VECTORS

FORCE

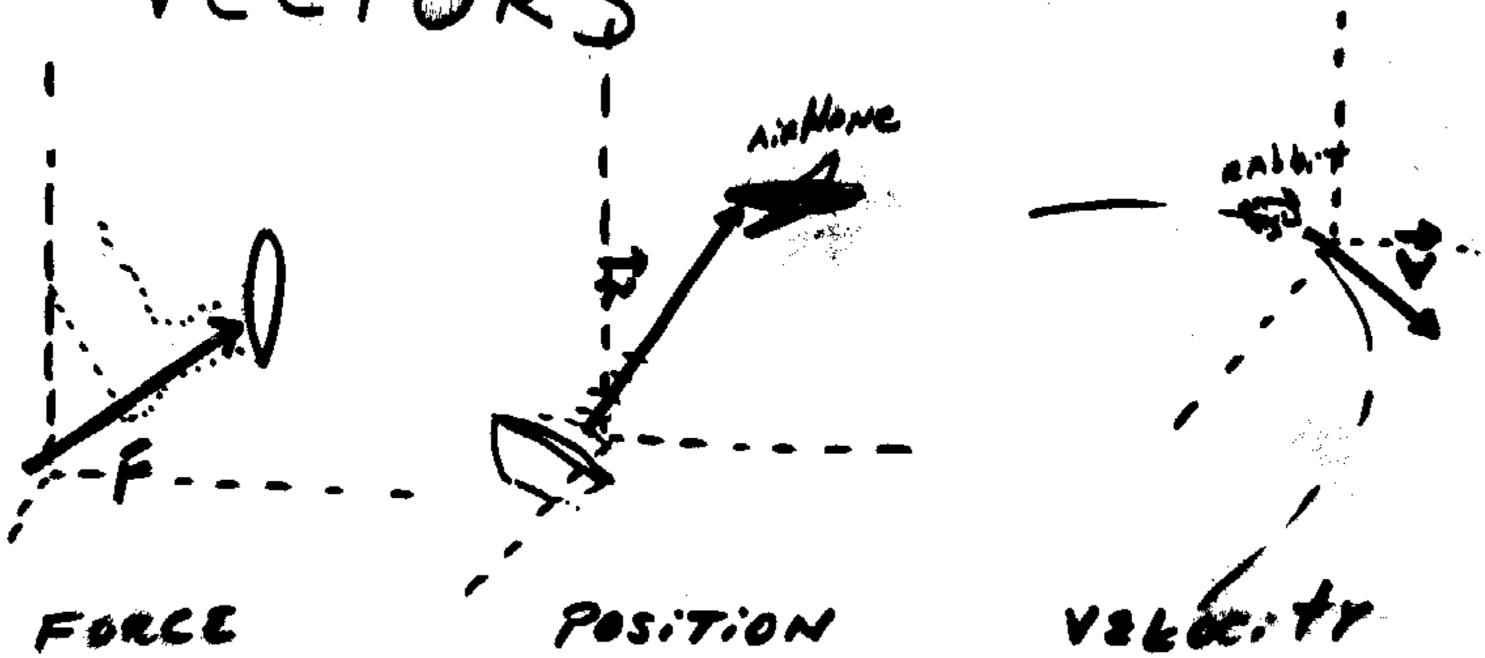
POSITION

VELOCITY

VECTOR QUANTITY { In order to be completely specified, we need to know: - MAGNITUDE
- DIRECTION

SCALAR QUANTITY { Just one number (which may be positive or negative) is enough to specify it

VECTORS



VECTOR QUANTITY: In order to be completely specified, we need to know:
- MAGNITUDE
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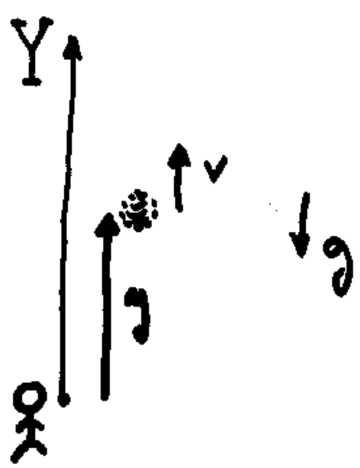
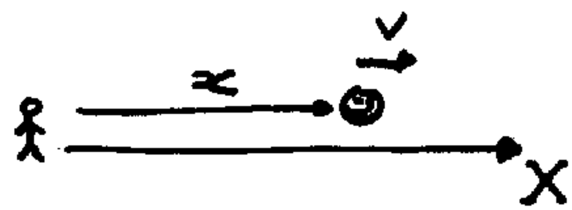
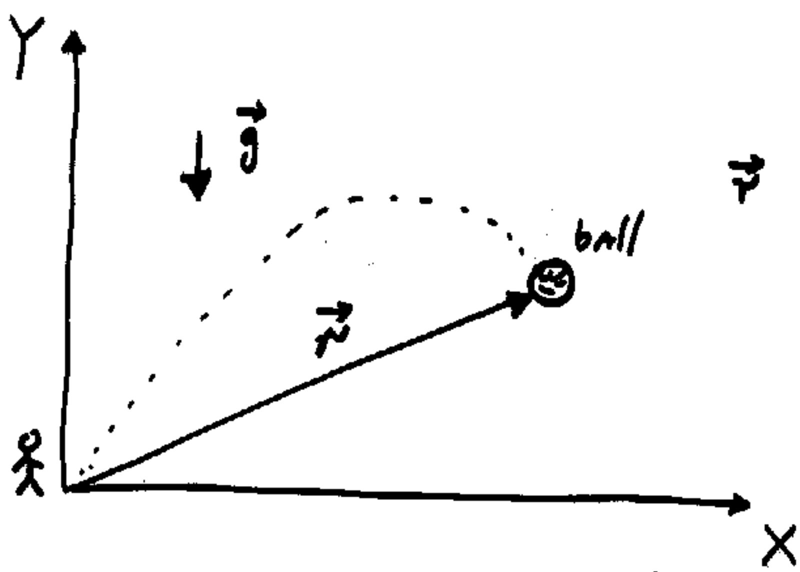
SCALAR QUANTITY: Just one number (which may be positive or negative) is enough to specify it



1D

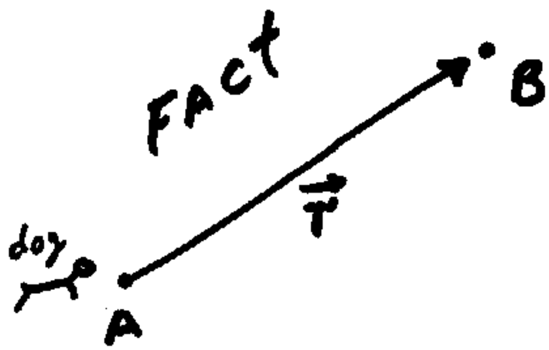


1D

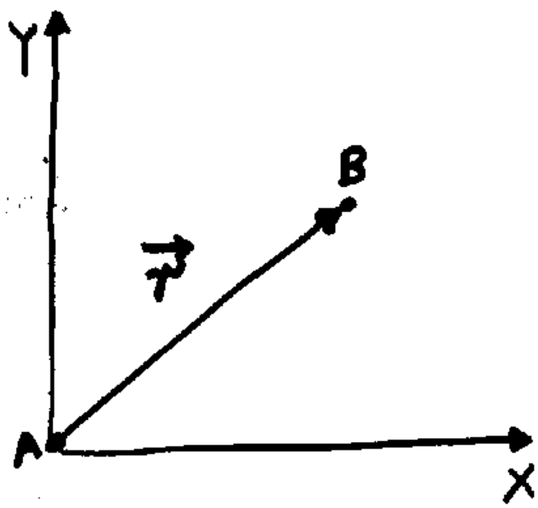


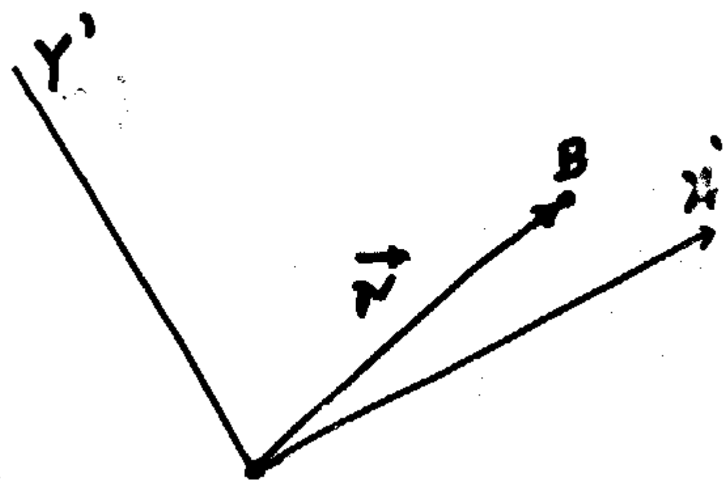
$\vec{r}$ vector position

2D

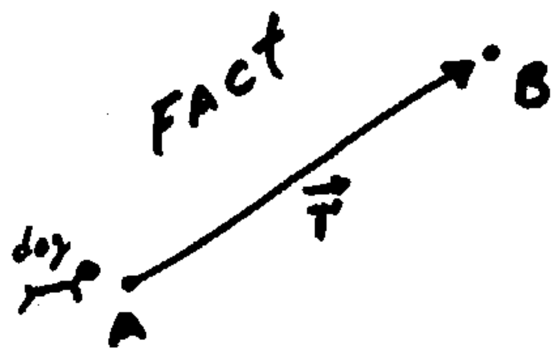


The same vector $\vec{r}$ can be represented in two different references:





VECTOR



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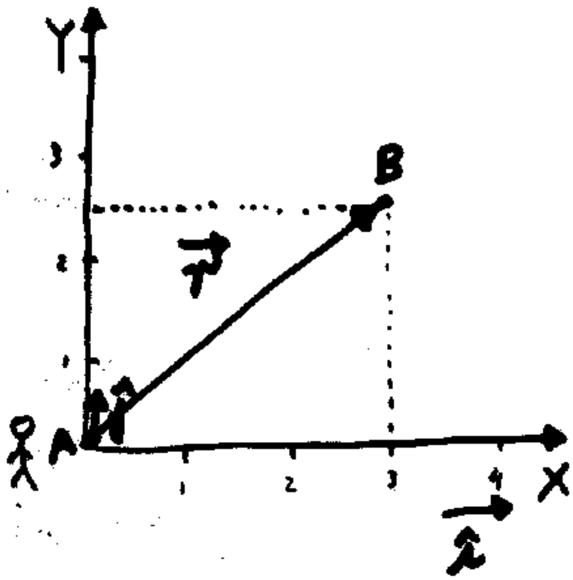
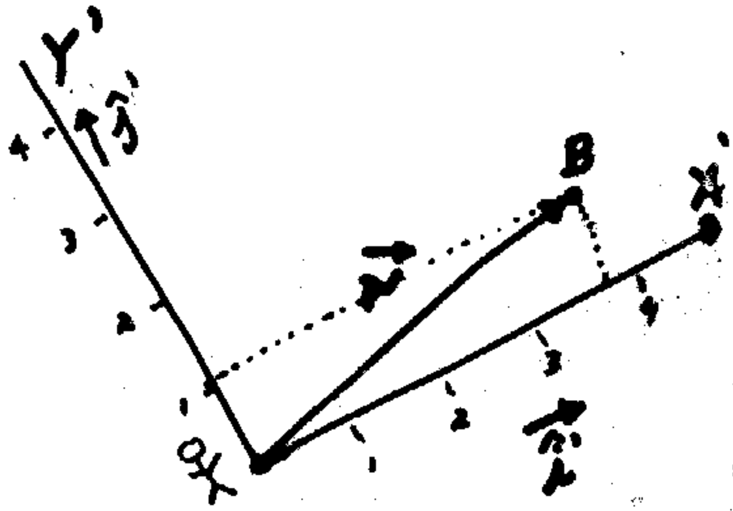




VECTOR COMPONENTS $(3\ ,\ 2.5)$

horiz comp — vert comp

$\vec{r} = 3\hat{i} + 2.5\hat{j}$

$(3.7\ ,\ 1.25)$ VECTOR COMPONENTS

horiz comp — vert comp

$\vec{r} = 3.7\hat{i}' + 1.25\hat{j}'$

VECTOR : Entity independent of the system of coordinates

VECTOR components:

they depend on the system of coordinates chosen to "measure" the VECTOR.

(3, 2.5)

according to one observer

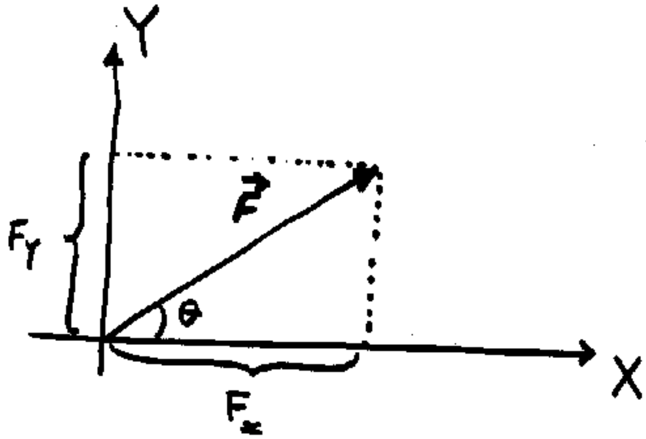
(3.7, 1.25)

according to the other observer

both are referring to the same VECTOR $\vec{r}$

FINDING VECTOR COMPONENTS

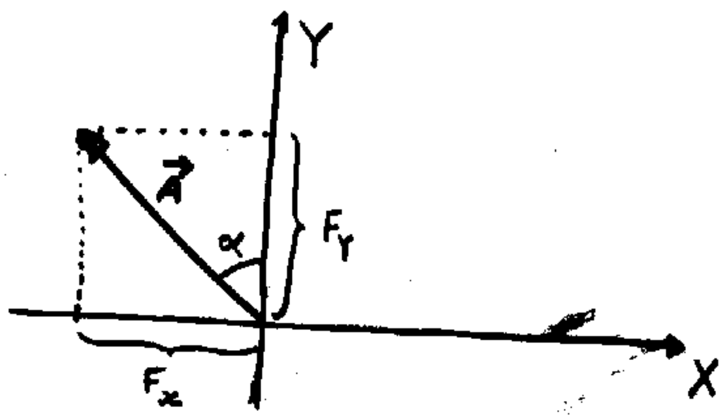
<u>Example</u>



$$F_x = F\cos\theta$$

$$F_y = F\sin\theta$$

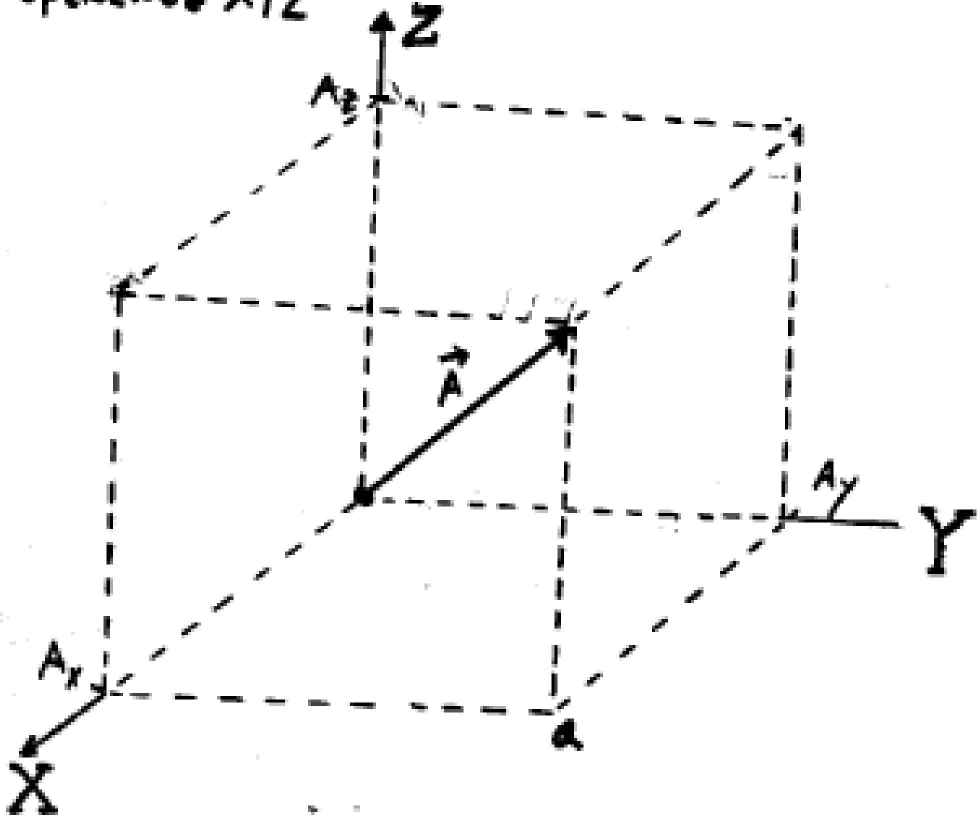
<u>Example</u>



$$F_x = -A\sin\alpha$$

$$F_y = A\cos\alpha$$

PROCEDURE to find the components of a vector $\vec{A}$ with respect to a given reference XYZ



Vector $\vec{A}$

and its components A_x

A_y

$\boldsymbol{A_z}$

Magnitude of $\vec{\boldsymbol{A}}$ = $|\vec{\boldsymbol{A}}| \equiv [\ (A_x)^2 + (A_y)^2 + (A_z)^2\ ]^{1/2}$

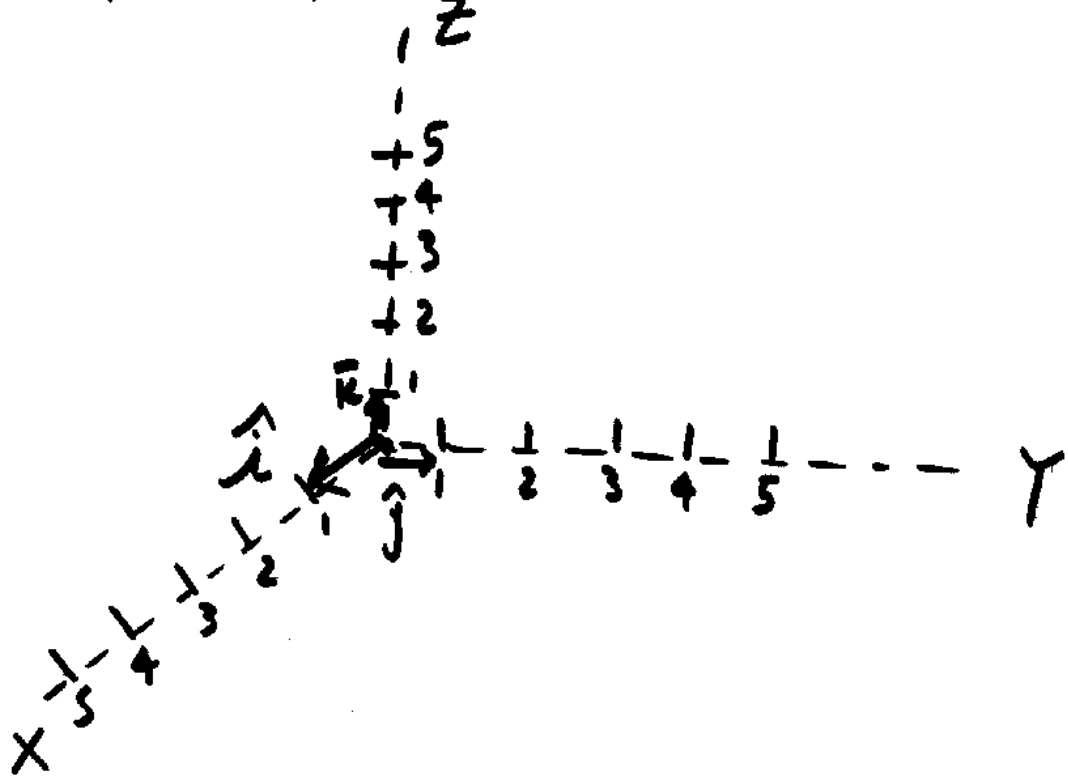
Unit vector

If $\sqrt{A_x^2 + A_y^2 + A_z^2} = 1$

then $\vec{A}$ is called unit vector

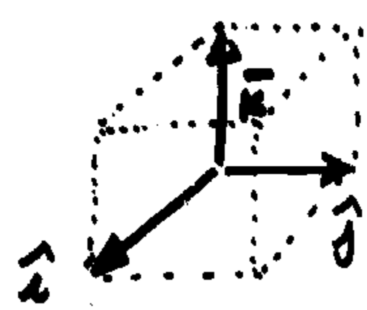
Definition

Examples of unit vector : $\hat{i}$
$\hat{j}$
$\hat{k}$



$\vec{r}$

$\hat{r}$



VECTOR ADDITION

by combining their components, axis by axis

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

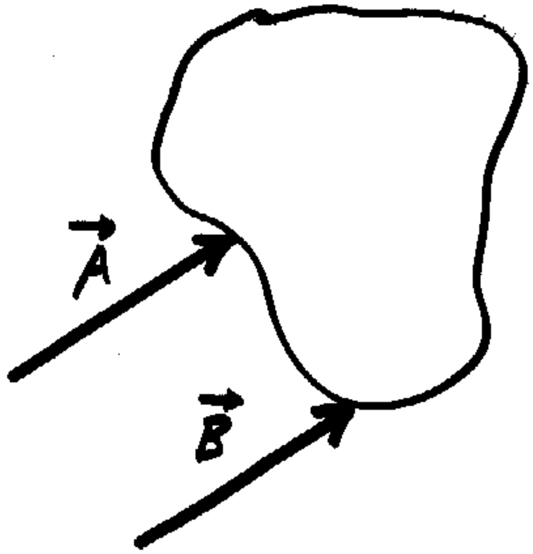
$$\left.\begin{aligned} &\boxed{\vec{A} + \vec{B} \equiv \vec{C}} \\ &\qquad = (A_x + B_x)\,\hat{i} \\ &\qquad\qquad + (A_y + B_y)\,\hat{j} \\ &\qquad\qquad + (A_z + B_z)\,\hat{k} \end{aligned}\right\}$$

This is:

$$\left.\begin{aligned} C_x &= A_x + B_x \\ C_y &= A_y + B_y \\ C_z &= A_z + B_z \end{aligned}\right\}$$

See also SAMPLE PROBLEM 3-4

EQUAL VECTORS



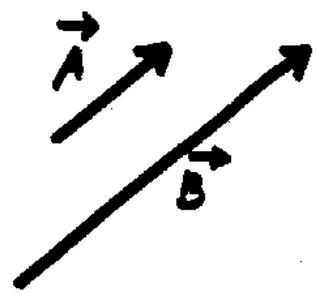
$$\vec{A} = \vec{B}$$

VECTOR VECTOR

OPPOSITE VECTORS

$$\vec{B} = -\vec{A}$$

PARALLEL VECTORS

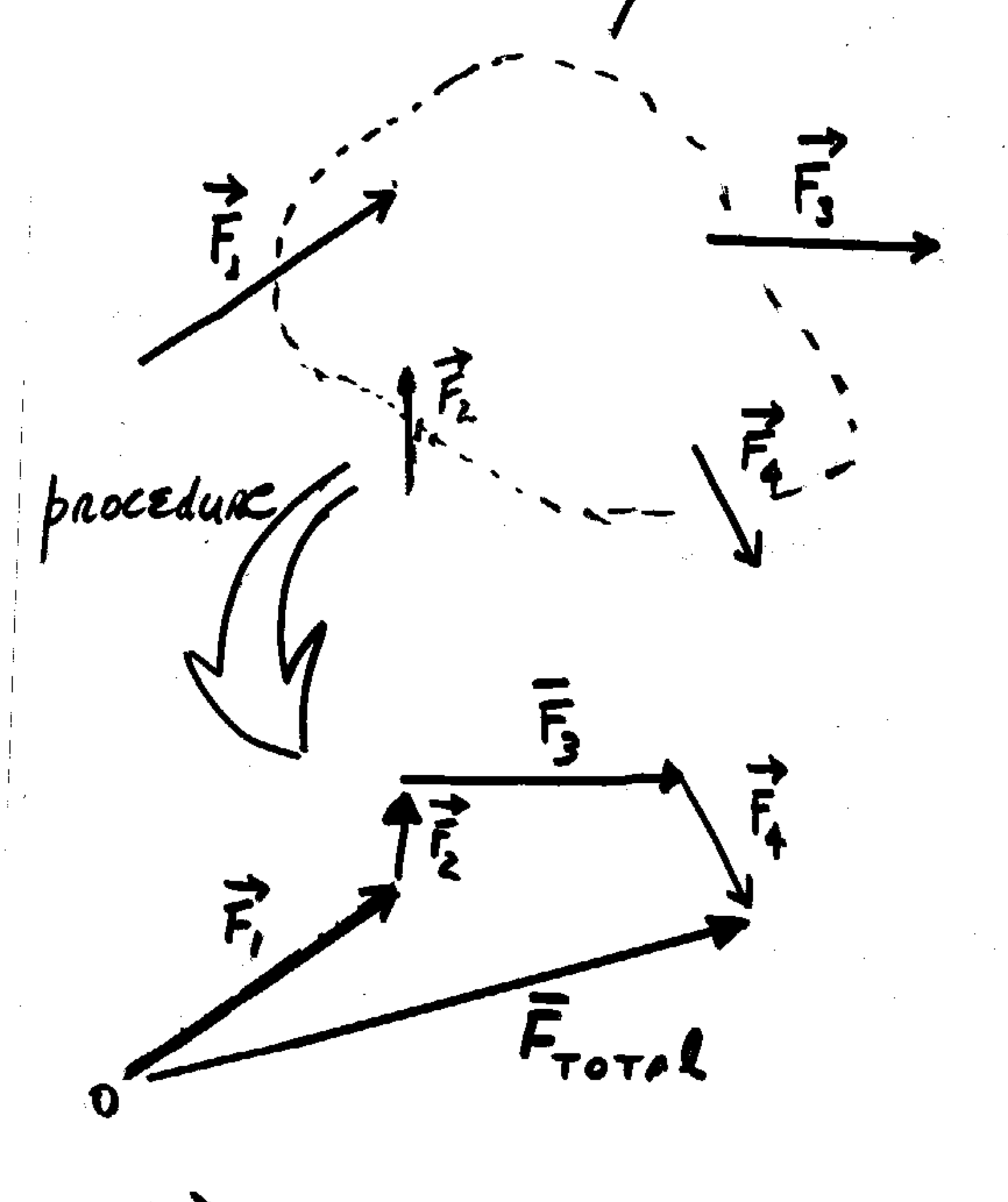


$$\vec{B} = \alpha \vec{A}$$

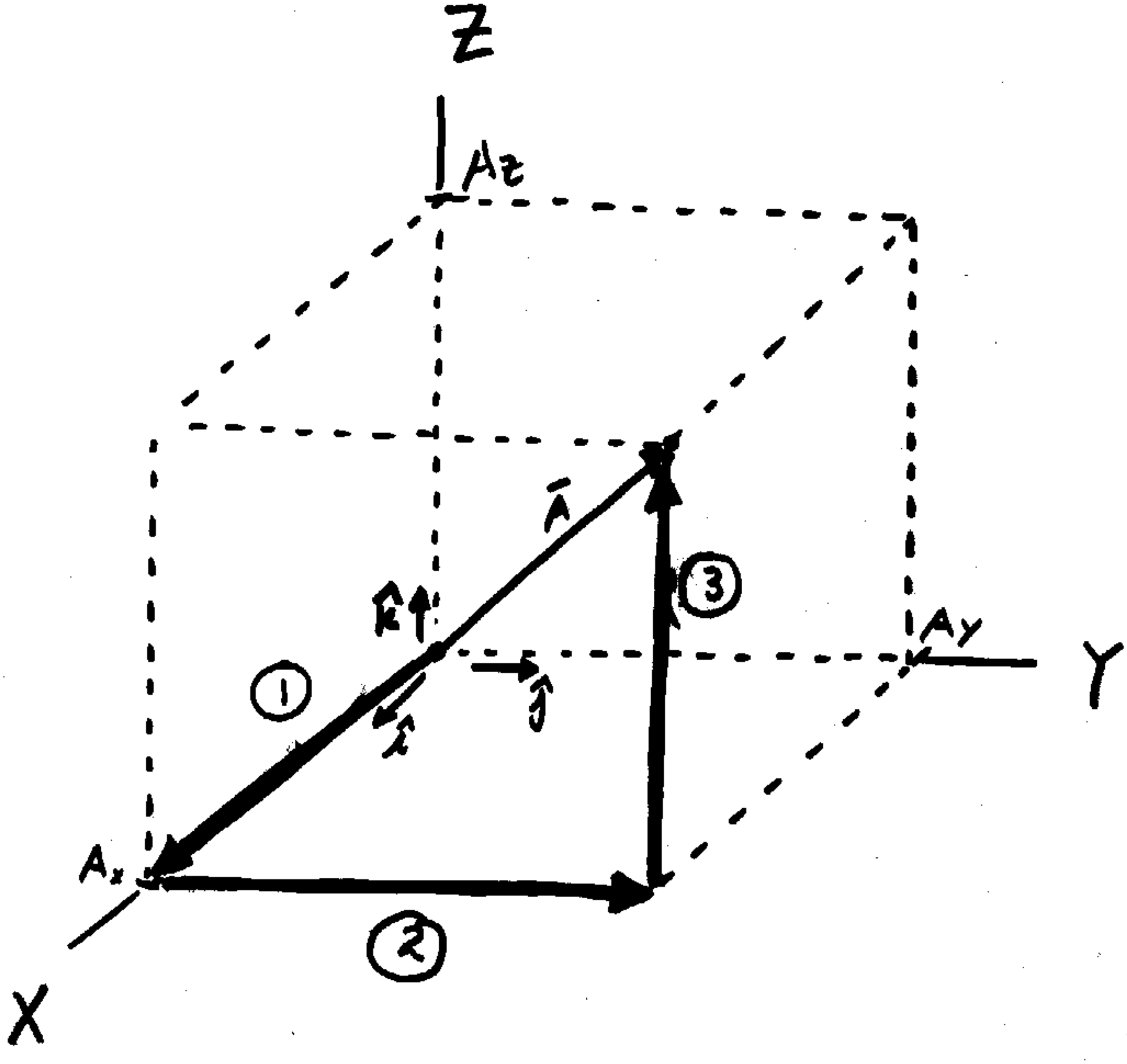
$$\vec{B} \parallel \vec{C}$$

VECTOR ADDITION

Tail-to-Tip method



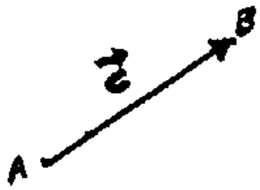
$$\vec{F}_{TOTAL} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$



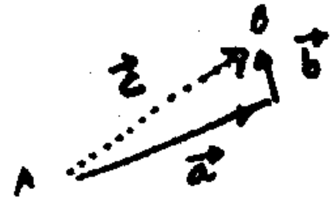
$$\vec{A} = \underbrace{A_x \hat{i}}_{①} + \underbrace{A_y \hat{j}}_{②} + \underbrace{A_z \hat{k}}_{③}$$

Vector $\vec{A}$ expressed as the sum of three vectors that are $\perp$ to each other

Example The same vector $\vec{c}$ expressed in different ways

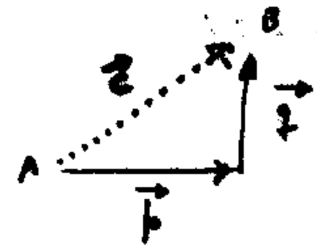


$\Rightarrow$



$\vec{c} = \vec{a} + \vec{b}$

OR



$\vec{c} = \vec{i} + \vec{j}$

OR

$\vec{c} = \vec{u} + \vec{v}$

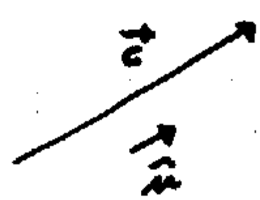
Example (important!)

Given a vector $\vec{c} = 3\hat{i} - 2\hat{j} + 4\hat{k}$
how to find a unit vector $\hat{u}$ pointing in the same direction of $\vec{c}$?

$$\hat{u} = \frac{\vec{c}}{|\vec{c}|}$$

← vector $\vec{c}$

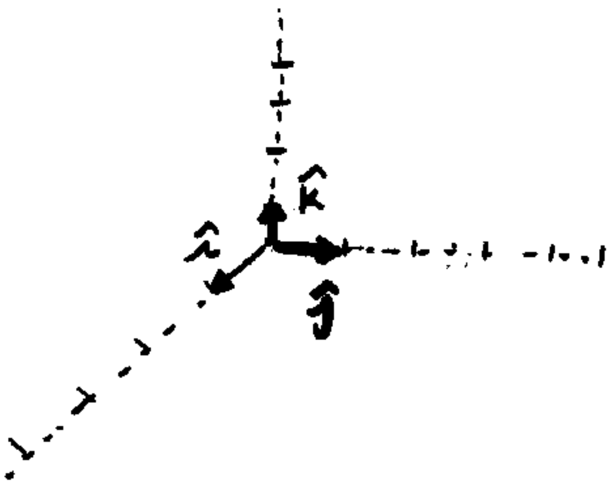
← magnitude of vector $\vec{c}$

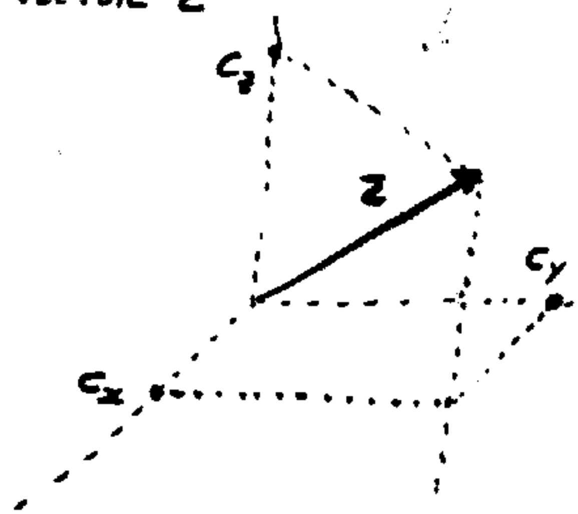


- Let's consider a vector $\vec{c}$

Once we choose a system of reference

, we start talking about the components of vector $\vec{c}$





Define your unit vectors $\hat{i}$, $\hat{j}$, $\hat{k}$

$$\vec{c} = c_x \hat{i} + c_y \hat{j} + c_z \hat{k}$$

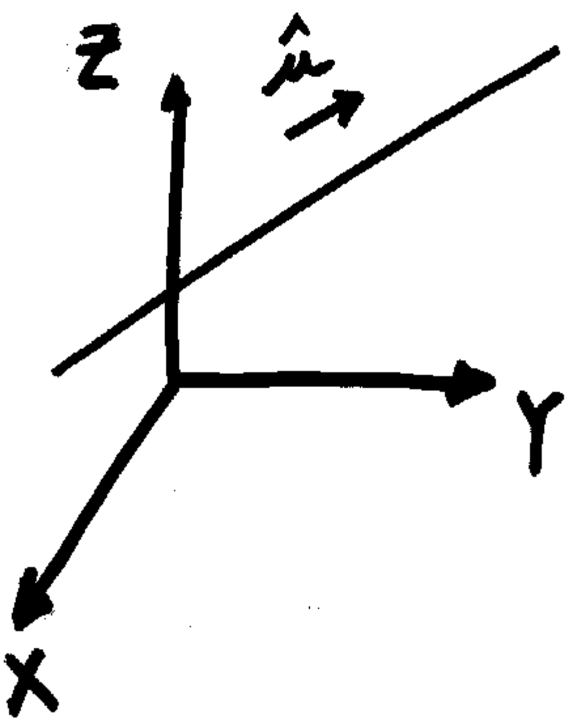
components of vector $\vec{c}$ with respect to a given system of reference.

- If $\vec{c} = 3\hat{i} - 2\hat{j} + 4\hat{k}$

  ↳ magnitude of $\vec{c}$ $\quad |\vec{c}| = \sqrt{(3)^2 + (-2)^2 + (4)^2} = 5.4$

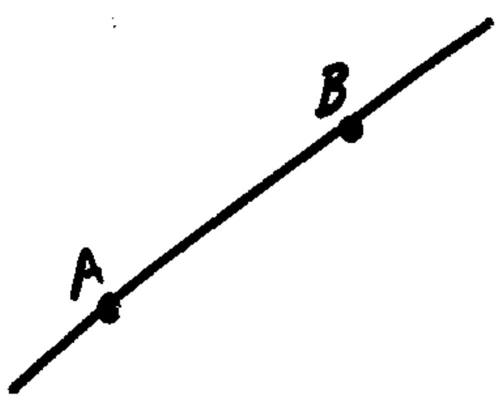
  ↳ unit vector pointing in the direction of $\vec{c}$ $\quad \hat{u} = \dfrac{\vec{c}}{|\vec{c}|} = \dfrac{3}{5.4}\hat{i} - \dfrac{2}{5.4}\hat{j} + \dfrac{4}{5.4}\hat{k}$

In general:

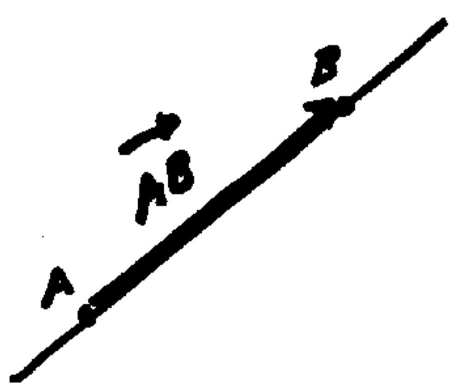


Given a straight line, how to find a unit vector $\hat{u}$ that runs parallel to that line?

PROCEDURE:



a) Pick up any two points A and B along the line.

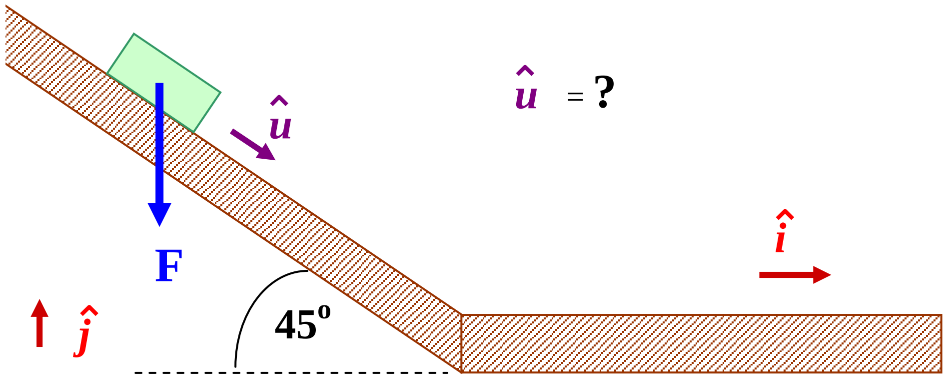


b) Find the vector $\overrightarrow{AB}$

c) $\hat{u}$ will be given by

$$\hat{u} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|}$$

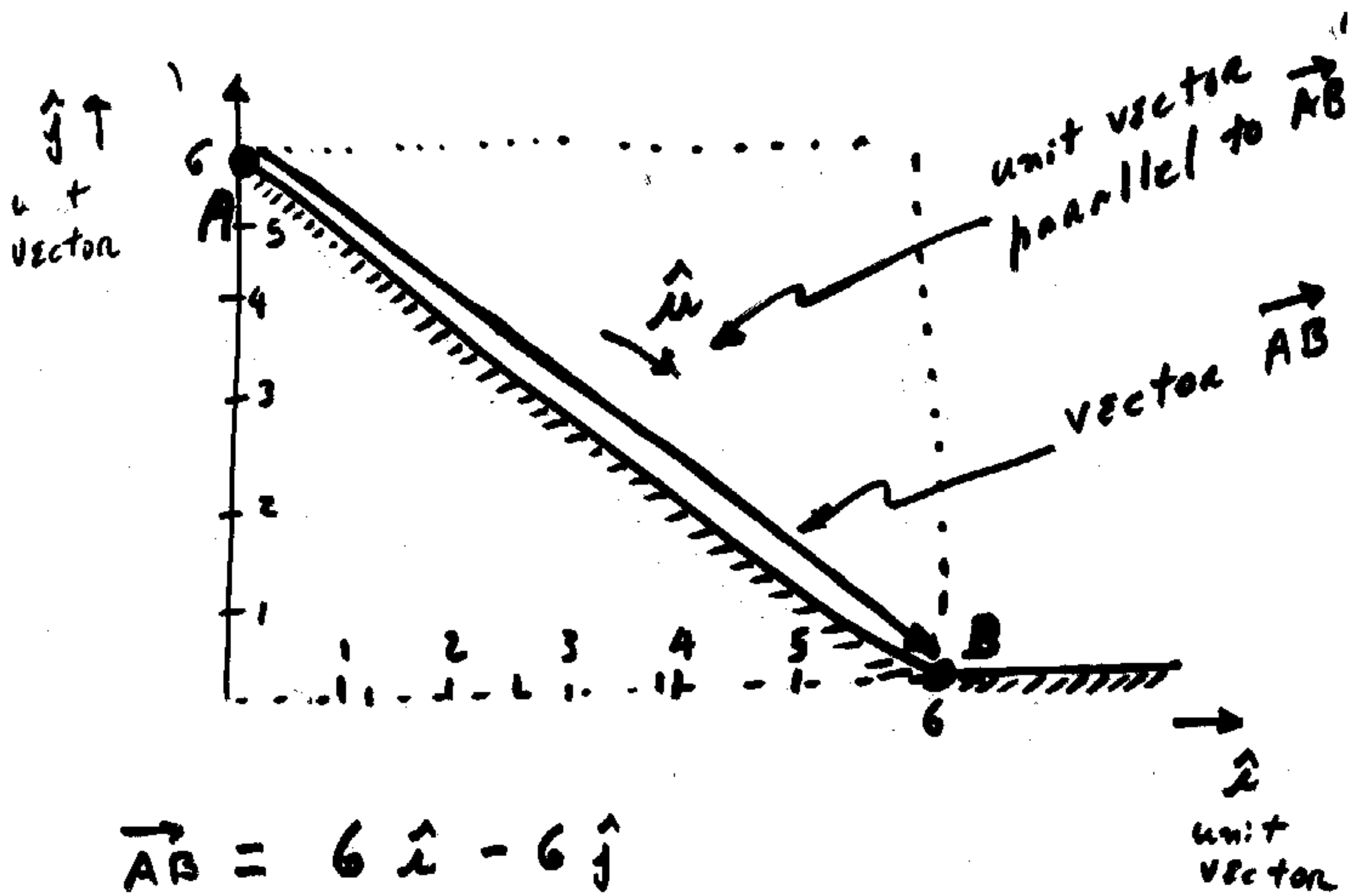
EXAMPLE



$\mathbf{F = 20\ \underbrace{Kg\text{-}f}_{\text{Unit of force}}}$

Express the vector $\hat{F}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$

Find the unit vector whose direction is parallel to the ramp.



$$\vec{AB} = 6\hat{i} - 6\hat{j}$$

$$|\vec{AB}| = \sqrt{6^2 + (-6)^2} = 6\sqrt{2}$$

$$\hat{u} = \frac{\vec{AB}}{|\vec{AB}|} = \frac{6\hat{i} - 6\hat{j}}{6\sqrt{2}}$$

$$= \frac{1}{\sqrt{2}}\hat{i} - \frac{1}{\sqrt{2}}\hat{j}$$

VECTORS MULTIPLICATION

① SCALAR PRODUCT $\vec{a} \bullet \vec{b}$

② VECTOR PRODUCT $\vec{a} \times \vec{b}$

NOTATION

SCALAR PRODUCT

Let $\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$

$\vec{b} = b_x \hat{i} + b_y \hat{j} + b_z \hat{k}$

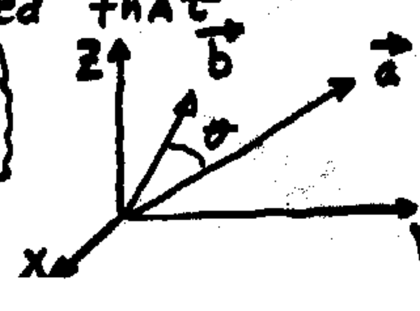
Definition: The scalar product between $\vec{a}$ and $\vec{b}$ is given by:

$$\boxed{\vec{a} \bullet \vec{b} \equiv a_x b_x + a_y b_y + a_z b_z}$$

Notice: $\vec{a} \bullet \vec{b}$ is a number

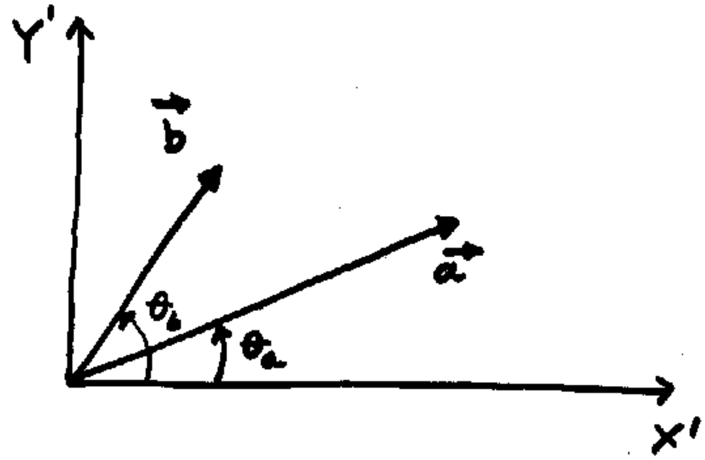
Additionally, it can be demonstrated that

$$\boxed{\vec{a} \bullet \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta}$$



PROOF:

Since any two vectors $\vec{a}$ and $\vec{b}$ define a plane, let's choose a reference whose axis X' and Y' are contained in that plane



$$\vec{a} = a_x \hat{i}' + a_y \hat{j}'$$

$$= \underbrace{a \cos\theta_a}\, \hat{i}' + \underbrace{a \sin\theta_a}\, \hat{j}'$$

$$\vec{b} = b_x \hat{i}' + b_y \hat{j}'$$

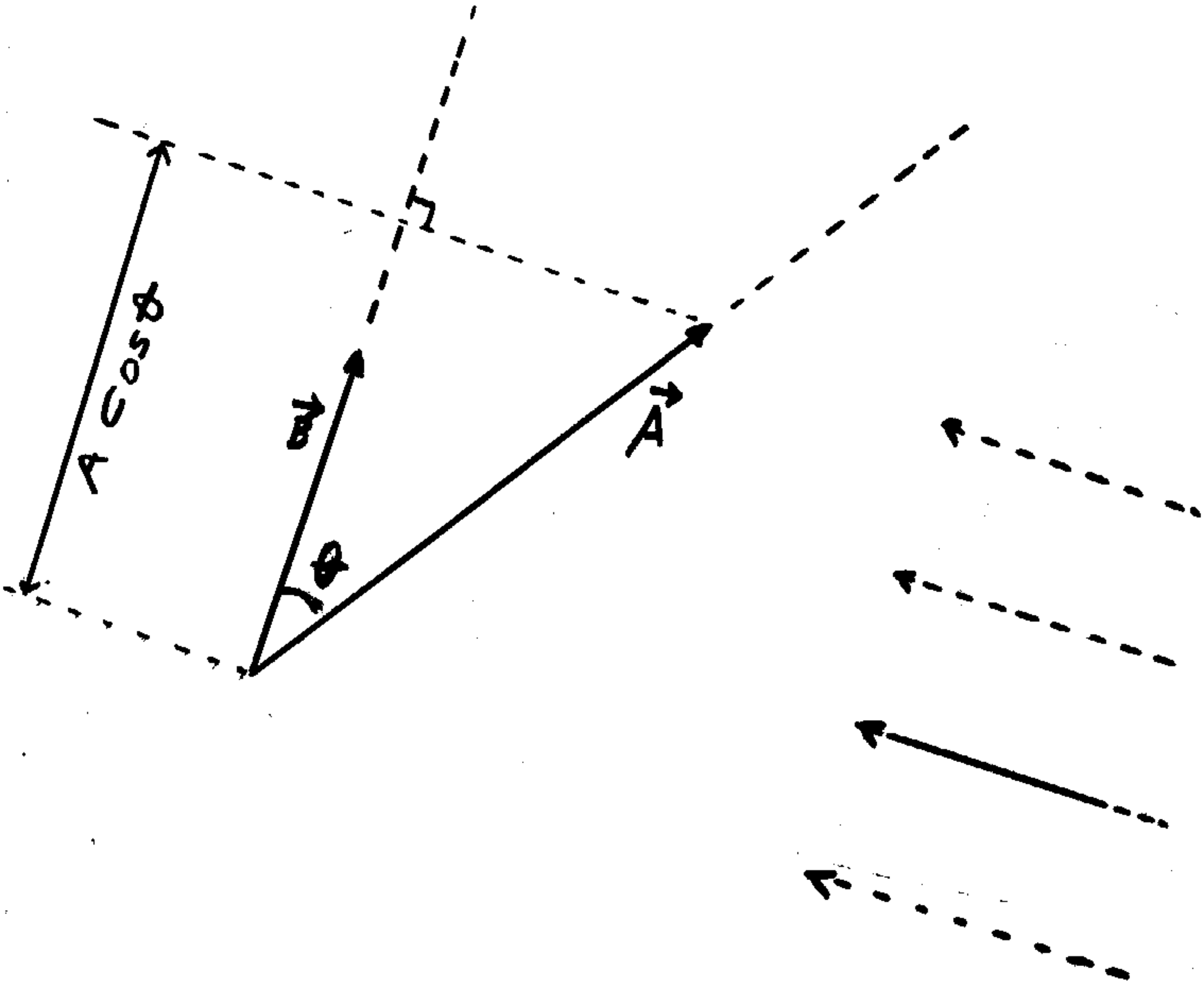
$$= \underbrace{b \cos\theta_b}\, \hat{i}' + \underbrace{b \sin\theta_b}\, \hat{j}'$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$$

$$= a b \cos\theta_a \cos\theta_b + a b \sin\theta_a \sin\theta_b$$

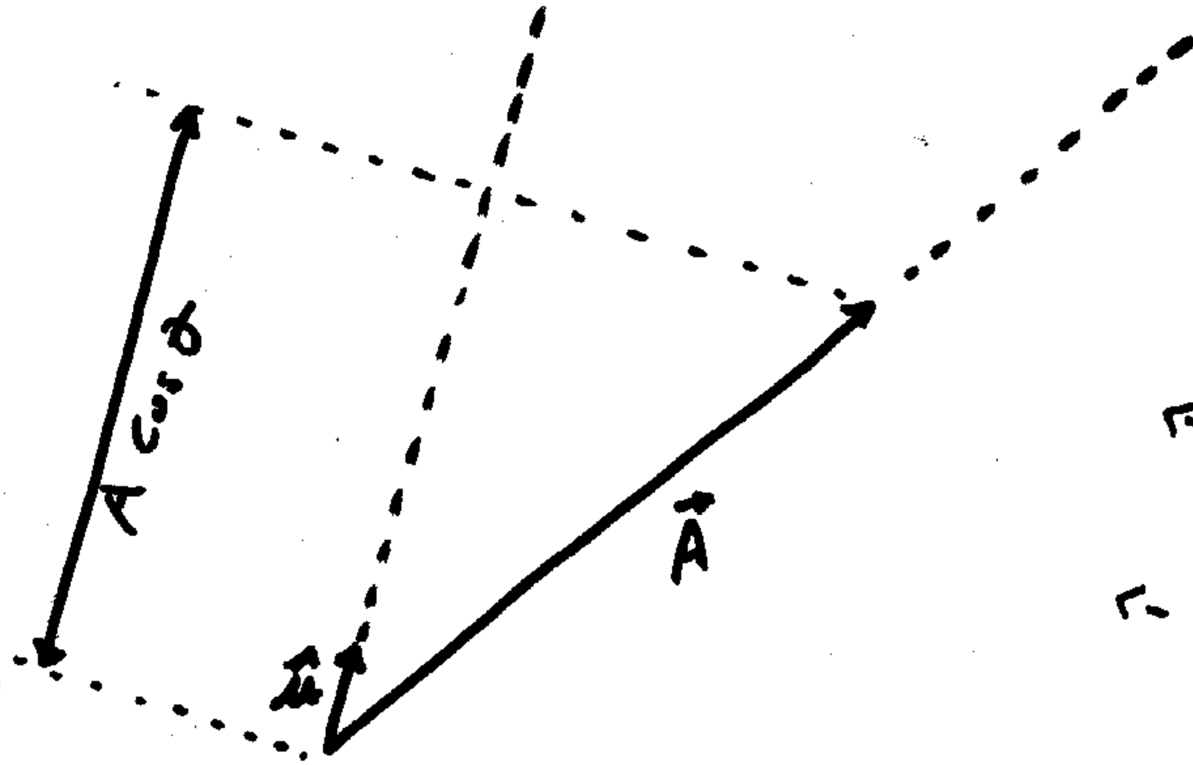
$$= a b \cos(\theta_b - \theta_a)$$

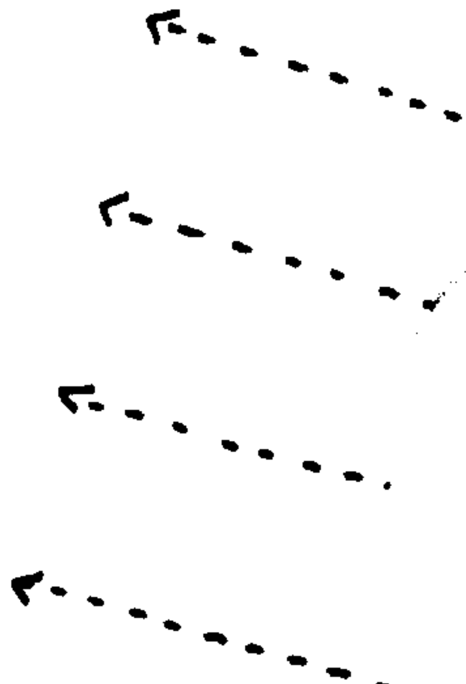
Geometrical interpretation



$$\underbrace{\vec{A} \cdot \vec{B}}_{\text{scalar product}} = BA\cos\theta$$

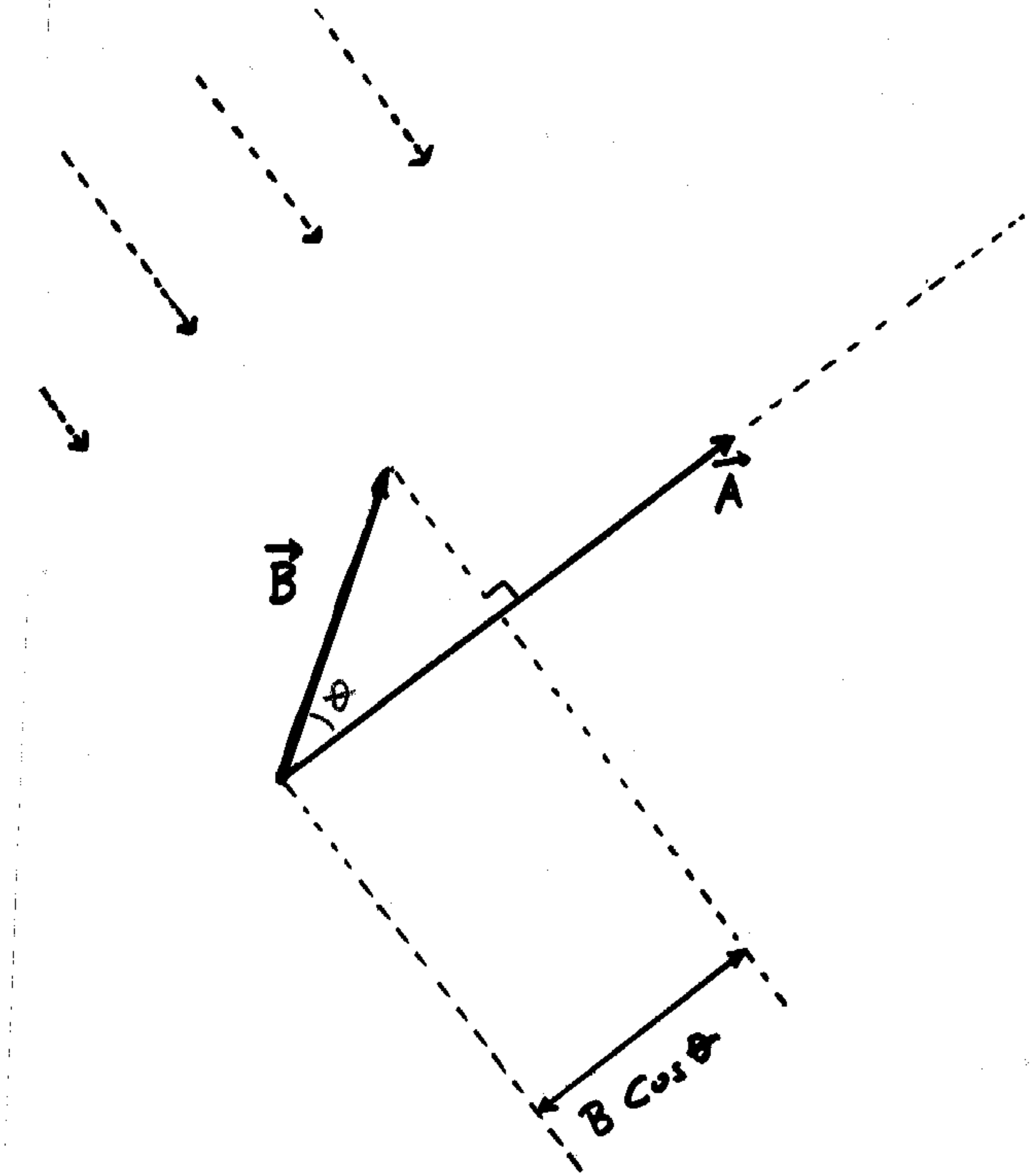
If $\vec{B}$ happened to be a unit vector we would have:





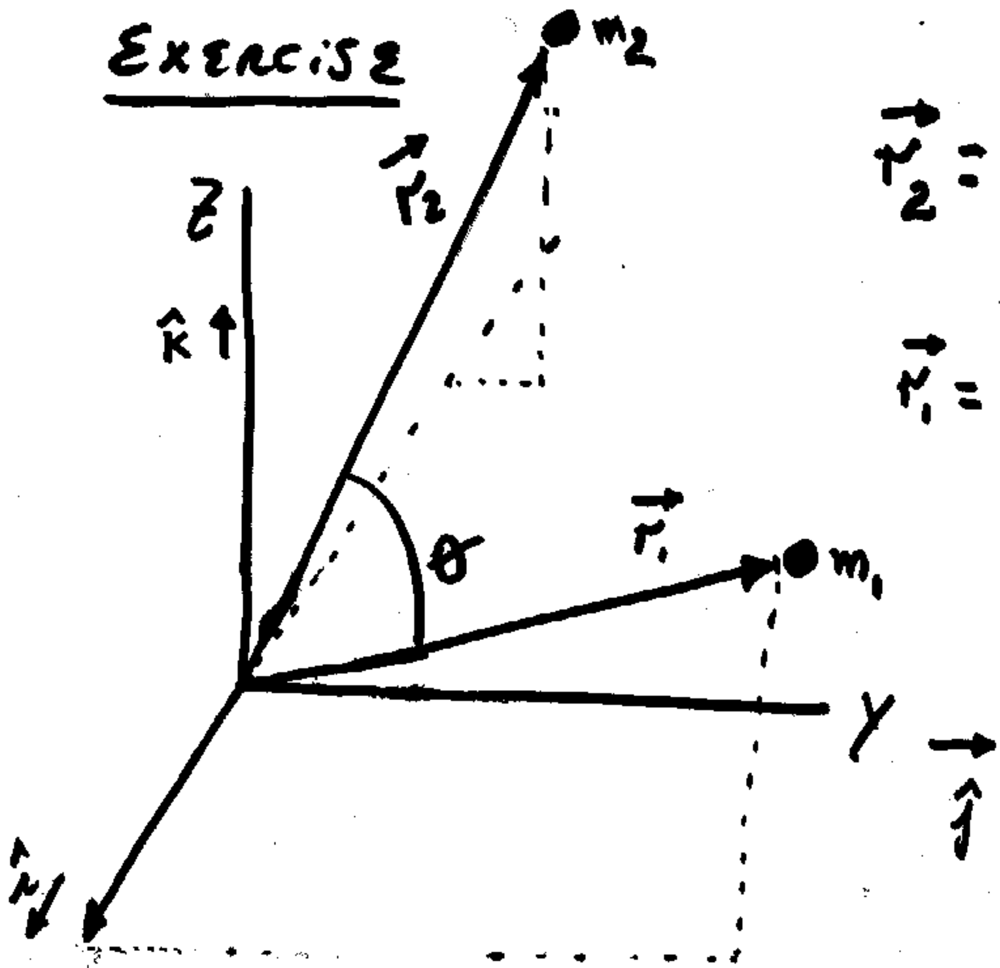
$$\vec{A} \cdot \hat{u} = A \times 1 \times \cos\theta$$
$$= A\cos\theta$$

So, $\vec{A} \cdot \hat{u}$ = component of vector $\vec{A}$ along the direction defined by the unit vector $\hat{u}$



$$\vec{A} \bullet \vec{B} = A B \cos\theta$$

EXERCISE

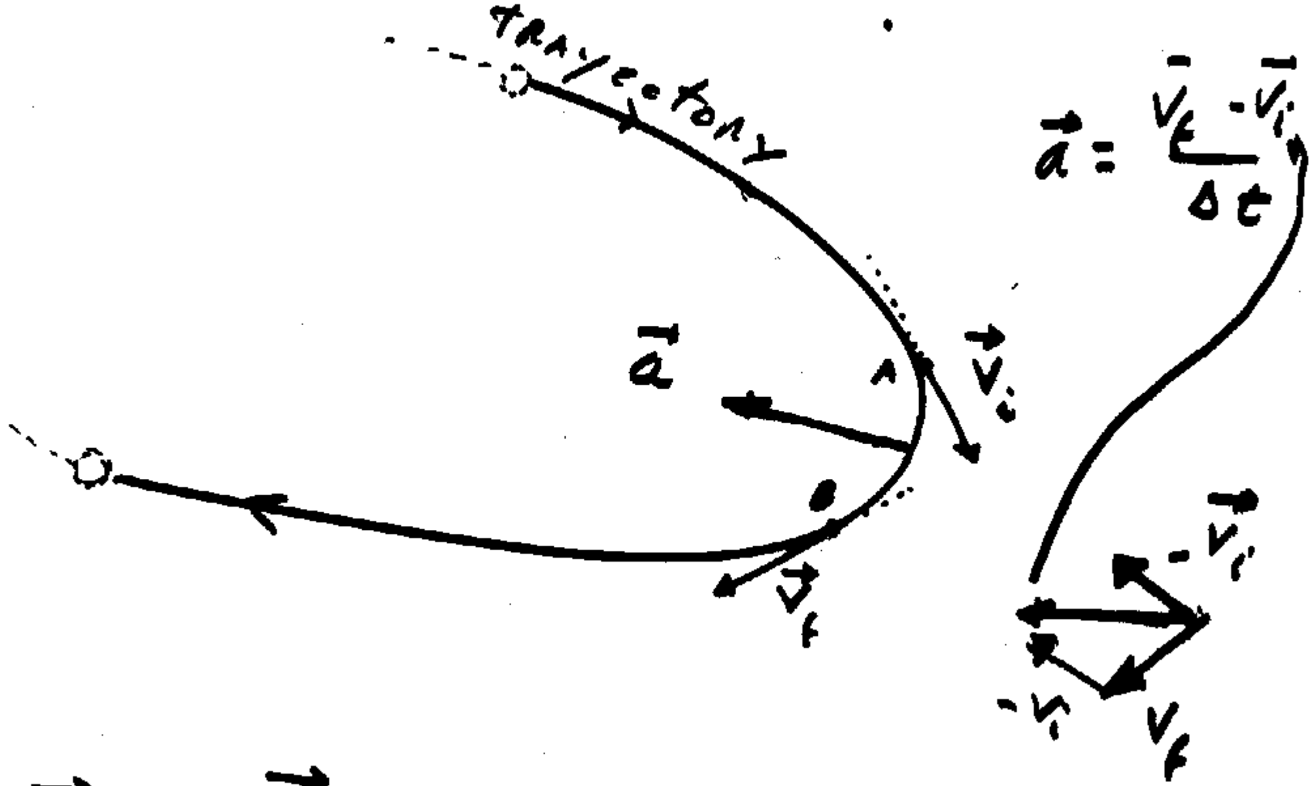


$$\vec{r}_2 = -4\hat{i} + 1\hat{j} + 5\hat{k}$$

$$\vec{r}_1 = 4\hat{i} + 5\hat{j} + 2\hat{k}$$

- Find the distance between the particles m_1 and m_2
- Find the angle between the vectors $\vec{r}_1$ and $\vec{r}_2$

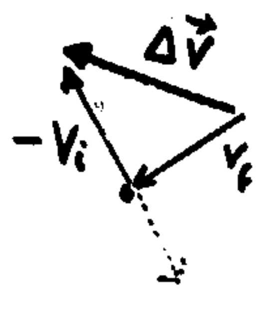
EXAMPLE: What is the <u>acceleration</u> of the particle as it moves from A to B?



$$\vec{V}_f + \vec{V}_i$$

$\leftarrow \vec{V}_f + \vec{V}_i$

$$\Delta\vec{V} \equiv \vec{V}_f - \vec{V}_i =$$



$$a = \frac{\Delta \vec{V}}{\Delta t} = \frac{\vec{V}_f - \vec{V}_i}{\Delta t}$$

$$= \vec{V}_f + (-\vec{V}_i) =$$

(picture in the fig. above the orientation of the acceleration vector $\frac{\Delta \vec{V}}{\Delta t}$