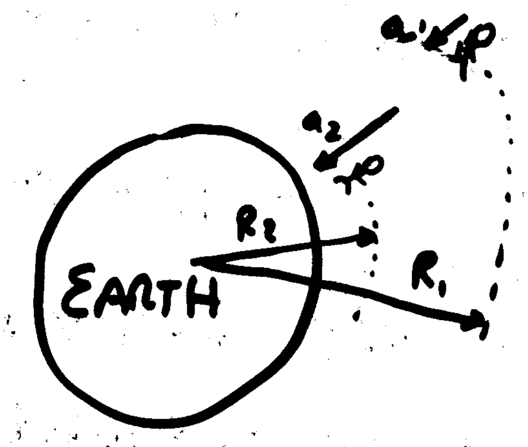
SPECIAL CASE:

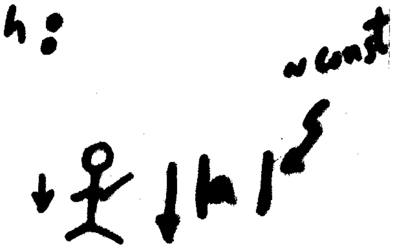
MOTION UNDER CONSTANT ACCELERATION

## FREE-FALL ACCELERATION




For large distances R from the earth, the accelerations $a_1$ and $a_2$ are very different. ($a_2 \gg a_1$)

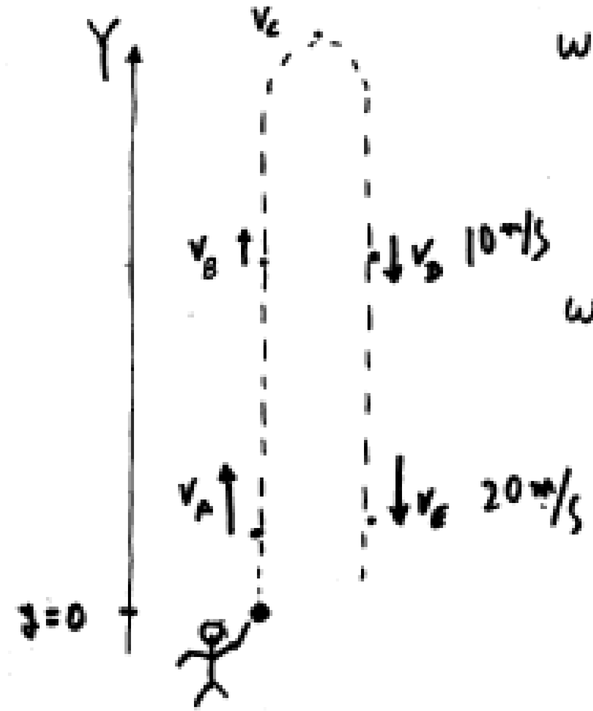
But, nearby the surface of the Earth: the acceleration does not change too much with distance, if the distance involved is ~ meters (assuming no effects from air viscosity)


- Objects falling down to earth increase their speed 9.8 $\frac{m}{s}$ every second

That is $\boxed{|a| = 9.8 \frac{m}{s^2} \equiv g}$

- $g$ is independent of the object's mass, shape, density, etc.




When the ball is on its way UP

acceleration is

When the ball is on its way DOWN

acceleration is

$$a = \frac{(-20\,m/s) - (-10\,m/s)}{\Delta} = \frac{-10\,m/s}{\Delta}$$

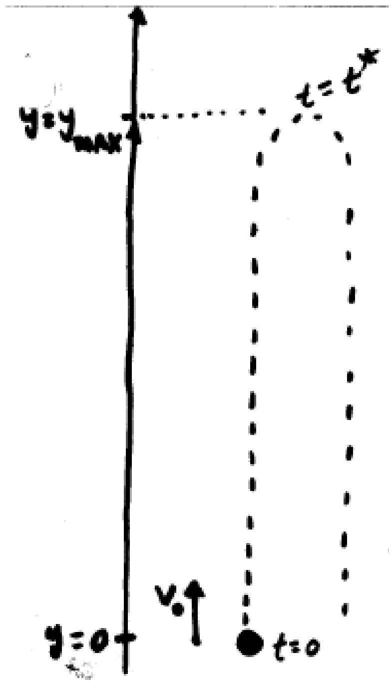
the 3 "magic" equations take the form:

$$\boxed{y = y_0 + v_0 t + \tfrac{1}{2}(-g)t^2} = y_0 + v_0 t - \tfrac{1}{2} g t^2$$

$$\boxed{v = v_0 + (-g)t} = v_0 - gt$$

$$\boxed{v^2 = v_0^2 + 2(-g)(y - y_0)} = v_0^2 - 2g(y - y_0)$$

**Example:** **A ball is thrown up with a initial velocity of 12 m/s. Evaluate the maximum height reached by the ball, as well as the time it takes to reach that peak.**




**Known quantities**

At $t$=0: $y = y_0 = 0$, $v = v_o = 12$ m/s

**Unknown quantities**

Max height $y_{max}$ reached by the ball

Time $t^*$ it takes to reach the max heigh

With the initial conditions at t=0 given by $y_0 = 0$ and $v_o = 12$ m/s the equation of motion takes the form:

$$y = v_0 t - \frac{1}{2} g t^2 = 12 \frac{m}{s} t - \frac{1}{2} g t^2 \longleftarrow g = 9.8\ m/s^2$$

$$v = v_0 - g t = 12 \frac{m}{s} - g t$$

$$v^2 = v_0^2 - 2 g y = \left(12 \frac{m}{s}\right)^2 - 2 g y$$

In particular, when $t = t^*$ the previous equations become:

$$y_{max} = 12\,\frac{m}{s}\,t^* - \frac{1}{2} g (t^*)^2$$

$$0 = 12\,\frac{m}{s} - g t^*$$

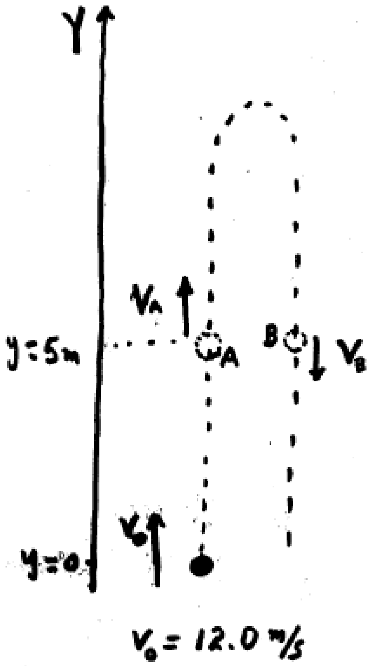
$$0 = \left(12\,\frac{m}{s}\right)^2 - 2g\,Y_{max}$$

gives $t^* = \dfrac{12\,\frac{m}{s}}{g} = 1.2$ sec

Knowing $t^*$ we can calculate $y_{max}$

$$Y_{max} = \frac{\left(12\,\frac{m}{s}\right)^2}{2g} = 7.3 \text{ m}.$$

**Question: What is the velocity of the ball when it passes through the coordinates y = 5 cm.**




*i* ) $y = y_0 + v_0 t - \frac{1}{2} g t^2$

$y_0$: 0 $\quad$ $v_0$: 12 m/s $\quad$ $g$: 9.8 m/s²

$$y = (12 \tfrac{m}{s})t - (4.9 \tfrac{m}{s^2})t^2$$

When $y = 5$ m

$$5 = 12t - 4.9t^2$$

or

$$4.9t^2 - 12t + 5 = 0$$

$$\boxed{\begin{array}{l} at^2 + bt + c = 0 \\ t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \end{array}}$$

$t = 0.53$ sec $\qquad$ $t = 1.92$

***ii*** **)** $v = v_o - g\,t$

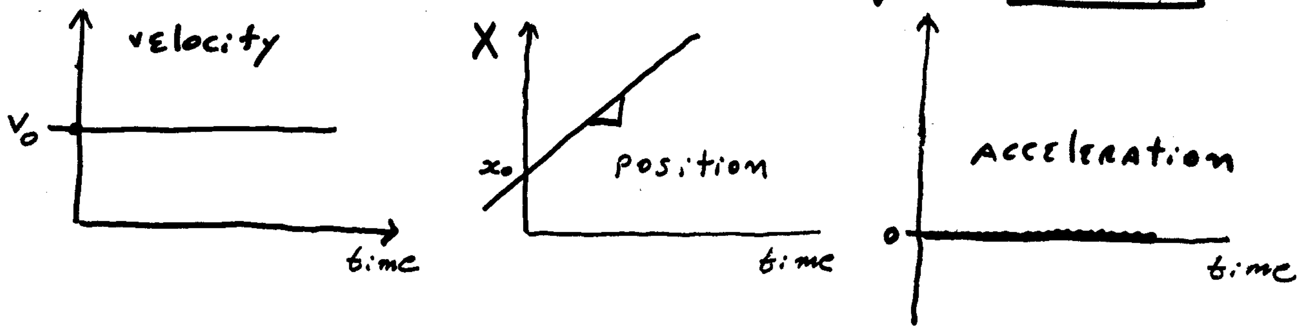
$\quad = 12$ m/s $- (9.8$ m/s$^2)\,t$

$t = 0.53\ s$ implies $\quad v = 6.8$ m/s

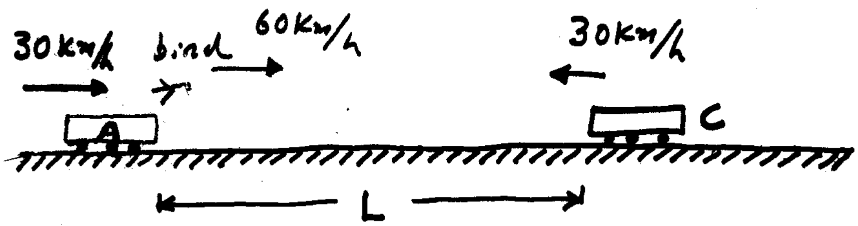
$t = 1.92\ s$ implies $\quad v = -6.8$ m/s

Review

- MOTION under CONSTANT VELOCITY $\boxed{V = V_0}$




$V = V_0$ $\qquad$ $x = x_0 + V_0 t$ $\qquad$ $a = 0$




$L = 60$ Km

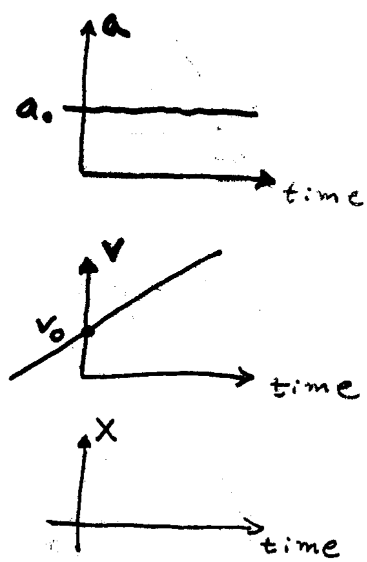
- Motion under CONSTANT ACCELERATION $\boxed{a = a_0}$

3 "magic" Equations

$$x = x_0 + V_0 t + \frac{1}{2} a_0 t^2$$

$$V = V_0 + a_0 t$$

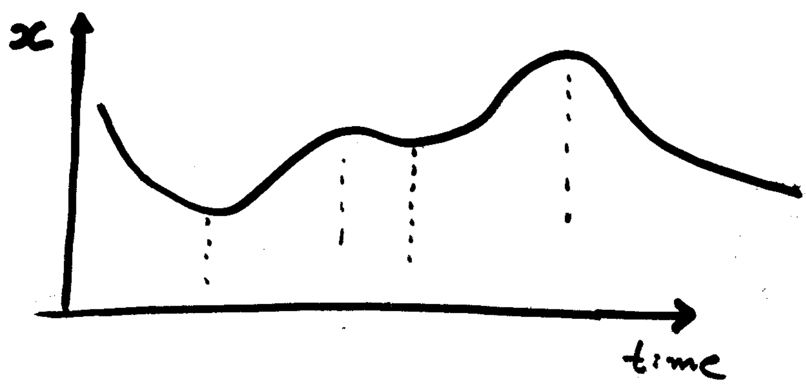
$$V^2 = V_0^2 + 2 a_0 (x - x_0)$$

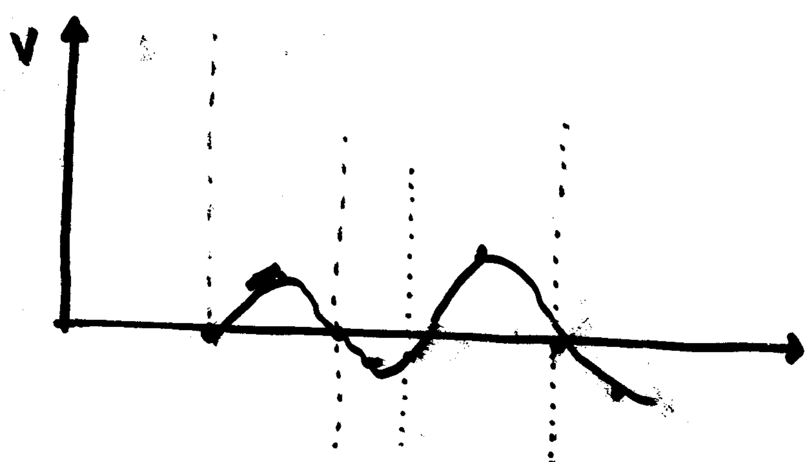


Review

$$v(t) = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \underset{\text{NOTATION}}{=} \frac{dx}{dt}$$

instantaneous velocity

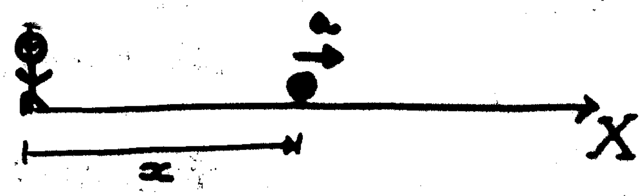


$$x = x_0 + v_0 t + \tfrac{1}{2} a t^2$$

$$v = v_0 + a t$$

$$v^2 = v_0^2 + 2a(x - x_0)$$




Motion under constant acceleration

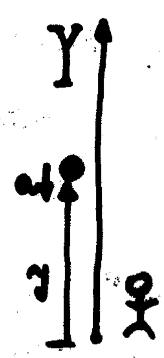
Notice:
They are essentially the same set of equations.
(the differences arise because of the particular system of reference chosen in the case below)

$$y = y_0 + v_0 t - \tfrac{1}{2} g t^2$$

$$v = v_0 - g t$$

$$v^2 = v_0^2 - 2g(y - y_0)$$

$g = 9.8\ m/s^2$




acceleration = $-g$

FREE-FALL motion

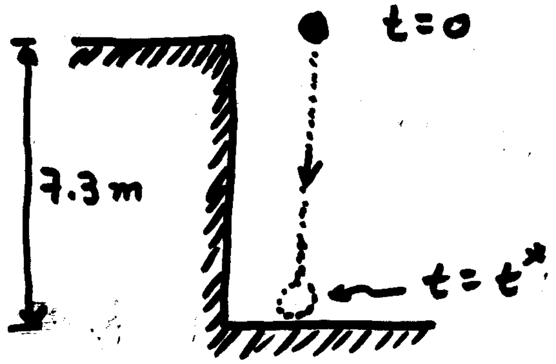
NOTE: Choose the origen of your system of coordinates ANYWHERE is more convenient for you

The result will be independent of your choice. Let's see

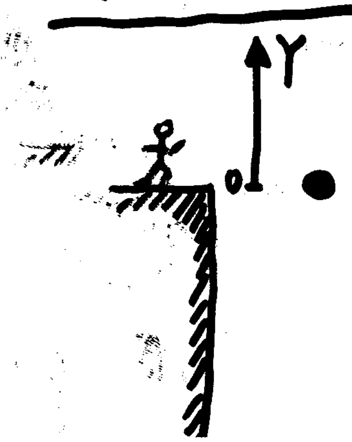
## EXAMPLE

**Fact:** A ball is dropped from the second floor of a building.

**QUESTION:** How long will it take for the ball to reach the floor?




**CHOICE #1** You decide to place the origen of your system of reference in the second floor.




At $t=0$: $y_0=0$ $v_0=0$

So, the eq. of motion have the form

$$\begin{cases} y = y_0 + v_0 t - \frac{1}{2} g t^2 = -\frac{1}{2} g t^2 \end{cases}$$

$$\begin{cases} y = -\frac{1}{2} g t^2 \\ v = -g t \end{cases}$$

when $t = t^*$,

the ball reaches the floor,

so $y = 0$ ?

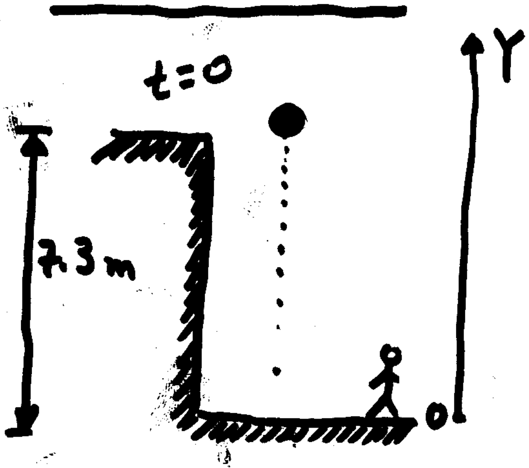
$y = -7.3 m$

$$-7.3 m = -\frac{1}{2}\left(9.8 \, m/s^2\right)\left(t^*\right)^2$$

$$\Rightarrow t^* = \frac{2 \times 9.8}{7.3} = 1.2 \text{ sec}$$

CHOICE #2

You decide to place the origen of your system of reference in the first floor.




At $t=0$: $Y_0 = 7.3 m$, $V_0 = 0$.

So, the eq. of motion are

$$\begin{cases} Y = Y_0 + V_0 t - \frac{1}{2} g t^2 = 7.3 - \frac{1}{2} g t^2 \\ V = V_0 - g t = -g t \end{cases}$$

When $t = t^*$, the ball reaches the floor, so $y = 0$ ✓

$y =$ ?

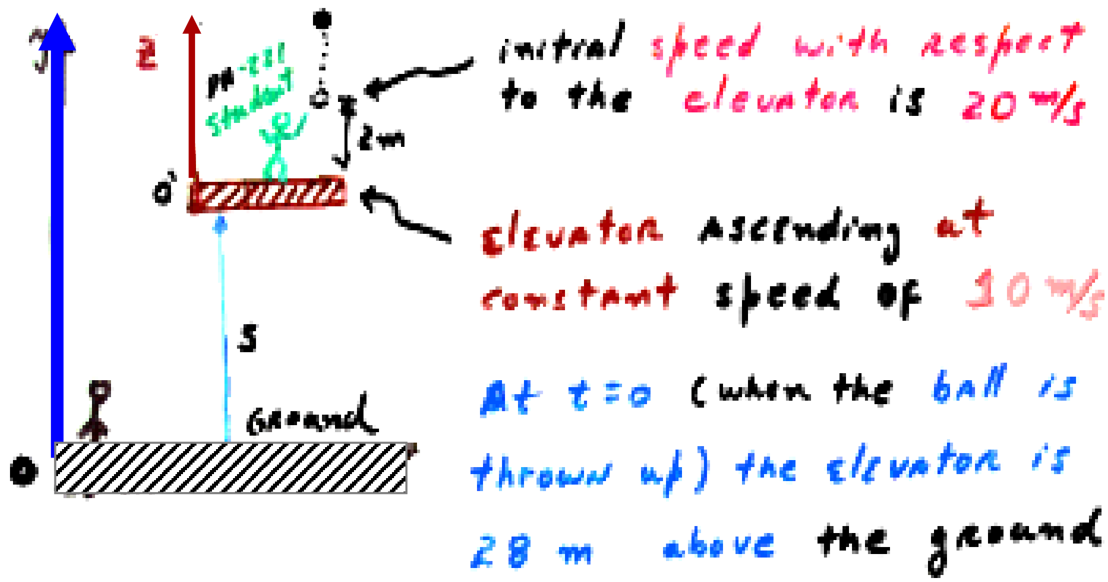
$$0 = 7.3^m - \frac{1}{2} g (t^*)^2$$

$$\Rightarrow \quad t^* = \frac{2 \times 7.3\,m}{9.8\,m/s^2} = 1.2 \text{ seconds}$$

**The following example is to illustrate how a given problem can be solved in two systems of references, still obtaining compatible results.**

## EXAMPLE

**A PH-211 student, who is at an elevator ascending at constant speed of 10 m/s (relative to the floor,) throw a ball vertically up with a initial velocity of 20 m/s (relative to the student in the elevator.) Question: What is the maximum height above the ground reached by the ball?**




- **Firs, let's figure out the equation of motion for the particle, with respect to an observer in the floor.**

$$y(t) = y_0 + v_0 t - \frac{1}{2} g t^2$$

$$\left.\begin{array}{lll} \text{At } t=0: & y_0 = 28\text{ m} & v_0 = 10\text{ m/s} \\ & \phantom{y_0} = 2\text{m} & \phantom{v_0} = 20\text{ m/s} \\ & \phantom{y_0} = 30\text{ m} & \phantom{v_0} = 30\text{ m/s} \end{array}\right\} ?$$

$$y(t) = \underline{y_0} + \underline{v_0}\, t - \frac{1}{2} g t^2 \longleftarrow \text{position the ball}$$

$$\text{At } t = 0, \quad y_0 = 28\,m + 2\,m = 30\,m$$

$$v_0 = \underbrace{10\,\frac{m}{s}}_{\text{velocity of the elevator}} + \underbrace{20\,\frac{m}{s}}_{\text{velocity of ball with respect to the elevator}}$$

$$= 30\,m/s$$

$$\left.\begin{aligned} y(t) &= 30\,m + 30\,\frac{m}{s}\, t - \frac{1}{2} g t^2 \\ v(t) &= \frac{dy}{dt} \Rightarrow v(t) = 30\,\frac{m}{s} - g t \\ v^2 &= \left(30\,\frac{m}{s}\right)^2 - 2g\cdot[y - 30\,m] \end{aligned}\right\} \text{Description of the ball's motion with respect to the ground}$$

- **When the ball reaches its maximum height, its velocity must be zero (with respect to the floor.)**

$$v^2 = (30\ m/s)^2 - 2g\,(y - 30\ m)$$

$$v^2 = 0 \qquad \text{when} \qquad y = y_{max}$$

$$(y - 30\ m) = (30\ m/s)^2 / 2g = 4.6 \times 10\ m$$

$$y = 7.6 \times 10\ m$$

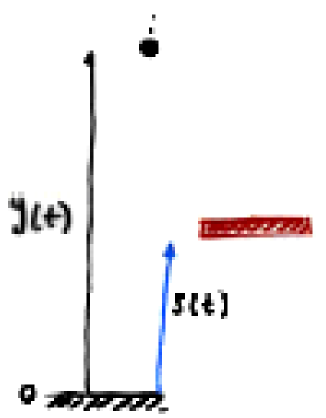
Question:

When the ball reaches Ymax (with respect to a reference to the floor) the velocity is zero (again, with respect to the floor.) But at that instant, the PH-211 student (who is riding the elevator) will state that the velocity of the ball is NOT zero. **Is the PH-211 correct?**

The answer is affirmative

**Question: How long does it take for the ball to return and hit the floor of the elevator?**
**That is, what is the total travel time of the ball since leaving the PH-211 student's hand and hitting the elevator's floor**

Method - 1




position of the ball

$$y(t) = 30 + 30t - \frac{1}{2} g t^2$$

$$s(t) = \underbrace{s_o}_{28\,m} + \underbrace{\boxed{v_o}}_{10\,m/s} t$$

$$s(t) = 28 + 10.\,t$$

position of the elevator floor

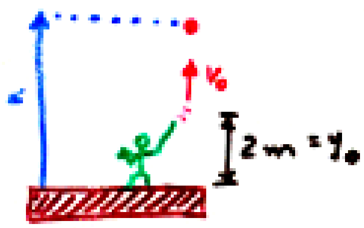
When the ball hits the elevator floor: $\Rightarrow$ $y(t) = s(t)$ $\quad t = ?$

$$30 + 30t - \frac{1}{2} g t^2 = 28 + 10t$$

$$0 = 4.9 t^2 - 20t - 2$$

## Method-2

Since the elevator is moving at constan velocity with respect to the ground floo the PH-221 student can completely ignore the ground reference and use his/her own reference that is attached to the elevator (warning: this wouldn't be true if the elevator were accelerating.)




$$z(t) = \underbrace{z_0}_{2m} + \underbrace{v_0}_{20\,m/s} t - \frac{1}{2} g t^2$$

$$z(t) = 2m + 20 \tfrac{m}{s} t - \frac{1}{2} g t^2$$

When the ball reaches the floor:

$z = 0$ and $t = ?$

$\Rightarrow \frac{1}{2} g t^2 - 20t - 2 = 0$

which gives $t = 4.2$ seconds

SAME ANSWER AS IN METHOD 1