

# GRAVITATIONAL FORCE

## Gravitation

determines star formation, planetary systems, galaxies, and the universe as a whole

was the first of the four fundamental forces to be described quantitatively.

Its effects are felt from microscopic to cosmological distances

The interest for the gravitation started very early on due to the fact that

It determines the structure of the solar system, a problem that intrigued people from the beginning of civilization.

It affect every object around us (physics textbook, you weight, causes the days and nights, etc.)

## How did Newton deduced the gravitation force law?

### Dependence on distance

- Assume the Moos rotate around the Earth following a uniform circular.

This implies that the Moon experiences a centripetal acceleration equal to  $a = v^2/r$

where  $r$  is the orbital radius, measured from the center of the Earth to the center of the Moon ( $r = 3.84 \times 10^8$  m. )

Notice  $v = 2\pi r/T$

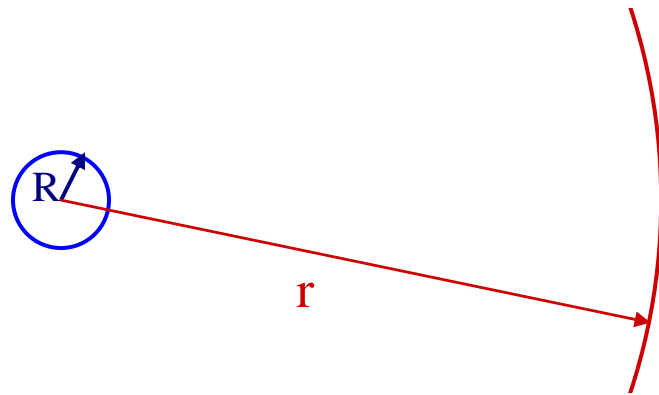
where  $T$  is the Moon's orbital period of revolution.

$T = 27.3$  days  $= 2.36 \times 10^6$  s

Hence,

$$\begin{aligned} \mathbf{a} \text{ (at a distance } r) &= v^2/r = 4\pi^2 r/ T^2 \\ &= 2.72 \times 10^{-3} \text{ m/s}^2 \end{aligned}$$

- The magnitude of the acceleration due to gravity near the surface of the Earth is  $9.81 \text{ m/s}^2$ . That is  $\mathbf{a}(\text{at } R) = 9.81 \text{ m/s}^2$ .
- $\mathbf{a}(\text{at } R) / \mathbf{a}(\text{at } r) = 9.81 / 2.72 \times 10^{-3}$   
 $= 3.60 \times 10^3$  (1)



On the other hand,

$$\begin{aligned} (r/R)^2 &= 3.84 \times 10^8 \text{ m} / 6.37 \times 10^6 \text{ m} \\ &\sim (60)^2 = 3600 \end{aligned} \quad (2)$$

Newton concluded:

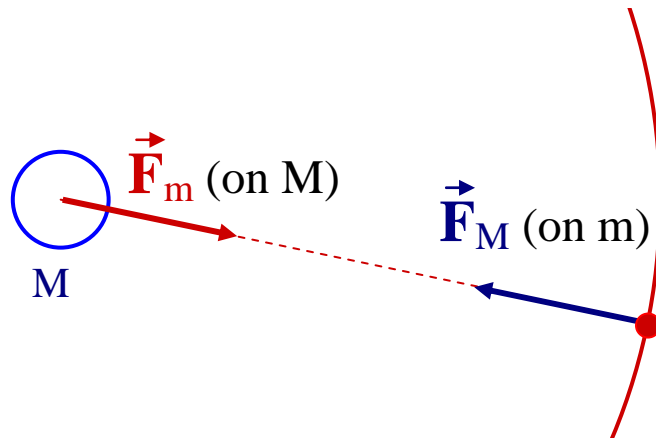
$$F_{\text{gravitation}} \sim 1/r^2 \quad (3)$$

### Dependence on distance

Since a common physical property of the Earth and the Moon is *mass*, it was reasonable that their masses also play a role in the gravitational force.

Let  $M$ : mass of the Earth

$m$ : mass of the Moon



From Newton's third law

$$\vec{F}_m \text{ (on M)} = - \vec{F}_M \text{ (on m)}$$

Hence, if the gravitational force depends on the mass, it should involve "M" and "m" in a symmetric way. That is, interchanging "m" and "M" should render the same result.

Possibilities:

$$F \sim (M + m)^n \quad \text{or} \quad F \sim (M m)^n$$

Newton's second law  $F=ma$  helps to discarded powers  $n>1$ .

Reason: From Galileo experiments, it is known that the acceleration of any mass  $m$  is independent of the mass  $m$ . (in vacuum, feathers and rocks fall with the same acceleration.)

However, the above expressions, when applied to find the acceleration of a mass  $m$  would give the following accelerations,

$$a \sim \frac{(M + m)^n}{m} \quad \text{or} \quad a \sim \frac{(Mm)^n}{m}$$

The requirement for  $a$  to be independent of  $m$  rules out the sum dependence (left expression above) of the

gravitational force.. It also requires that the power in the product dependence (second expression above) to be equal to  $n = 1$ . That is

$$F \sim Mm \quad (4)$$

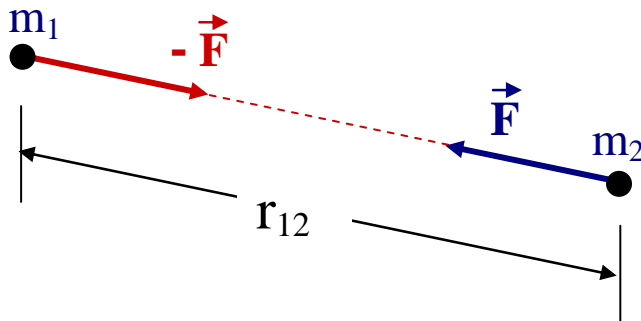
From (3) and (4)

$$F \sim \frac{Mm}{r^2} \quad (5)$$

## Newton's Law of Universal Gravitation

Newton's law describes the remarkable observation that two **pointlike** particles in the universe exert a mutually attractive force on each other, which is proportional to the product of their masses and inverse proportional to the square of their separation distance.

The constant of proportionality  $G$  is obtained in the laboratory by directly measuring the slight force between two known masses separated by a known distance



$$F_{12} = G \frac{m_1 m_2}{(r_{12})^2} \quad (6)$$

$$G = 6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2$$

## Gravitational mass and inertial mass

The masses that appear in the law of universal gravitation are called **gravitational masses**.

$$F_{12} = G m_{1,\text{grav}} m_{2,\text{grav}} / (r_{12})^2$$

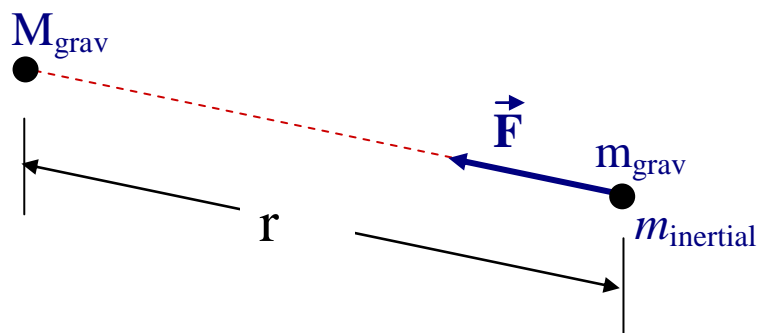
This law indicates how to calculate the gravitational force, via the corresponding gravitational masses associated to the particles. The gravitational masses do not need to be necessarily in motion to calculate this force.

On the other hand, the mass  $m$  that appears in the Newton's second law is the **inertial mass**.

$$F = m_{\text{inertial}} a$$

This law indicates how a particle (via its inertial mass) responds to a force.

Newton was concerned whether the gravitational mass property and the inertial mass property (for the same body) were the same



Let's assume the gravitational force is the total force acting on a given mass (the one on the right in the figure above.)

Accordingly,

$$F = G M_{\text{grav}} m_{\text{grav}} / r^2 \quad \text{and}$$

$$F = m_{\text{inertial}} a$$

Hence, the acceleration of the mass on the right side of the figure above is,

$$a = \frac{G M_{\text{grav}}}{r^2} \frac{m_{\text{grav}}}{m_{\text{inertial}}}$$

The acceleration depends on the ratio of the gravitational and inertial masses.

But experiments indicate that all objects fall with the same acceleration, the latter having a numerical value equal to  $GM_{\text{grav}}/r^2$ . Very careful experiments indicate

$$\frac{m_{\text{grav}}}{m_{\text{inertial}}} = 1.000\ 000\ 000\ 000(?)$$