UNIFORM CIRCULAR MOTION

speed → CONSTANT

Velocity → Is not constant

(that's why there exist an acceleration)

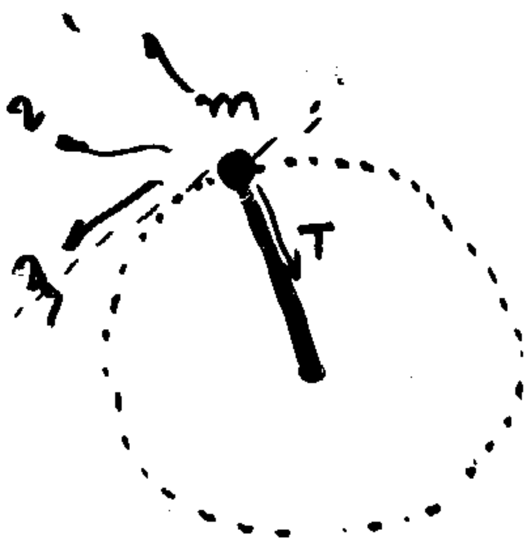
ACCELERATION → $\dfrac{V^2}{R}$

CENTRIPETAL ACCELERATION

The centripetal acceleration is caused by a CENTRIPETAL FORCE

$$F = m\frac{V^2}{R}$$

CENTRIPETAL FORCE



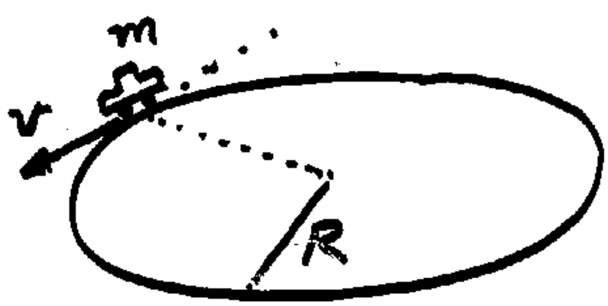
Mass m attached to a string

The centripetal force is provided by the tension force T

Satellite S orbiting the earth

The centripetal force is the gravitational attraction F_g excerted by the earth on the satellite.

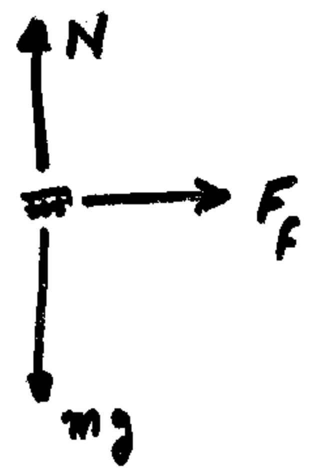
EXAMPLE



CAR traveling at a constant speed v along a flat circular road of radius R

What does provide the CENTRIPETAL FORCE ?

Free-body diagram



No vertical motion ⇒ $N = mg$

No radial slipping ⇒

$$F_f = m \frac{v^2}{R}$$

Is $F_f = \mu N$?

Should we use μ_s or μ_k ?

On the verge of slipping $\Rightarrow$ $\underbrace{F_f}_{m\frac{v^2}{R}} = \mu_s \underbrace{N}_{mg}$

(*)

NOTICE: m will cancel out

$$\frac{v^2}{R} = \mu_s g \quad \text{or} \quad \boxed{\mu_s = \frac{v^2}{Rg}}$$

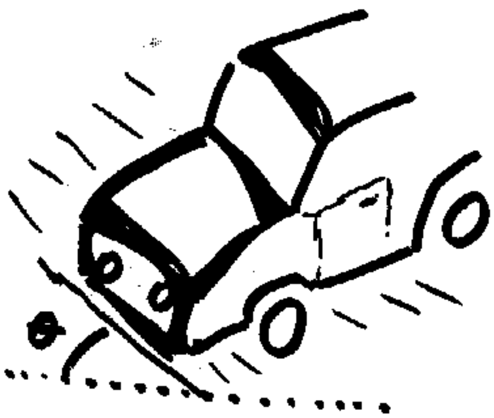
- The higher speed v the higher μ_s (the higher friction force needed to avoid sliding)

- For a given v and R the required μ_s to avoid sliding is the same for a small car or for a Big truck

EXAMPLE:

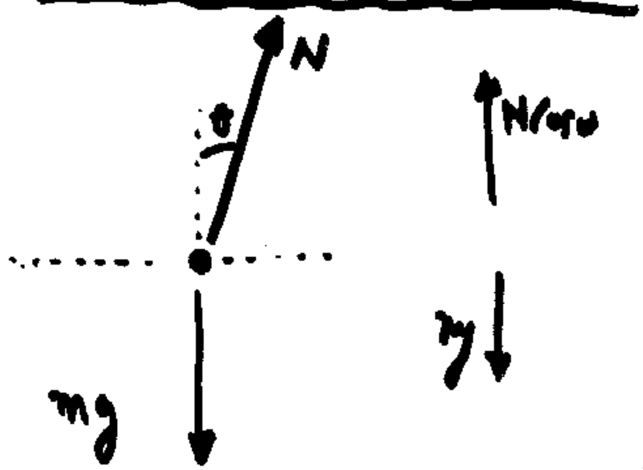
Car traveling at constant speed v along a banked road ($\angle\,\theta$)

FOR A GIVEN v
WHAT BANK ANGLE MAKES
RELIANCE ON FRICTION UNNECESSARY

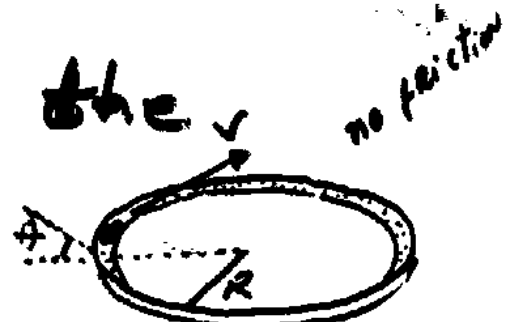


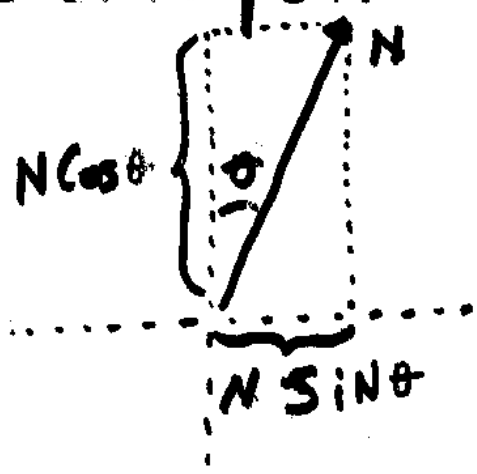
no friction

Free-body diagram



Which force will provide the centripetal force?





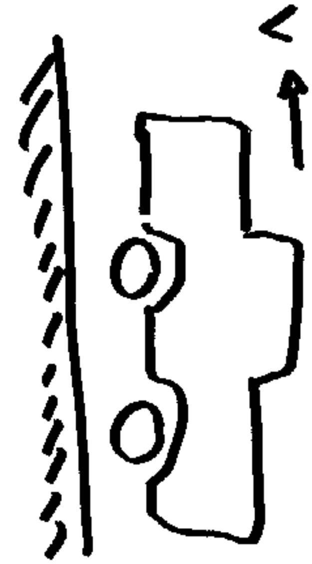
$$N \sin\theta = \frac{m v^2}{R}$$

Higher v
higher θ needed.

NO VERTICAL MOTION implies:

$$N \cos\theta = mg$$

$$N \sin\theta = \frac{mv^2}{R}$$



$$\boxed{\tan\theta = \frac{v^2}{gR}}$$

Notice, from the two previous examples, that the two quantities

μ_s for an unbanked road

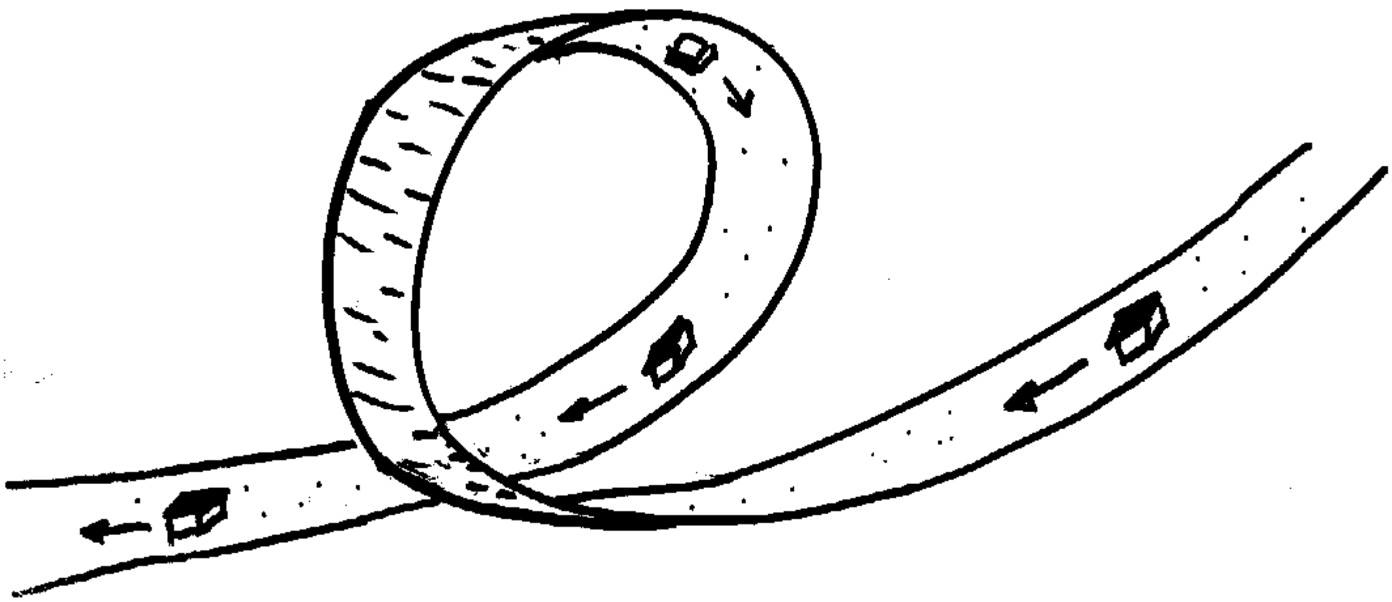
$\tan\theta$ for a banked road

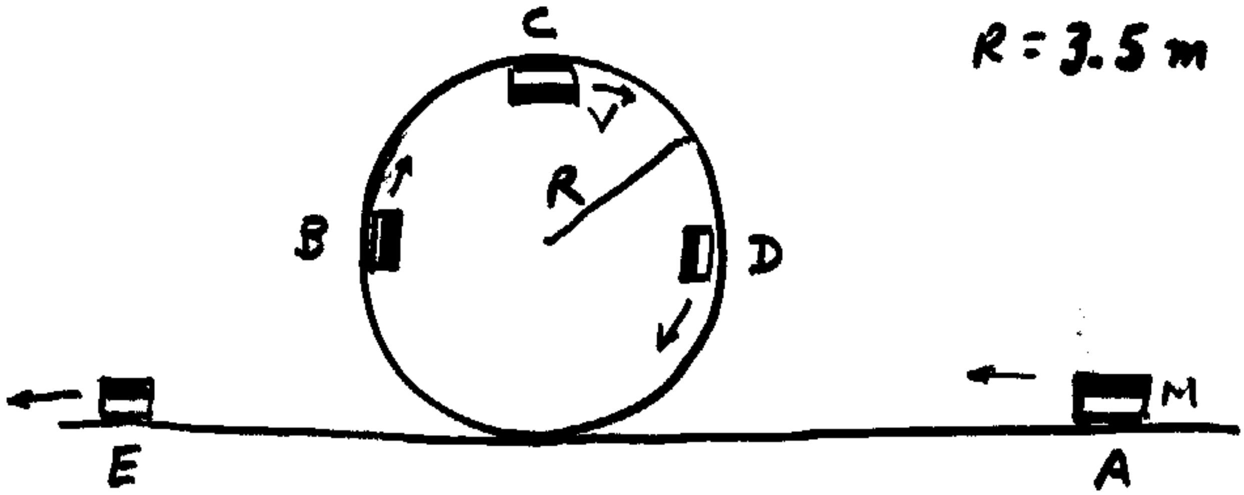
are numerically equal.

That is: THE ROAD MUST CAUSE A CERTAIN CENTRIPETAL FORCE, either through μ_s or building it with a bank angle

Practice:

- Sample problem 6-9 (page 110)
- Problems 36E, 37E chapter 6



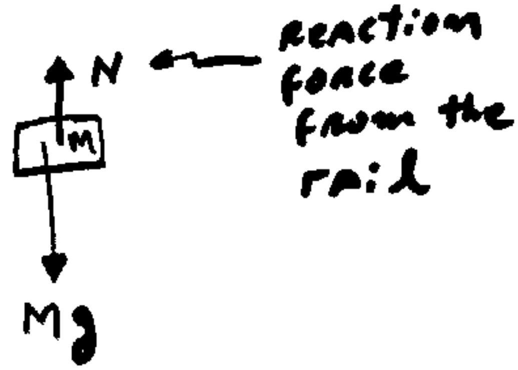


$R = 3.5\ m$

Will the block be able to
move around, without loosing

We have identified a few positions "A", "B", "C", "D" and "E" along the rail.

At "A"



$N_A = ?$

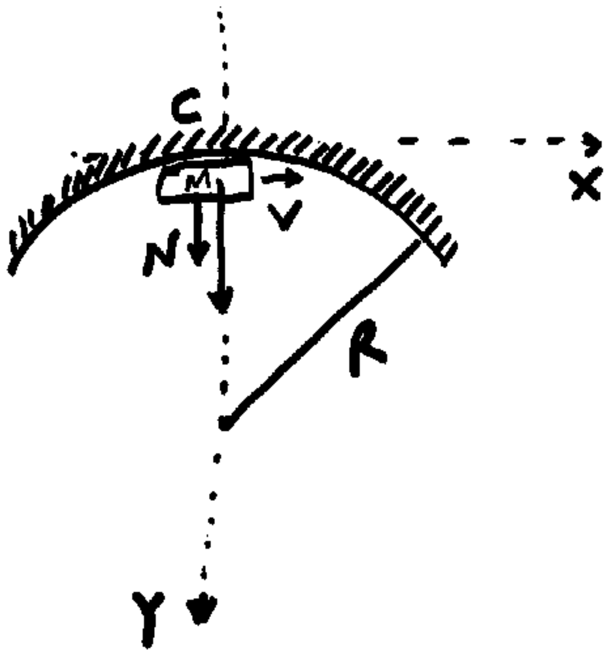
At "B"

If the block were still in contact with the rail, there would be a reaction force

At "C"

If block is in contact with the rail, there will be a Normal force

$N \neq 0$ means contact between the block and the rail



V = ?

What should be the value of V such that the block remains in contact with the rail?

when the block passes by "C":

Forces in the Y direction

$$Mg + N$$

Acceleration in the Y direction

$$a = \frac{V^2}{R}$$

Newton's 2nd law $F = ma$

$$(Mg + N) = M\left(\frac{V^2}{R}\right)$$

OR

$$N = M\frac{v^2}{R} - Mg$$

the higher speed, the higher N needed (to keep the block moving along the circular path)

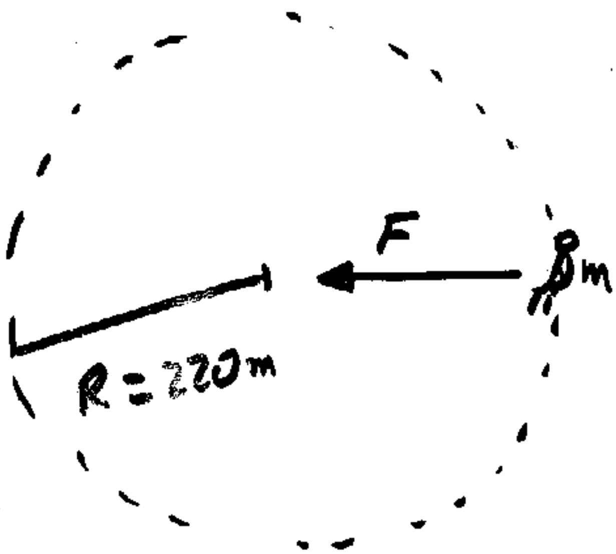
The minimum possible speed V such that block of mass M barely remains in contact with the rail is:

$$\not{M}\frac{v^2}{R} = \not{M}g$$

$$v^2 = Rg$$

$$v_{min} = \sqrt{Rg}$$

Example: (Textbook, 63P page 129) 5th Ed.



F force needed to act on you, in order to make you turn around the circumference of radius $R = 220m$ and at a speed $v = 94\ km/h$

$$F = m\frac{v^2}{R}$$

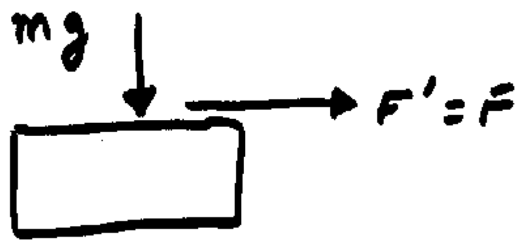
Such force F is provided by the friction force that your seat exerts on you

From the seat acting on you

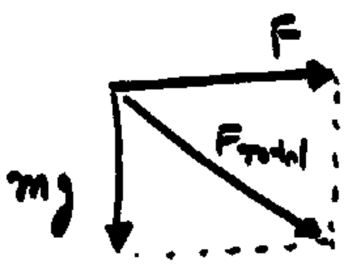
According to Newton's Third Law, there is a reaction force acting on the seat

$F' = F$ (magnitude)

In addition to F' you are also pushing down the seat with your weight



So:



$$F_{TOTAL} = \sqrt{F^2 + (mg)^2}$$