Practice problems Chapter 2

17) $x = 9.75 + 1.5\, t^3$ (1)

[they really mean $x = 9.75\text{ cm} + 1.5\text{ cm}\left(\frac{t}{\text{sec}}\right)^3$]

velocity $v = \frac{dx}{dt} = 0 + 1.5\frac{d}{dt}t^3 = 1.5 \times 3t^2 = 4.5\,t^2$ (2)

[We really mean $V = 4.5\,\frac{\text{cm}}{\text{sec}} \times \left(\frac{t}{\text{sec}}\right)^2$]

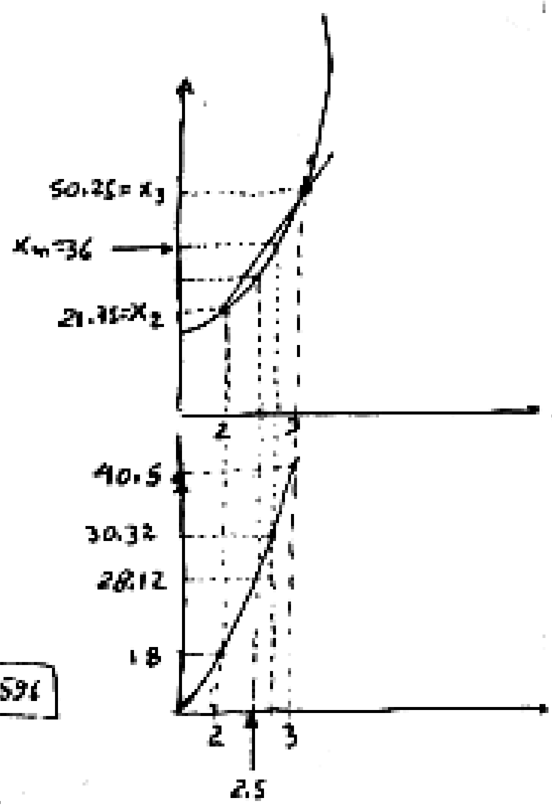
As far as we work with x in cm and t in sec we are OK in using (1) and (2) without worrying about the units.

a) when $t = 2 \longrightarrow x = x_2 = 9.75 + 1.5\,(2)^3 = 21.75$ cm

$t = 3 \longrightarrow x = x_3 = 9.75 + 1.5\,(3)^3 = 50.25$ cm

Average velocity $= \frac{x_3 - x_2}{\Delta t} =$

$= \frac{21.75 - 50.25}{1} = 28.5$ cm/sec



b) When $t = 2\text{sec} \longrightarrow V = 4.5 \times (2)^2 = 18$ cm/sec

c) When $t = 3\text{sec} \longrightarrow V = 4.5 \times (3)^2 = 40.5$ cm/sec

d) When $t = 2.5\text{sec} \rightarrow V = 4.5 \times (2.5)^2 = 28.12$

e) Midway between x_2 and x_3 is x_m

$x_m = x_2 + \frac{x_3 - x_2}{2} =$

$= 21.75 + \frac{50.25 - 21.75}{2} = 36$ cm

Also,

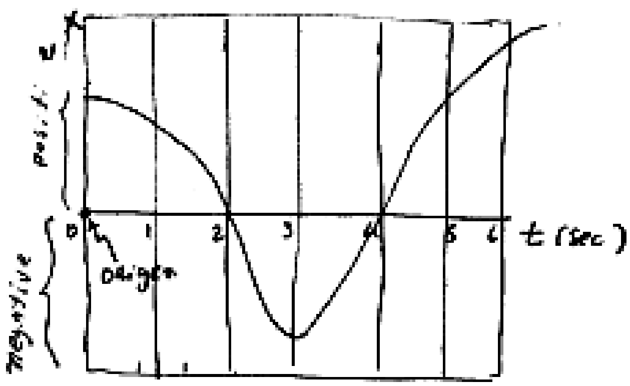
when $x = x_m = 36$, $t = t_m = ?$

$x_m = 9.75 + 1.5\, t_m^3$

$36 = 9.75 + 1.5\, t_m^3 \xRightarrow[\text{solving for } t_m]{} \boxed{t_m = 2.596}$

when $t = t_m = 2.596 \longrightarrow v = 4.5 \times t_m^2 = 30.32$ m/s

Problem: The graph shown is for an armadillo that scampers left (negative direction) and right (positive direction) along an x-axis. A) When, if ever, is the animal to the left of the origin on the axis? When, if ever, is the velocity b) negative, c) positive, or d) zero?



a) During the interval from 2 sec to 4 sec the animal is located at the LEFT of the origen

b) The velocity is negative whenever the slope is negative in the graph x vs t.
Thus: From 0 to 3 sec the velocity vector is pointing to the negative side of the x axis.

c) From $t = 3$ sec to $t = 7$ sec $\longrightarrow$ velocity is positive

d) The velocity is instantaneously zero at $t = 0$ sec, $t = 3$ sec and $t = 7$ sec.

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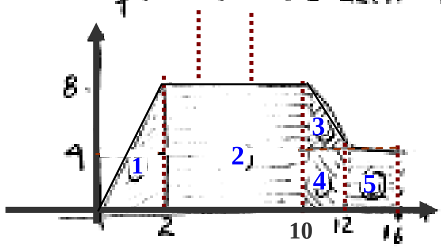
67) The net displacement of the runer is numerically equal to the area below the curve $v(t)$.



thus

$$\Delta x = \underbrace{\frac{8 \times 2}{2}}_{①} + \underbrace{8 \times (10-2)}_{②} + \underbrace{\frac{(12-10)\times(8-4)}{2}}_{③} + \underbrace{4 \times (12-10)}_{④} + \underbrace{4 \times (16-12)}_{⑤} = 100 \text{ m}$$

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30)

$$\text{ACCELERATION} = \frac{\Delta V}{\Delta t} = \frac{V_{final} - V_{initial}}{\Delta t}$$

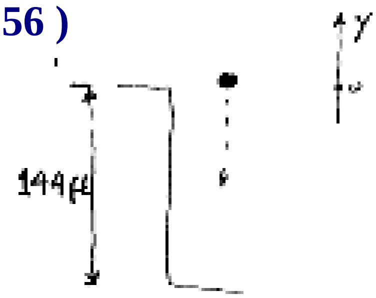
$$a = \frac{0 - 1020 \text{ km/h}}{1.4 \text{ sec}} = -728.6 \frac{\text{km}}{\text{h} \cdot \text{sec}}$$

$$a = -728.6 \frac{\cancel{\text{km}}}{\cancel{\text{h}} \cdot \text{sec}} \times \underbrace{\frac{1000 \text{ m}}{1 \cancel{\text{km}}}}_{=1} \times \underbrace{\frac{1 \cancel{\text{h}}}{3600 \text{ sec}}}_{1} = -\frac{728.6}{3.6} \frac{\text{m}}{\text{sec}^2}$$

$$\boxed{a = -202.4 \frac{\text{m}}{\text{sec}^2}}$$

$$|a| = 202.4 \frac{\text{m}}{\text{sec}^2} \times \underbrace{\left(\frac{g}{9.81 \text{ m/sec}^2}\right)}_{=1} = 20.6 \, g$$

56)



First stone

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

$y_0 = 0$, $v_0 = 0$, $a = -9.8\ m/s^2 = -32\ ft/s^2$

$$\boxed{y = -\tfrac{1}{2}\left(32\ \tfrac{ft}{s^2}\right) t^2}$$

When the stone reaches the floor $t = t_1$ and $y = -144\ ft$

$$-144\ ft = -\tfrac{1}{2}\left(32\ ft/s^2\right) t_1^2$$

$$\Rightarrow \quad t_1 = \sqrt{\frac{2 \times 144}{32}\ s^2} = 3\ sec$$

Second stone

$$y = y_0 + v_0 t + \tfrac{1}{2} a t^2$$

$y_0 = 0$, v_0: Don't know, $a = -32\ ft/s^2$

When $t = 2\ sec$ the stone reaches the floor $y = -144\ ft$

Thus

$$-144\ ft = 0 + v_0 \times (2\ sec) - \tfrac{1}{2}\left(32\ ft/s^2\right)(2\ sec)^2$$

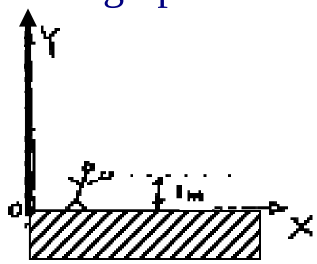
$$-144\ ft = 2 v_0 \cdot sec - 64\ ft$$

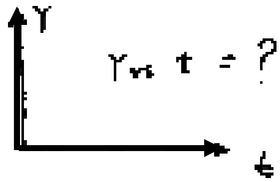
$$\Rightarrow \quad v_0 = \frac{-144 + 64}{2}\ \frac{ft}{sec} = -40\ ft/sec$$

Practice problem

A ball is thrown upwards with v_o = 10 m/s

a) Make a graph of **Y vs t**





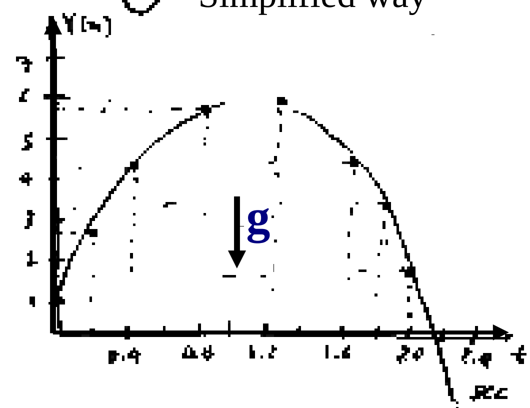
$$Y = \underbrace{Y_o}_{1\,m} + \underbrace{v_o}_{10\text{ m/s}} t + \frac{1}{2}\underbrace{a}_{-9.8\text{ m/s}^2} t^2$$

$$Y = 1\,m + (10\,\tfrac{m}{s})\,t - (4.9\,\tfrac{m}{s^2})\,t^2 \leftarrow (1) \leftarrow \text{correct way}$$

But, as ar as we keep "y" in meters and "t" in seconds, we write:

$$Y = 1 + 10t - 4.9t^2 \longleftarrow (2)$$ Simplified way

t	Y
0.2	2.8
0.4	4.2
0.8	5.86
1.2	5.34
1.6	4.2
1.8	3.12
2.0	1.4
2.4	-3.22



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b) *How long does it take to the ball to reach the to reach the max height*

$$V = V_0 + a(t - t_0) = \boxed{10\,\frac{m}{s} - \left(9.81\,\frac{m}{s^2}\right)\cdot t} \quad (3)$$

$V_0 = 10\,m/s$, $a = -9.8\,\frac{m}{s^2}$, $t_0 = 0$

we write

$$\boxed{V = 10 - 9.81\,t} \quad (4)$$

When the ball reaches the max height,
$t = t_1$ and $v = v_1 = 0$

$$0 = 10 - 9.81\,t_1 \longrightarrow \boxed{t_1 = 1.02\ s} \quad (5)$$

Time the ball takes to reach the max height

c) *What is the max height reached by the ball*

$y = y_o + v_o - (1/2)g\,t^2$

$y = 1m + (10\ m/s)t - (4.9\ m/s^2)\ t^2$

In the simplified version
$y = 1 + 10\ t - 4.9\ t^2$

In the simplified version
$y = y_{max}$ when $t = t_1 = 1.02$ s

$y_{max} = 1 + 10.2 - 4.9(1.02)^2$
$y_{max} = 6.1$ m

Alternatively we could have chosen the following expression

$$v^2 = v_0^2 + 2a(y - y_0)$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

$$v^2 = 100\left(\frac{m}{s}\right)^2 - 19.6\,\frac{m}{s^2}\cdot(y - y_0)$$

Simplified notation

$$v^2 = 100 - 19.6\cdot(y - y_0)$$

when $y = y_{max}$, $v = 0$

$$0 = 100 - 19.6\,(y_{max} - 1)$$

$$y_{max} - 1 = \frac{100}{19.6} = 5.1$$

$\Rightarrow$ $y_{max} = 6.1$ m

What is the position of the ball when $t = 2t_1$?

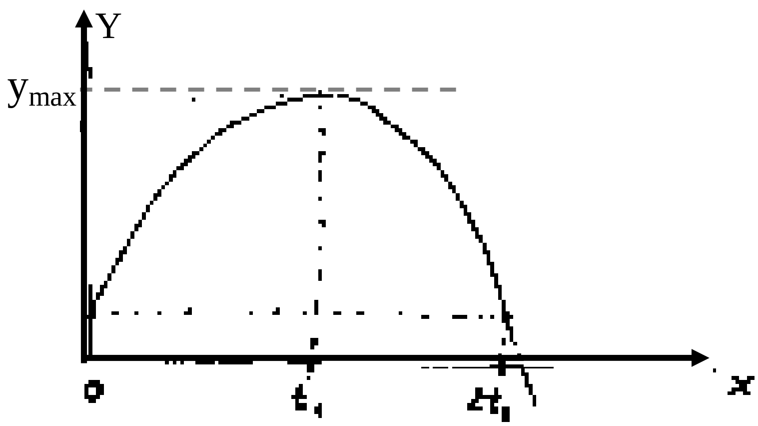
Using expression 1 :

$y = 1m + (10\ m/s)(\mathbf{2t_1}) - (4.9\ m/s^2)(\mathbf{2t_1})^2$

$y = 1m + (10\ m/s)(\mathbf{2}x\mathbf{1.02s}) - (4.9\ m/s^2)(\mathbf{2}x\mathbf{1.02s})^2$

$= 1m + 20.4m - 20.39m = 1m$ **Position At $t = 2t_1$**

That is



Conclusion:

t_1 : time it takes the ball to reach its max height

$2t_1$: time it takes the ball to come back to its initial height