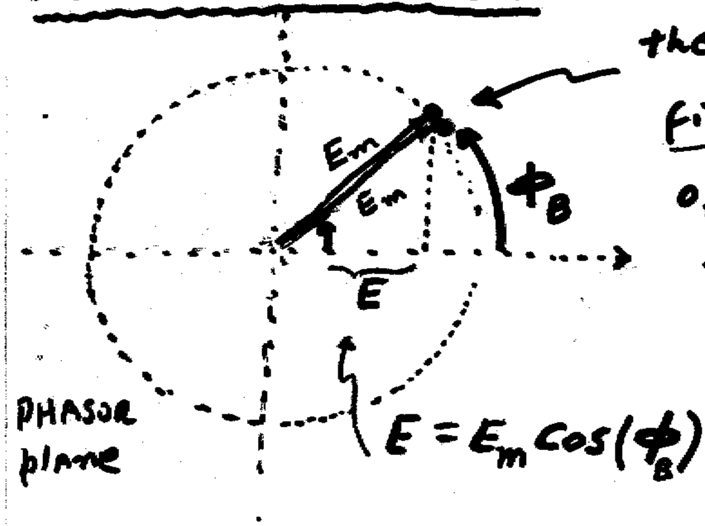
At $x = x_B$

the PHASE of both waves is given by

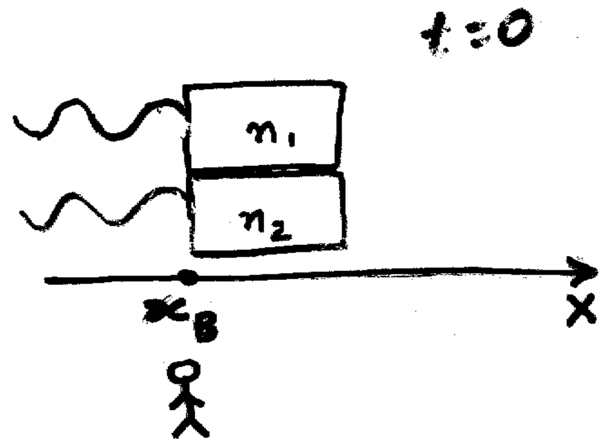
$$\phi_B = (k x_B - \omega t + \delta)$$

(as the time changes, so does the phase)

At $t = 0$ and at x_B :

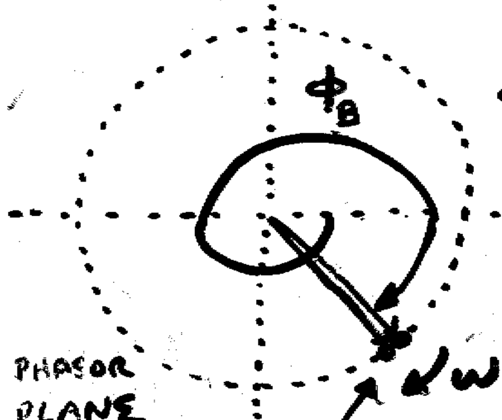


the actual electric fields are the projection of these vectors along the horizontal axis.



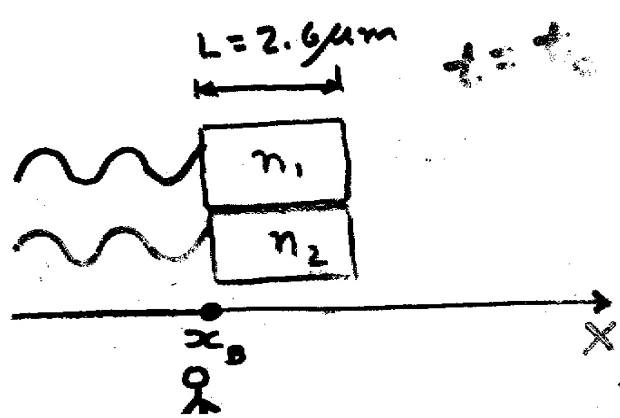
At $t = 0$: $\phi_B = k x_B + \delta$

At $t = t_0$ and at $x = x_B$:

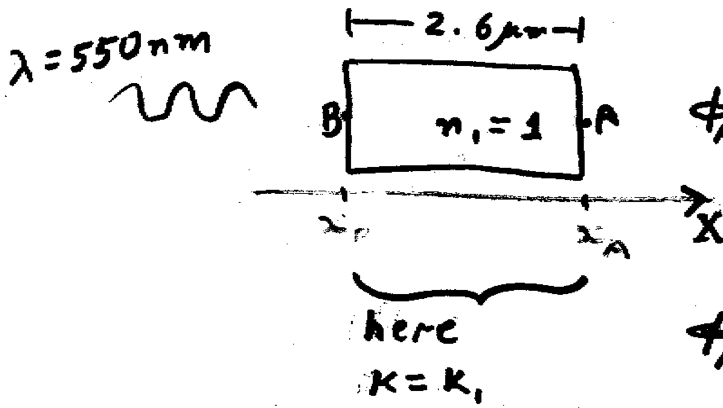


$\phi_B = (k x_B - \omega t_0 + \delta)$

these two vectors are traveling in phase. same ϕ --- both waves



At a fixed value of $t = t_0$



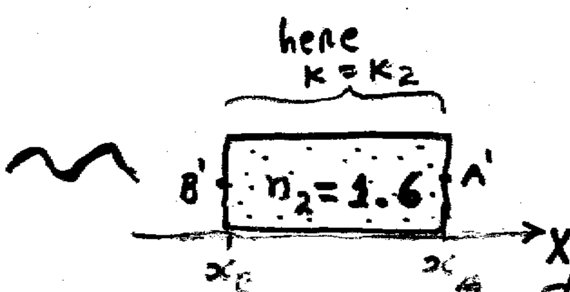
$$\phi_A = \phi_B + K_1 L$$

$$\phi_A = \phi_B + \frac{2\pi}{\lambda/n_1} \cdot L$$

$$= \phi_B + \frac{2\pi}{\lambda} n_1 L$$

$$= \phi_B + \frac{2\pi}{550\text{nm}} \times 1 \times 2600\text{nm}$$

$$\boxed{\phi_A = \phi_B + 2\pi \times 4.73}$$



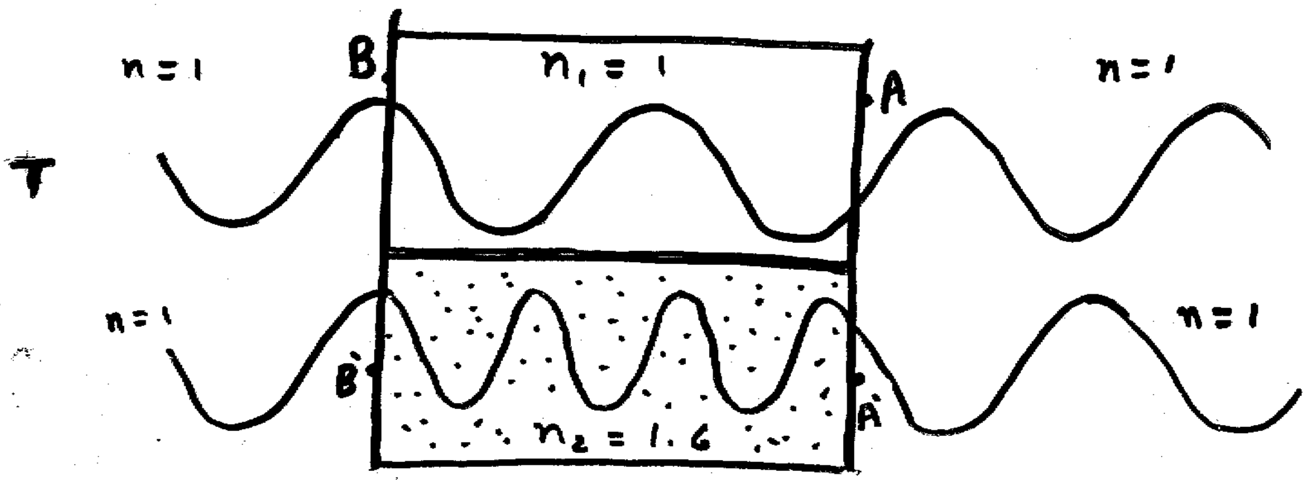
$$\phi_{A'} = \phi_{B'} + K_2 L$$

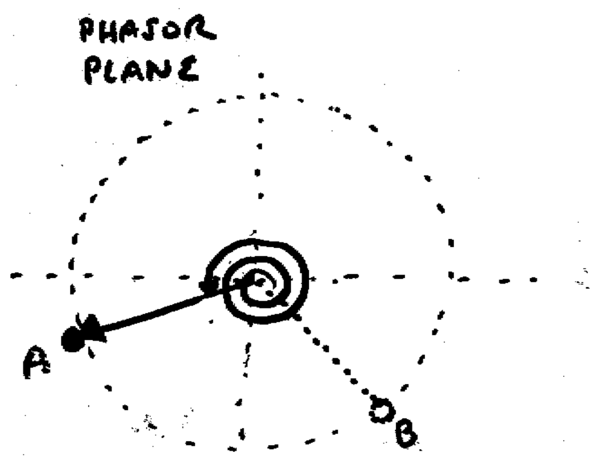
$$\phi_{A'} = \phi_{B'} + \frac{2\pi}{550\text{nm}} \times 1.6 \times 2600\text{nm}$$

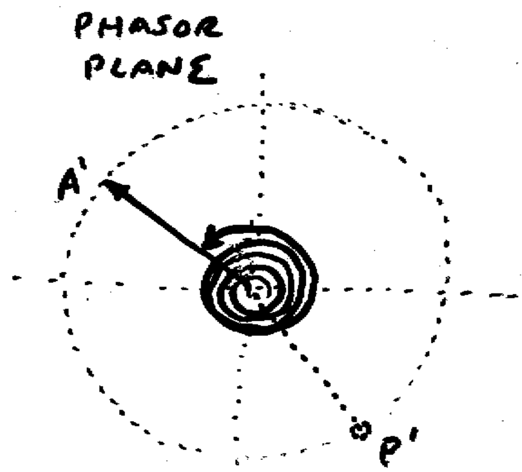
$$\boxed{\phi_{A'} = \phi_{B'} + 2\pi \times 7.56}$$

At a fixed value of $t = t_0$

ANOTHER VIEW







$\phi_A = \phi_B + \frac{2\pi}{\lambda/n_1} \times L$

$\phi_A = \phi_B + 2\pi \times \underbrace{4.73}_{\text{almost 5 turns}}$

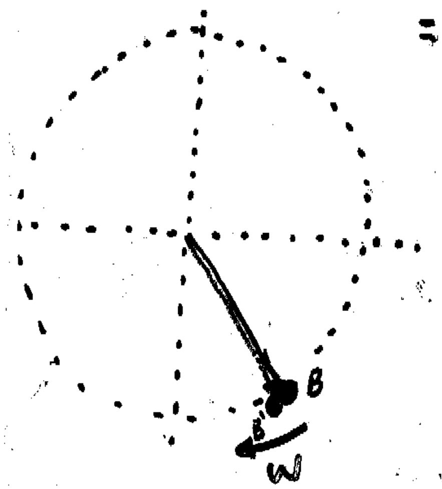
$\phi_{A'} = \phi_{B'} + \frac{2\pi}{\lambda/n_2} \times L$

$\phi_{A'} = \phi_{B'} + 2\pi \times \underbrace{7.56}_{\approx 7\frac{1}{2}\text{ turns}}$

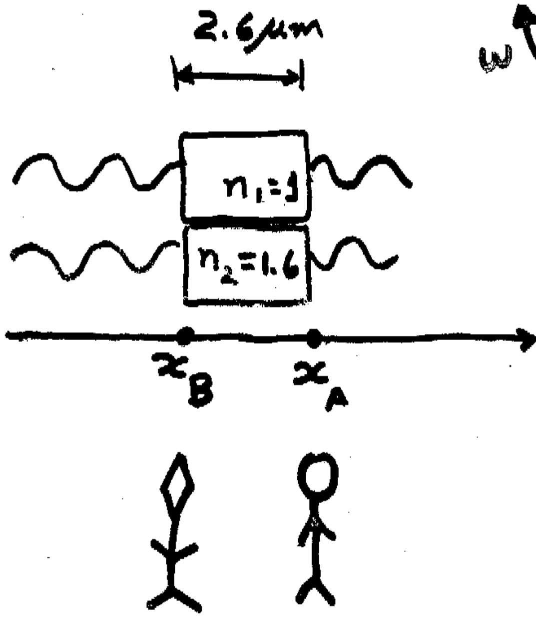
$$\phi_{A'} = \phi_A + 2\pi \times (7.56 - 4.7)$$

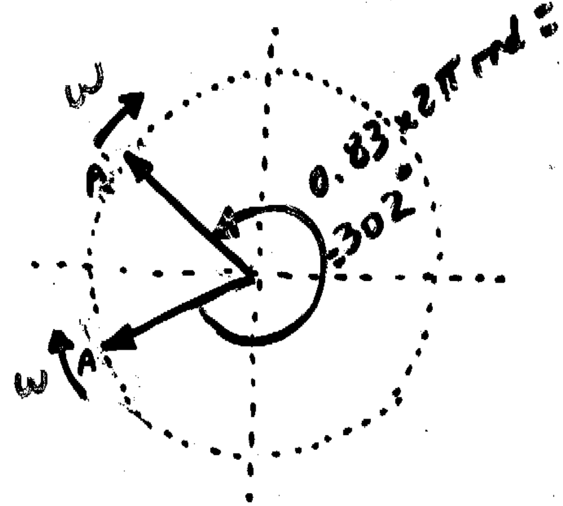
$$= \phi_A + 2\pi \times 2.83 = \phi_A + 2\pi \times 2 + 2\pi \times 0.83 \text{ rad}$$

$$= \phi_A + 302^\circ$$

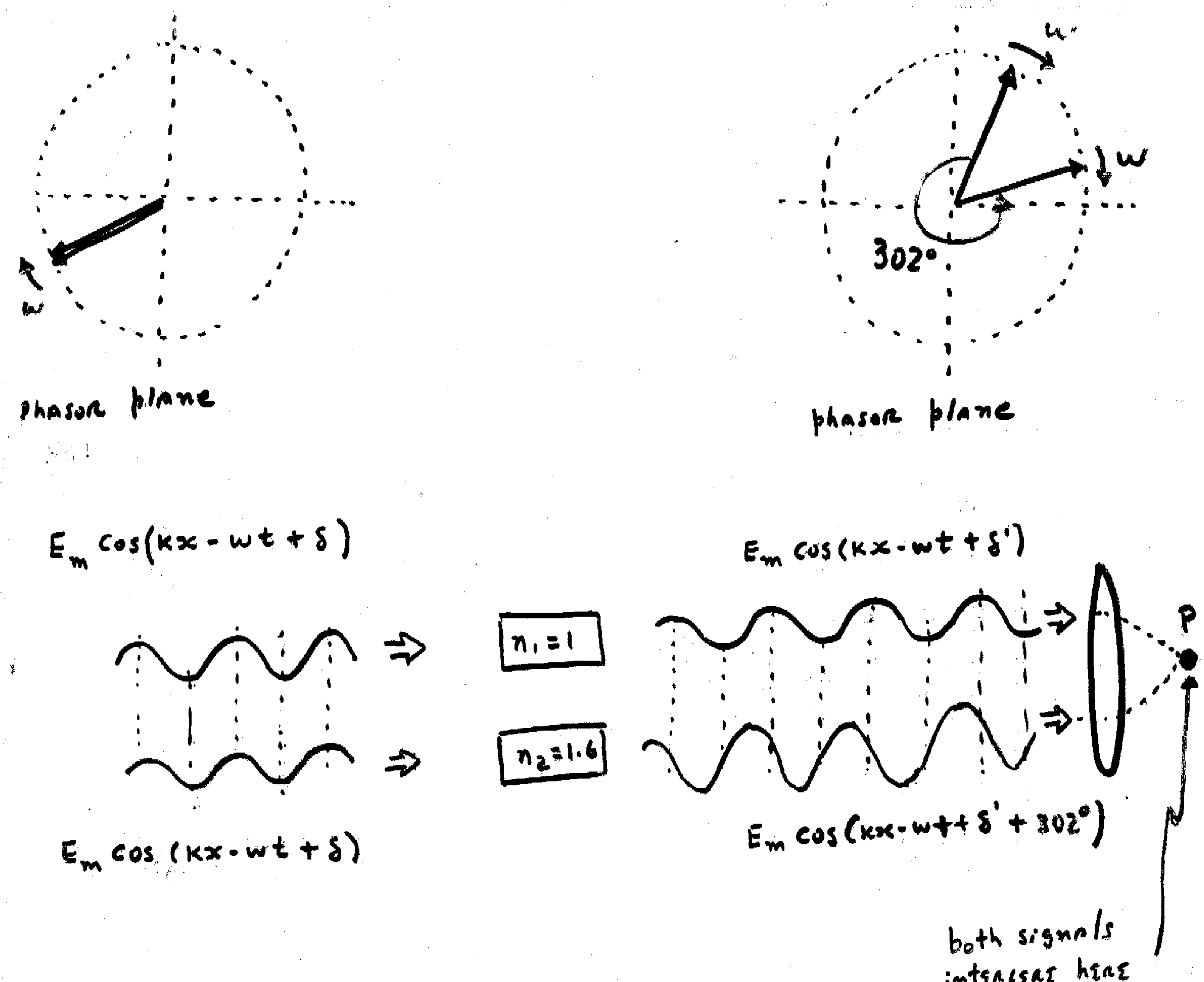


This is what the observer figures at x_B





This is what the observer figures at x_A


ω
phasor plane
ω'
ω
302°
phasor plane
$E_m \cos(kx - \omega t + \delta)$
$n_1 = 1$
$n_2 = 1.6$
$E_m \cos(kx - \omega t + \delta)$
$E_m \cos(kx - \omega t + \delta')$
P
$E_m \cos(kx - \omega t + \delta' + 302°)$
both signals
interfere here

the interference signal at P can be found either

a) ANALYTICALLY

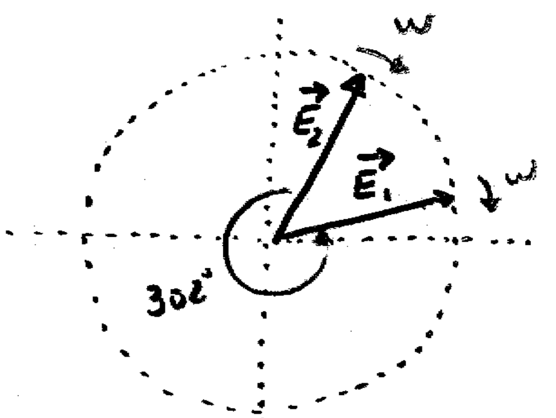
$$E_{TOTAL} = E_m \cos(kx - \omega t + \delta') + E_m \cos(kx - \omega t + \delta' + 302^\circ)$$

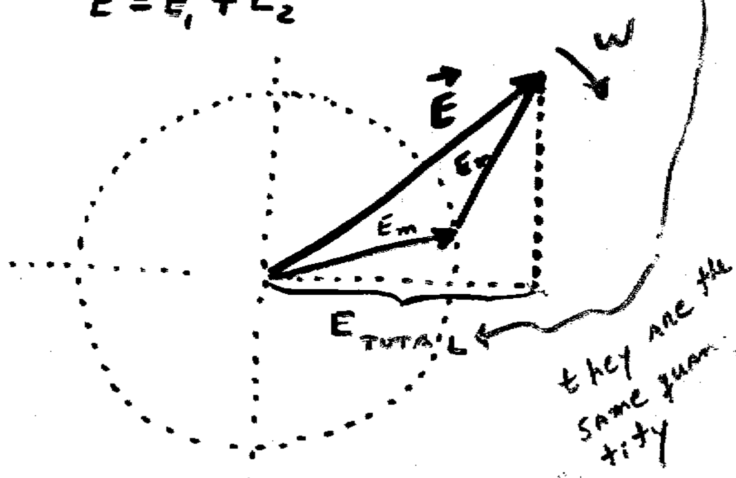
$$= 2 E_m \cos\left(\frac{302^\circ}{2}\right) \cos\left(kx - \omega t + \delta' + \frac{302^\circ}{2}\right)$$

or

b) GRAPHICALLY (using phasors)

$\vec{E} = \vec{E}_1 + \vec{E}_2$

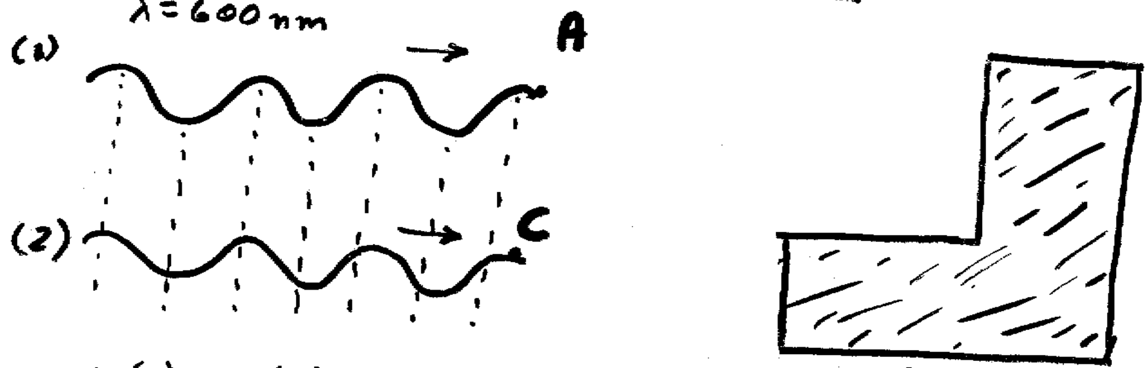




You add $\vec{E}_1$ and $\vec{E}_2$ (as if they were 2 dimensional vectors) to obtain vector $\vec{E}$. E_{TOTAL} is the horizontal component of $\vec{E}$.

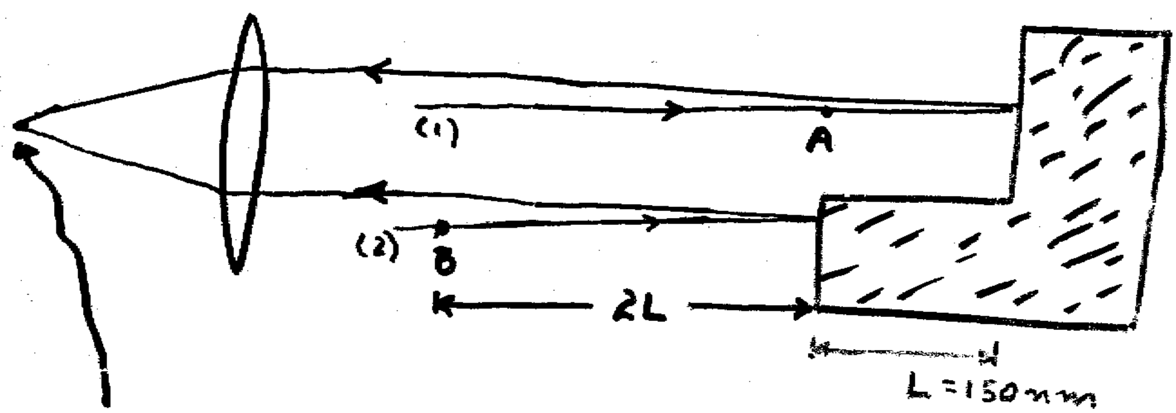
(Notice, $\vec{E}$ rotates also with angular speed ω; the magnitude $|\vec{E}|$ remains constant

PHASE SHIFT between waves that travel different distances



waves (1) and (2) are in phase

Two waves initially in phase can also change their relative phase difference by making them to travel different distances.



point of interference between wavefronts A and B

- Notice, the lens allows the interference between

  the wavefront A of wave (1)

  the wavefront B, running behind, of wave (2)

  where the distance between A and B is 2L.

- The phase difference $\Delta = \Phi_A - \Phi_B$ is

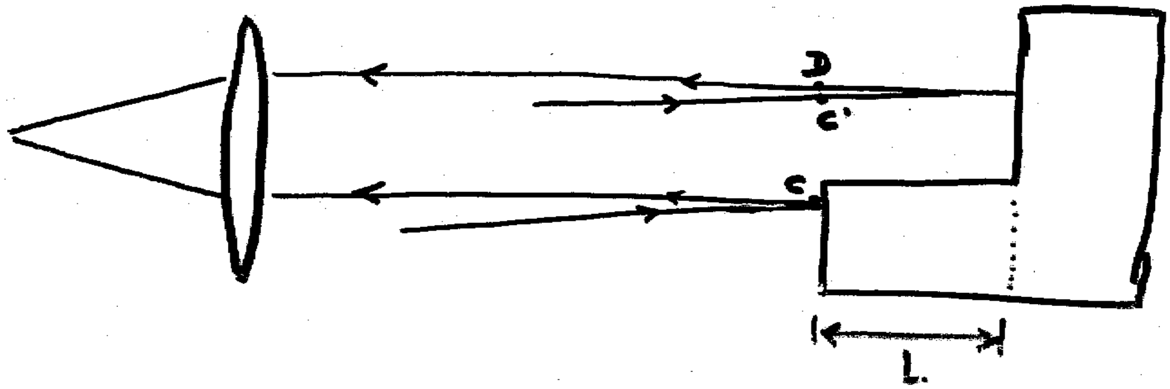
$$\Delta = K \, 2L$$

$$= \frac{2\pi}{\lambda} 2L$$

$$\boxed{\Delta = 4\pi \frac{L}{\lambda}}$$

If $L = 150\,nm$, $\lambda = 600\,nm$ $\quad \Delta = \pi \quad$ destructive interference

Alternatively, we could have also used the following graph. 24



and argue that C and D are the wavefronts to interfere.

Since $\phi_D = \phi_{C'} + K\,2L$

and

$\phi_{C'} = \phi_C$

then $\Delta = \phi_D - \phi_C = K\,2L$

Practice : Problem 9P, 10P, 8P $\begin{pmatrix} \text{CH-36} \\ \text{p. 884} \end{pmatrix}$
*