# FORCE and POTENTIAL ENERGY

- Conservative forces:

    1) gravitational force $\quad U = mgz$

    2) spring force $\quad U = \frac{1}{2}kx^2$

    ⋮

    In general:

    $F$ force $\qquad U = U(x)$ potential energy

- Relationship between $F$ and $U$:

$$F = -\frac{dU}{dx}$$

- Conservation of the mechanical energy

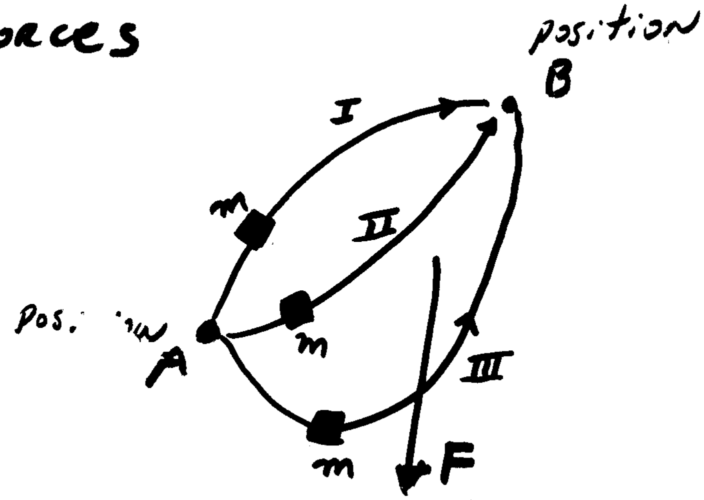
$$E = \frac{1}{2}mv^2 + U$$

| FORCE | POTENTIAL ENERGY |
| --- | --- |
| $\vec{F}$ | $U$ |
| (VECTOR) | (scalar) |
| motion description $\Downarrow$ | $\Downarrow$ |
| $\vec{F} = m\vec{a}$ | $E = K + U$ |

$$F = -\frac{dU}{dx}$$

# REVIEW

## Conservative forces

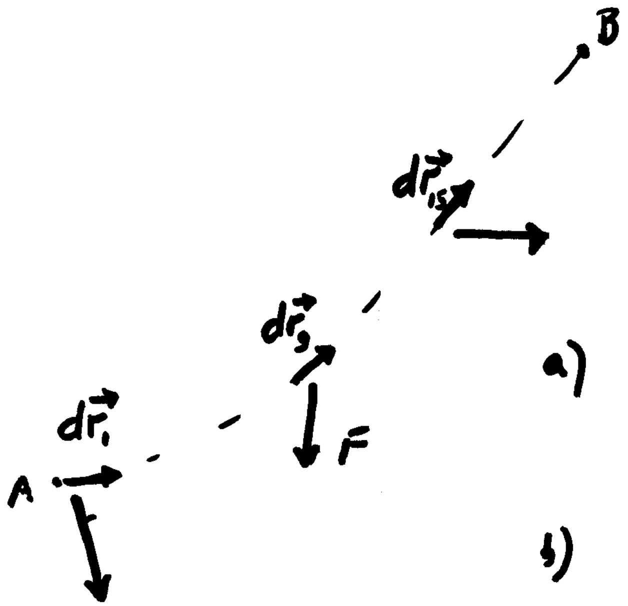



Work done by a conservative force $\vec{F}$ does not depend on the particular path followed by the block of mass m

$$W_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = \#$$

number

This number is independent of the path connecting A and B




a) $\vec{F}_n \cdot d\vec{r}_n$

b) $\sum_n \vec{F}_n \cdot d\vec{r}_n$

c) $\int \vec{F} \cdot d\vec{r}$

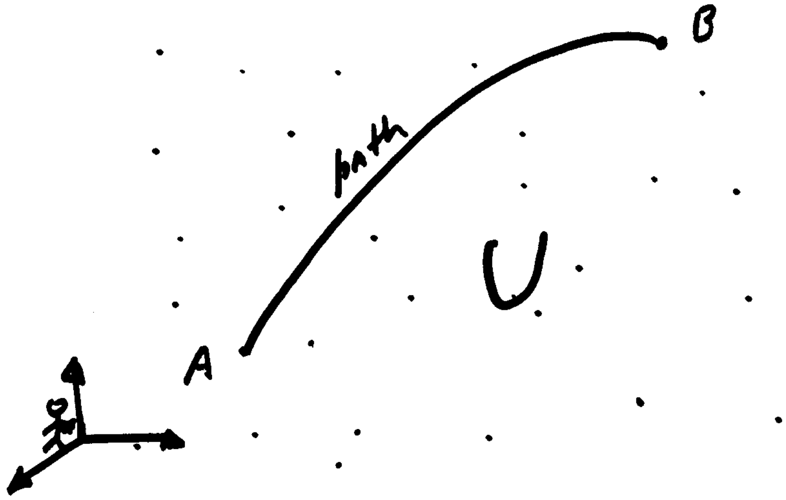
d) $\int$ = anti- derivative

integral

$$\left( \text{Example} \int x = \frac{x^2}{2} \right)$$

For a conservative force $\vec{F}$

there exists a corresponding potential energy $U$ such that




$$\underbrace{\tfrac{1}{2} m v_A^2 + U_A}_{\text{mechanical energy at position A}} = \underbrace{\tfrac{1}{2} m v_B^2 + U_B}_{\text{mechanical energy at position B}}$$

Remember:

Conservative force $\vec{F}$ $\xleftrightarrow[\text{associated with}]{}$ Potential energy $U$

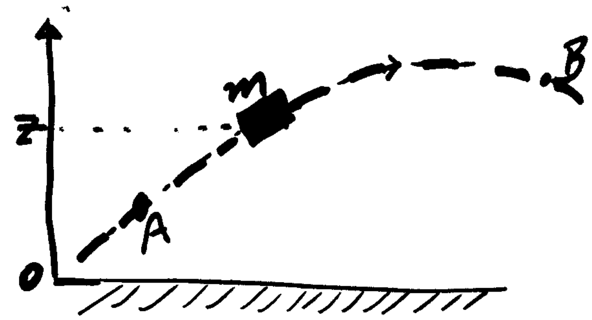
A by product of this is:

Conservation of the <u>MECHANICAL ENERGY</u>

$$\tfrac{1}{2} m v^2 + U = E \longleftarrow \text{is a constant}$$

Examples of conservative forces

1) Gravitational force



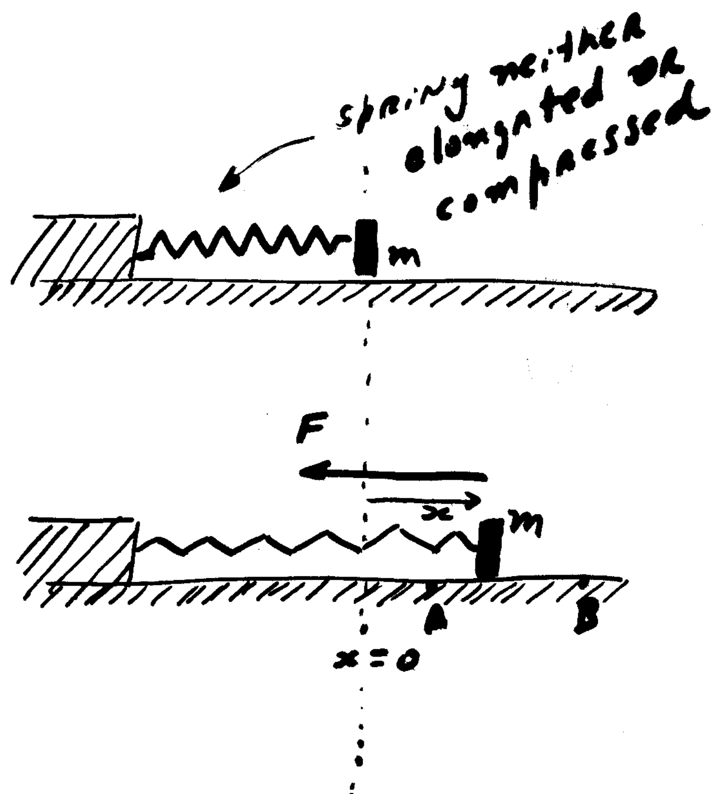

$$\vec{F} = -mg\,\hat{k}$$

$$U = mgz$$

At any 2 points A and B along the trajectory:

$$\frac{1}{2}mv_A^2 + mgz_A = \frac{1}{2}mv_B^2 + mgz_B$$

2) Spring force




$$F = -kx$$

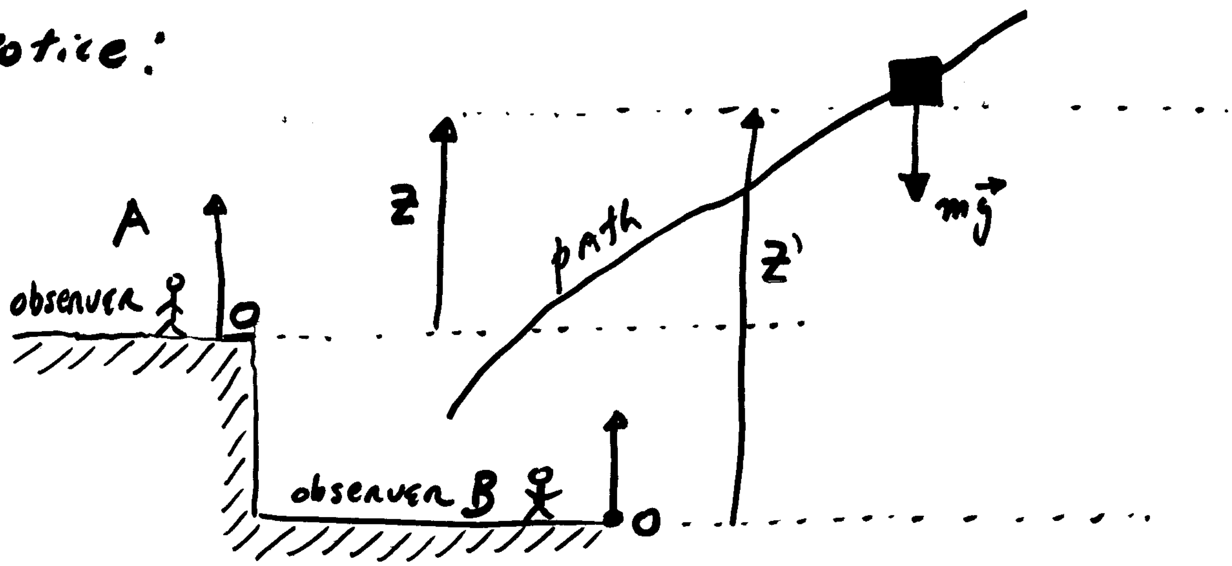
$$U = \frac{1}{2}kx^2$$

$k \equiv$ spring constant

At any 2 points along the trajectory followed by the mass m we'll have:

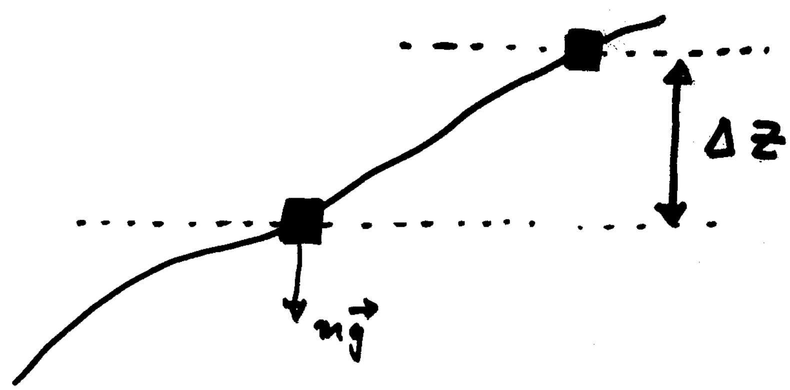
$$\frac{1}{2}mv_A^2 + \frac{1}{2}kx_A^2 = \frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2$$

Notice:




Notice Observer "A" measures a different potential energy than observer B

However

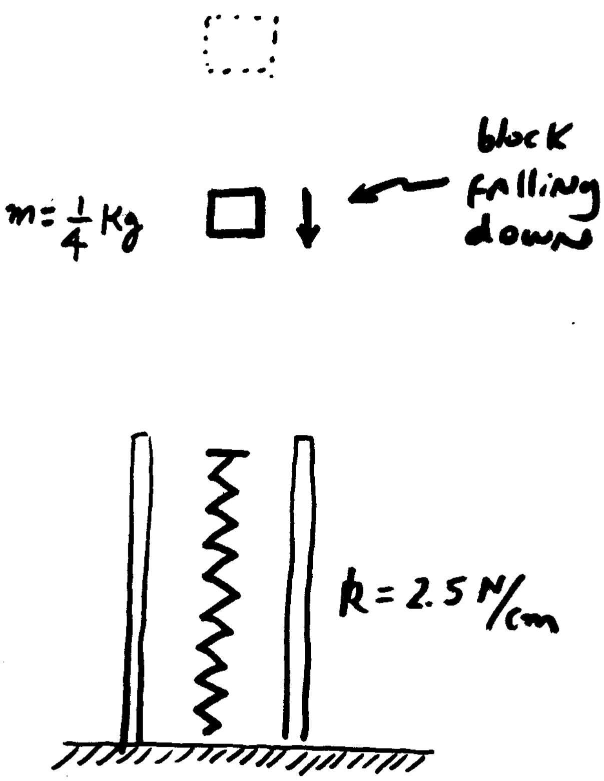



Both observers will measure the same change in potential energy

So, you can choose the <u>origen</u> of your system of reference at a point where you judge is more convenient

Examples of conservative forces

3) We can have the case where a block of mass "m" is under the influence of the gravitational force and the spring force

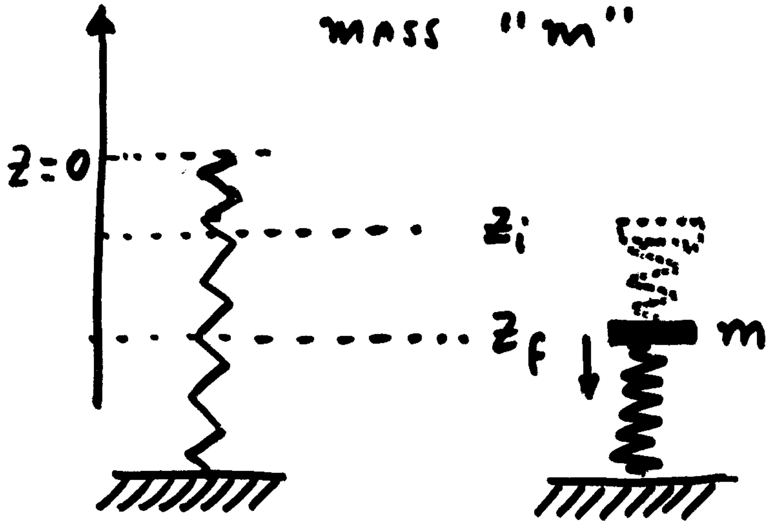


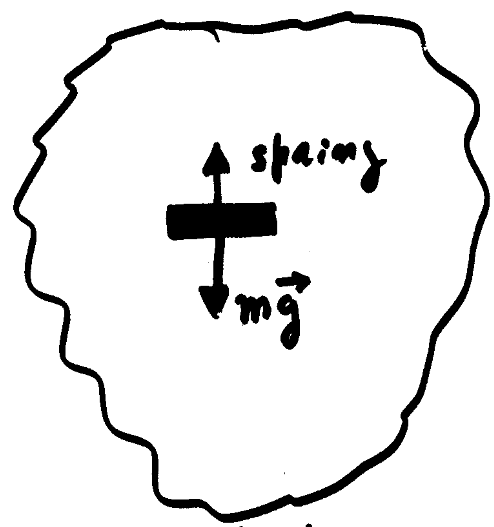

In this region, only the gravitational force acts on the block

In this region both the gravitational and the spring forces will act on the block

Practice problem: In the illustration above, the block falls down, then impacts with the spring, and compresses it 12 cm. What was the speed of the block just a the moment in impacted the spring?

**CASE** Gravitational force and spring force act simultaneously on a block of mass "m"




free-body diagram

$$K_f - K_i = W_{TOTAL}$$

$$= W_{spring} + W_{gravit}$$

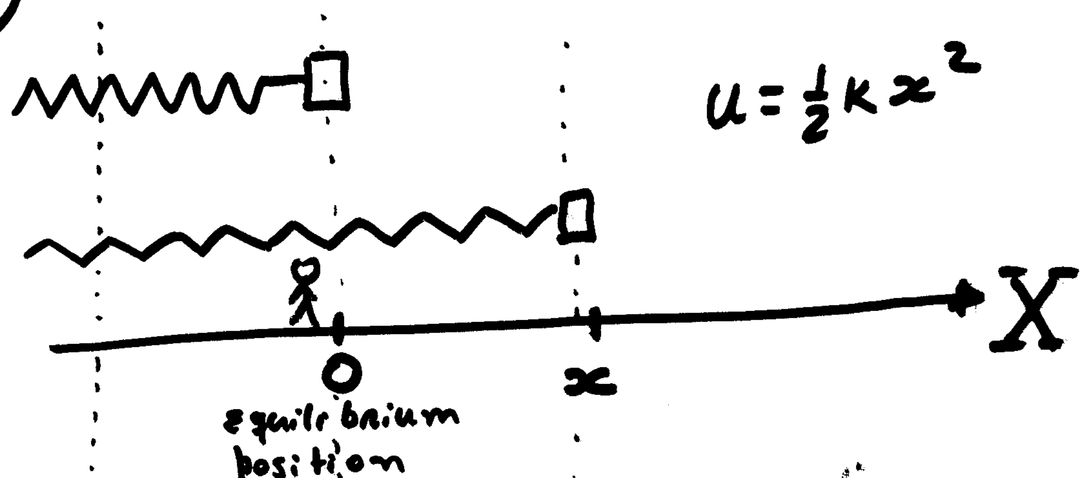
$$= -\frac{1}{2}k\left(z_f^2 - z_i^2\right) + \left\{-mg\,(z_f - z_i)\right\}$$

Rearranging terms we obtain

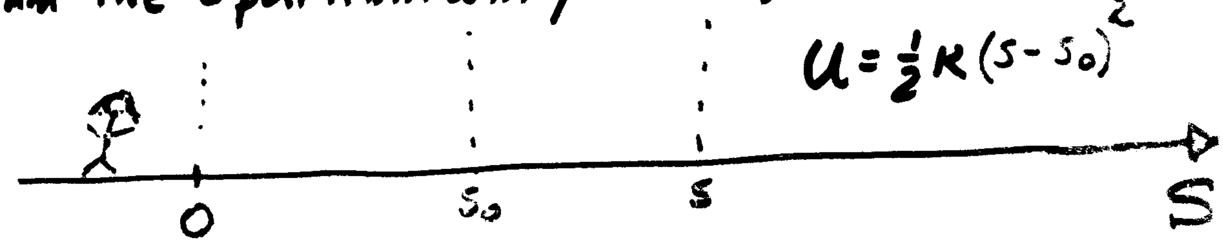
$$\boxed{K_f + \frac{1}{2}k z_f^2 + mg z_f = K_i + \frac{1}{2}k z_i^2 + mg z_i}$$

Warning:

the expression for the spring potential energy $\frac{1}{2}kx^2$ assumes that the spring is neither stretched or compressed when $x=0$. That is, the origen of of our reference is located at the position where the spring is not stretched, nor compressed (i.e. equilibrium position)

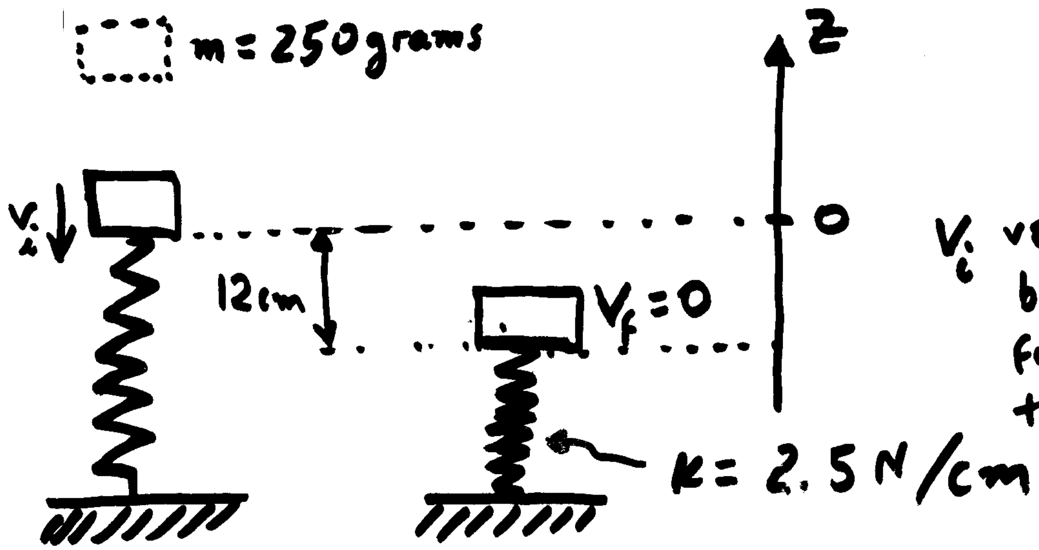



Of course, we are free to choose our origen anywhere we want. But, if we choose the origen at a position different than the equilibrium position, then



Problem #54
Chapter 7






$V_i = ?$

$V_i$ velocity of the block just before it hits the spring

$$\frac{1}{2}mv^2 + mgz + \frac{1}{2}kz^2 = E$$

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At $z = 0$ $\quad E_1 = \frac{1}{2}mv_i^2 + 0 + 0$ $\quad$ I

---

At $z = -12\,cm$
$= -0.12\,m$

$$E_2 = 0 + mg(-0.12\,m) + \frac{1}{2}\frac{2.5\,N}{0.01\,m}(-0.12\,m)^2$$

II

Notice, we have expressed k into proper units!

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$E_1 = E_2$. Solving for V:

$V_i = 3.4\ m/s$

ANOTHER METHOD TO SOLVE THIS PROBLEM

a) work done on the block by the weight

$$W = m\vec{g} \cdot \vec{d} = mg\,(0.12\,m)\,\cos 0^\circ$$

$$= (0.25\,Kg)(9.8\,m/s^2)(0.12\,m)$$

$$= 0.3\,\underbrace{Kg\,m/s^2}_{\text{Newton}}\,m$$

$$= 0.3\text{ Joule} \qquad > 0$$

b) work done on the block by the spring force $W_{i\to f} = -\frac{1}{2}k\,(s_f^2 - s_i^2)$

$$-\frac{1}{2}k\,z_f^2 = -\frac{1}{2}\,\frac{2.5\,\text{Newt}}{0.01\,\text{mt}}\,(-0.12\,m)^2$$

$$= -1.8\text{ Joule} \qquad < 0$$

c) $K_f - K_i$ = WORK (TOTAL WORK)

$$0 - \frac{1}{2}mV_i^2 = 0.3\,J - 1.8\,J = -1.5\,J$$

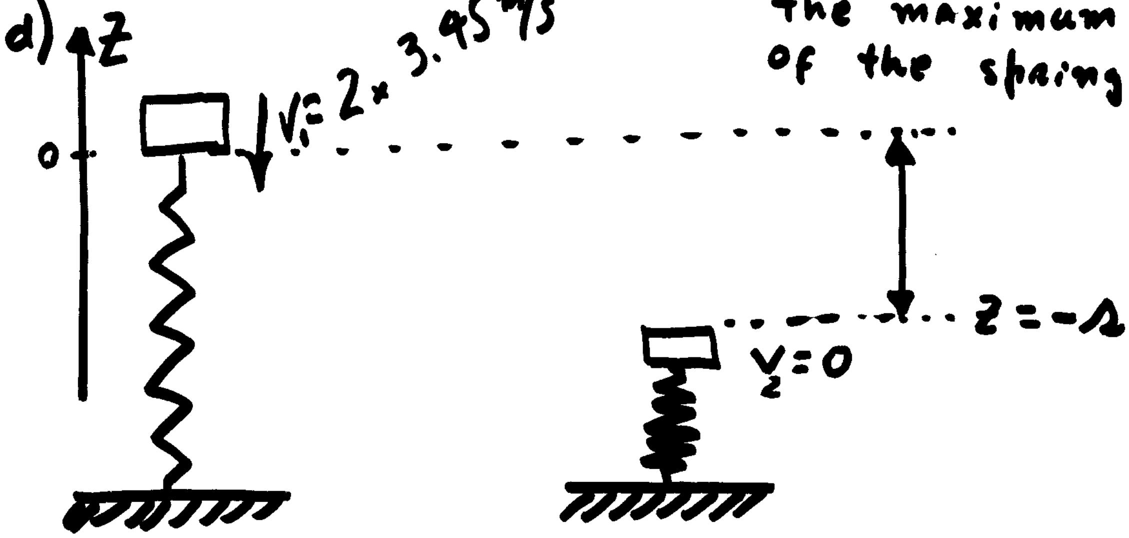
$$\Rightarrow v^2 = \frac{3\,J}{0.25\,Kg}$$

HW: demonstrate that $J/Kg = m^2/s^2$

$v = 3.45\ m/s$

d) If the speed of impact is doubled, what will be the maximum compression of the spring?

d)



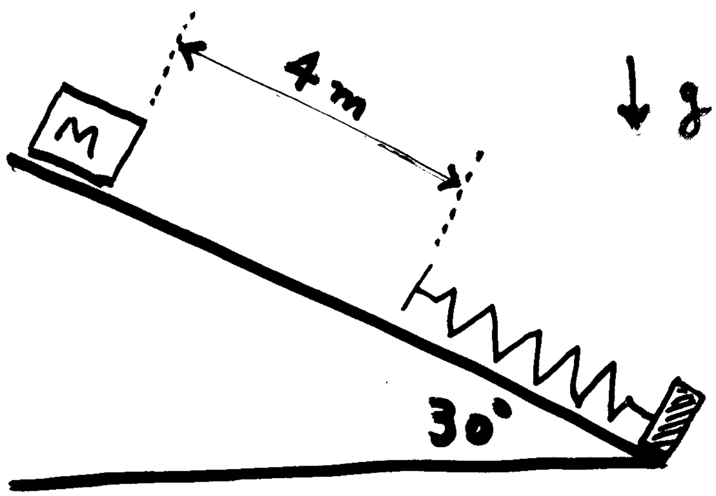

$$E_1 = \frac{1}{2} m \left(2 \times 3.45 \text{ m/s}\right)^2$$

$$E_2 = mg(-s) + \frac{1}{2} K s^2$$

$E_1 = E_2 \Rightarrow \; \ldots\ldots \; 125 s^2 - 2.45 s - 5.95 = 0$

$$\Rightarrow \boxed{s = 0.23 \text{ m}}$$

Exercise: A 2 Kg block is realeased 4 m from a massless spring of $K = 100$ N/m that is fixed along a frictionless plane inclined at 30°



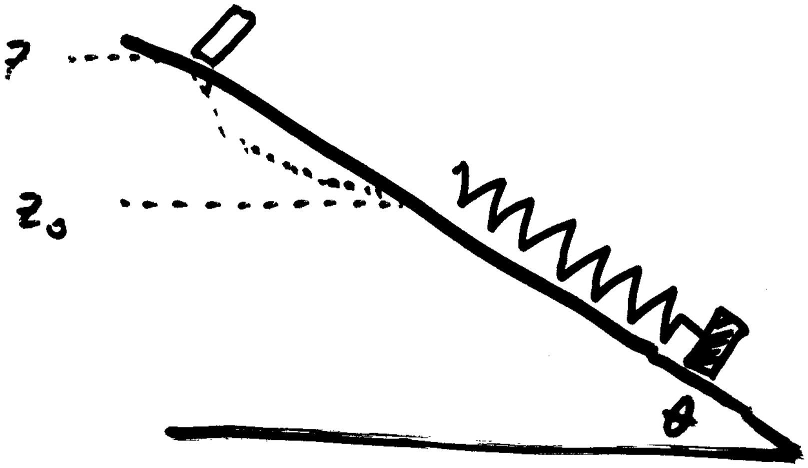

a) Find the max compression of the spring

b) If the plane is rough rather than frictionless ($\mu_K = 0.2$), find the maximum compression.

c) For each case a) and b) above, how far up the incline will the block travel after leaving the spring?

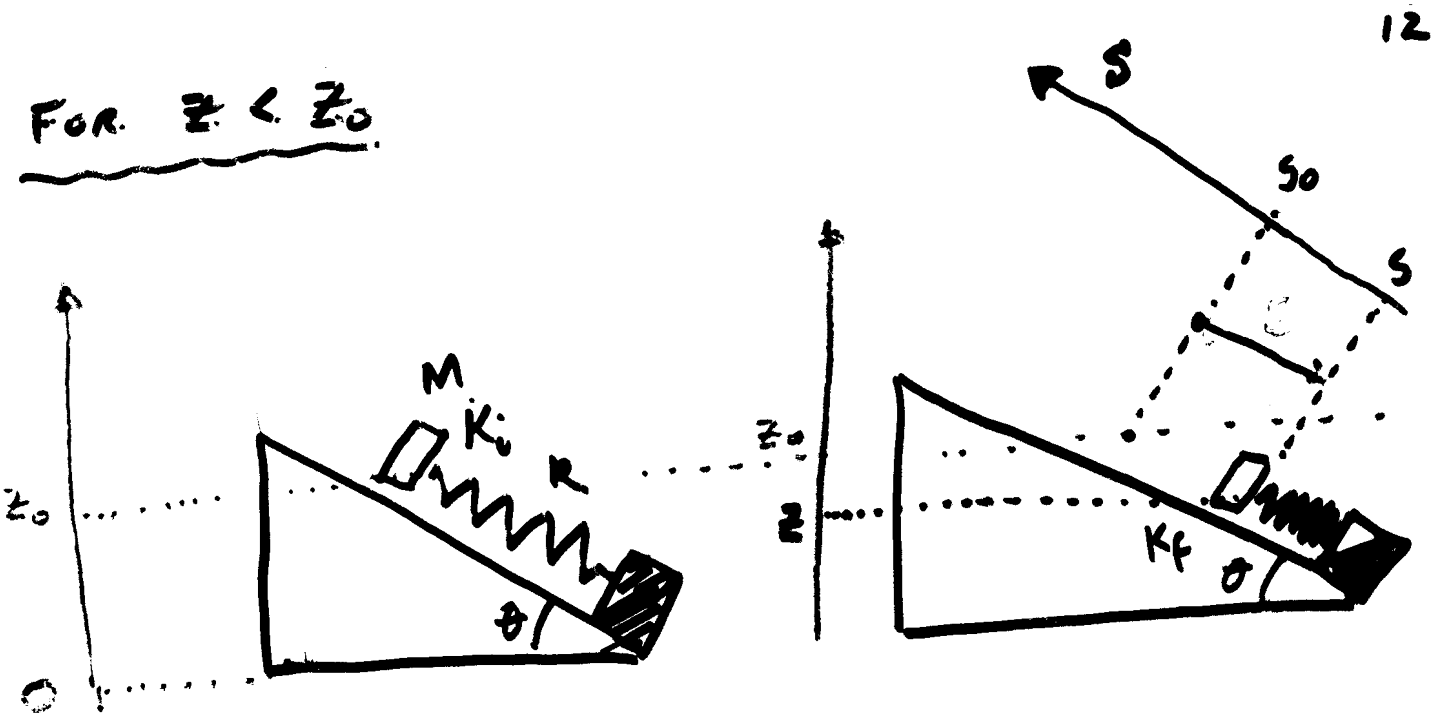
a) 0.989 m,
b) 0.783 m
c) —, 1.59 m
d)

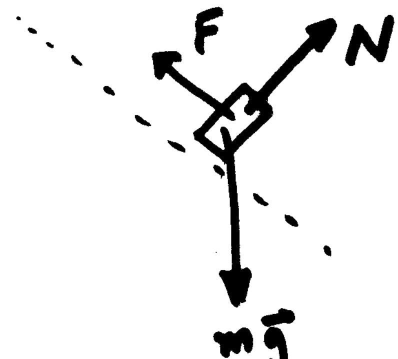
FOR $z > z_0$




$$K_z + mg\,z = K_{z_0} + mg\,z_0$$

## FOR $z < z_0$



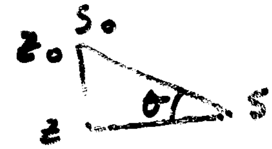

$$\Delta K = W_{TOTAL} = K_f - K_i$$

$$= W_{normal} + W_F + W_{grav}$$

$$= 0 - \tfrac{1}{2}k(s-s_0)^2 - mg(z-z_0)$$

Apparently we have two variables $z$ and $s$.

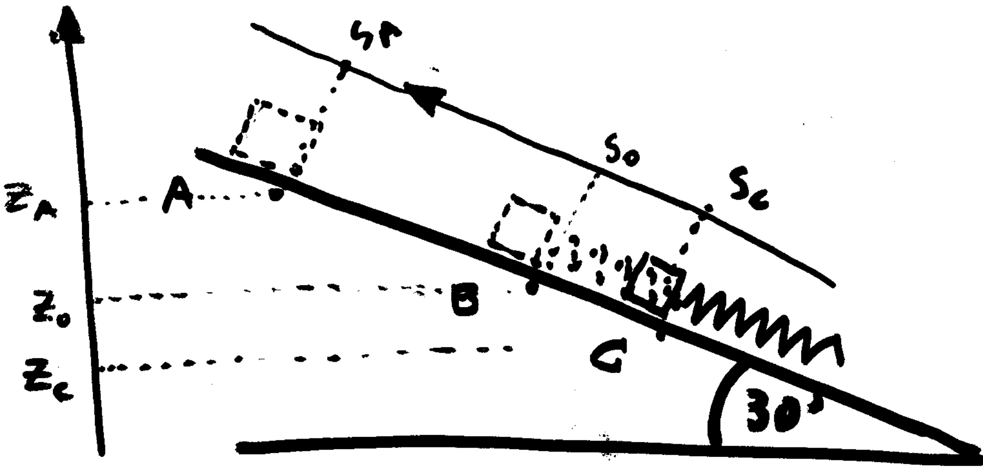
However notice




$$\frac{z_0 - z}{s_0 - s} = \sin\theta$$

or $$\frac{z - z_0}{s - s_0} = \sin\theta$$

$$\Delta K = -\tfrac{1}{2}k\frac{(z-z_0)^2}{\ldots} - mg(z - z_0) = K_f - K_i$$




$s_A - s_0 = 4\ m$

$s_0 - s_c = d = ?$

$k = 100\ \frac{N}{m}$

a) Find $K_B$

$$\underbrace{K_A}_{0} + mg\,z_A = K_B + mg\,z_0$$

(Notice, the spring is not involved in this region)

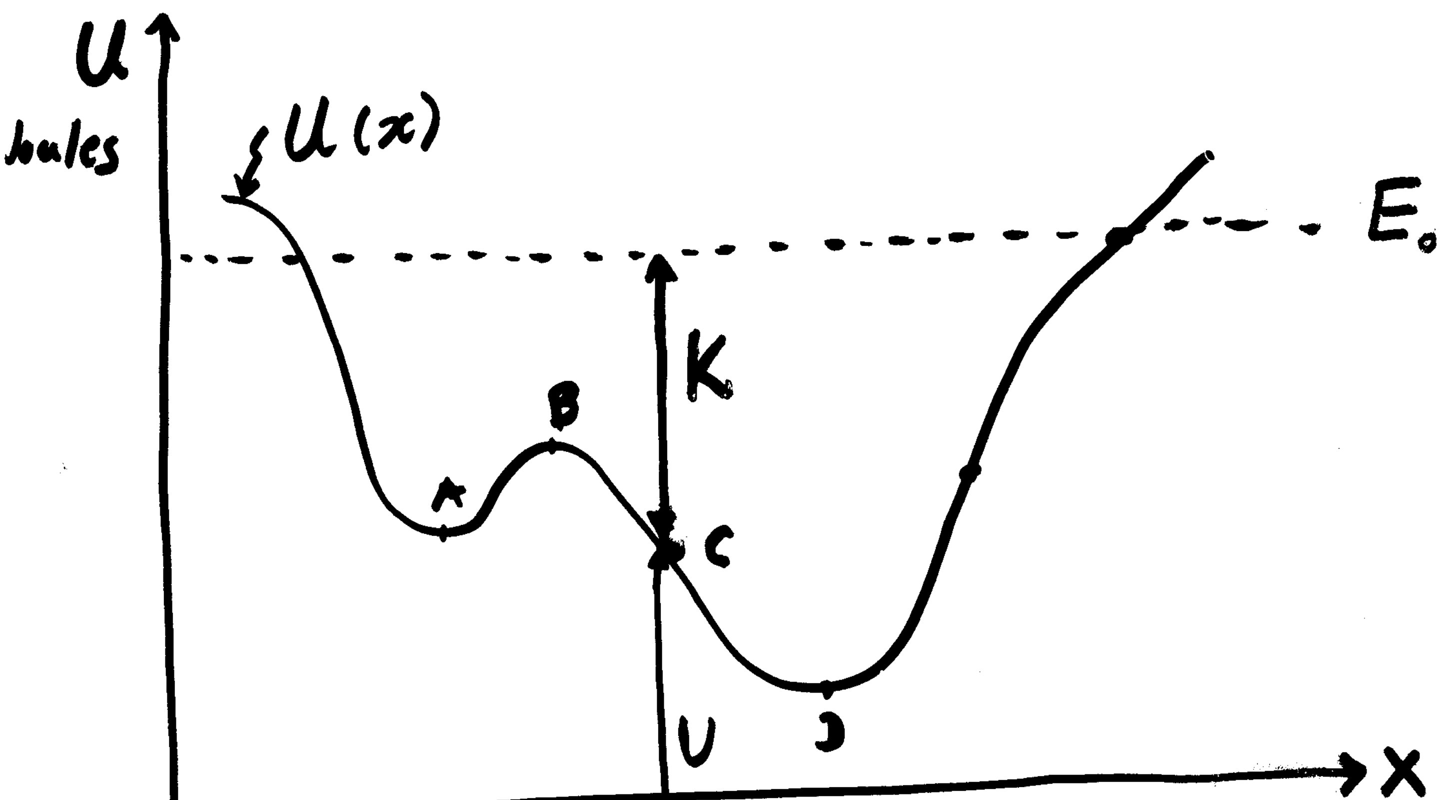
$$K_B = mg(z_A - z_0)$$
$$= mg(s_A - s_0)\sin 30° \approx 40 \text{ Joules}$$

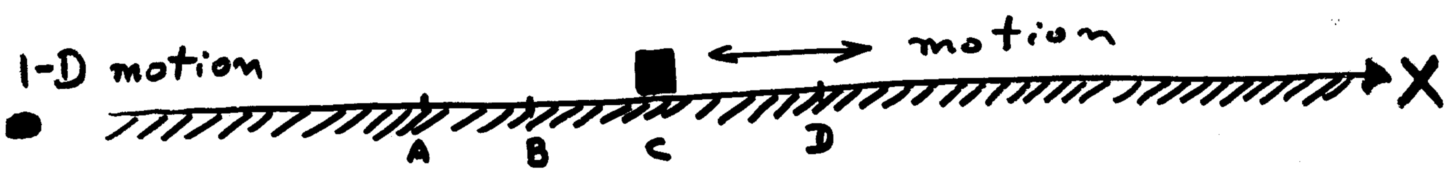
b) $$\Delta K = \underbrace{K_c}_{0} - K_B = -\tfrac{1}{2}k(s_c - s_0)^2 + mg(z_0 - z_c)$$
$$= \text{WORK}$$

(Notice: Both $\vec{g}$ and the spring force act on the block)

$$-K_B = -\tfrac{1}{2}k(s_0 - s_c)^2 + mg(s_0 - s)\sin 30°$$
$$-40\text{J} = -\tfrac{1}{2}k\,d^2 + 10\,d$$
$$50\,d^2 - 10\,d - 40 = 0$$
$$\Rightarrow d \approx 1 \text{ meter.}$$

# Reading a POTENTIAL ENERGY CURVE



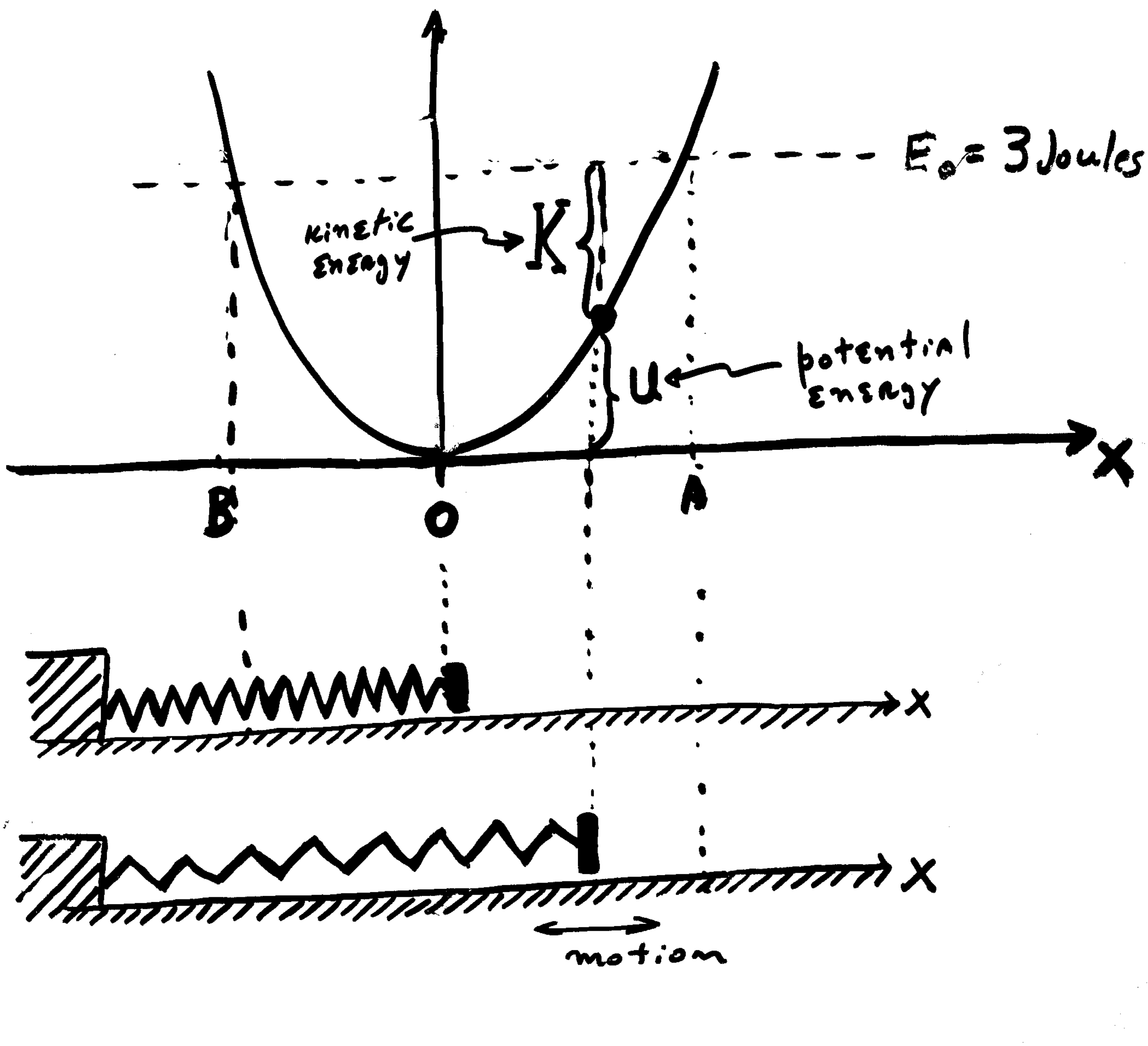




- $E_o = U(x) + K$

- Force $F = -\dfrac{dU}{dx}$

<u>Example</u> Given $U(x) = \frac{1}{2} k x^2$

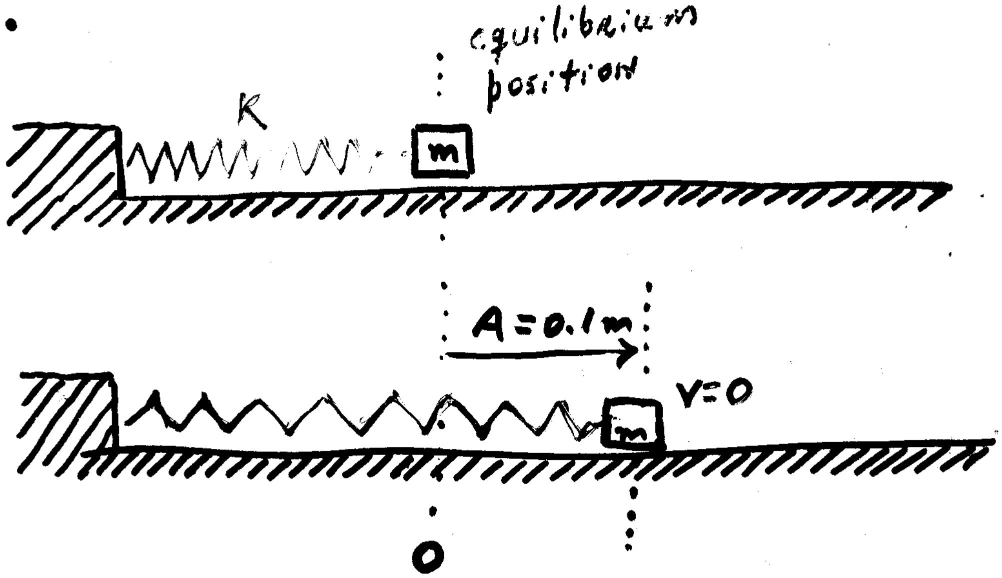
if the mechanical energy $E_0$ of the particle is also given, we'll have




$$K + U = E_0 \quad ; \quad F = -\frac{dU}{dx}$$

# EXAMPLE

A block of mass $m = \frac{1}{4}$ kg is attached to a spring ( $R = 2.5$ N/cm ). The spring is stretched 0.10 m from its equilibrium position and released from rest. Analize the motion of the block using a potential energy curve.

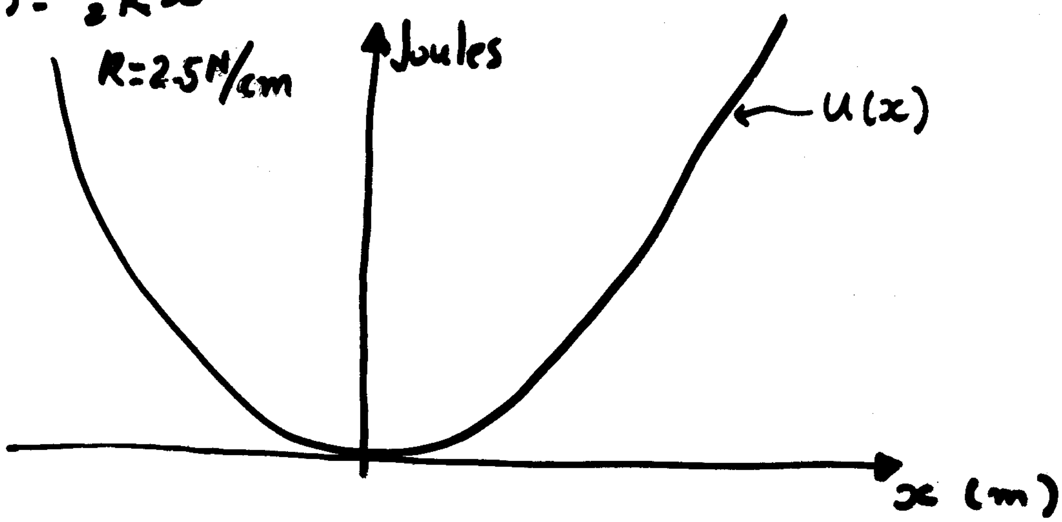



SOLUTION:

a) First, we draw the U vs $x$ graph.

$U(x) = \frac{1}{2} R x^2$

| $x$ | $U$ |
|---|---|
| 0.05 | 0.31 J |
| 0.10 | 1.25 J |
| 0.15 | ⋮ |
| 0 | |
| ⋮ | |



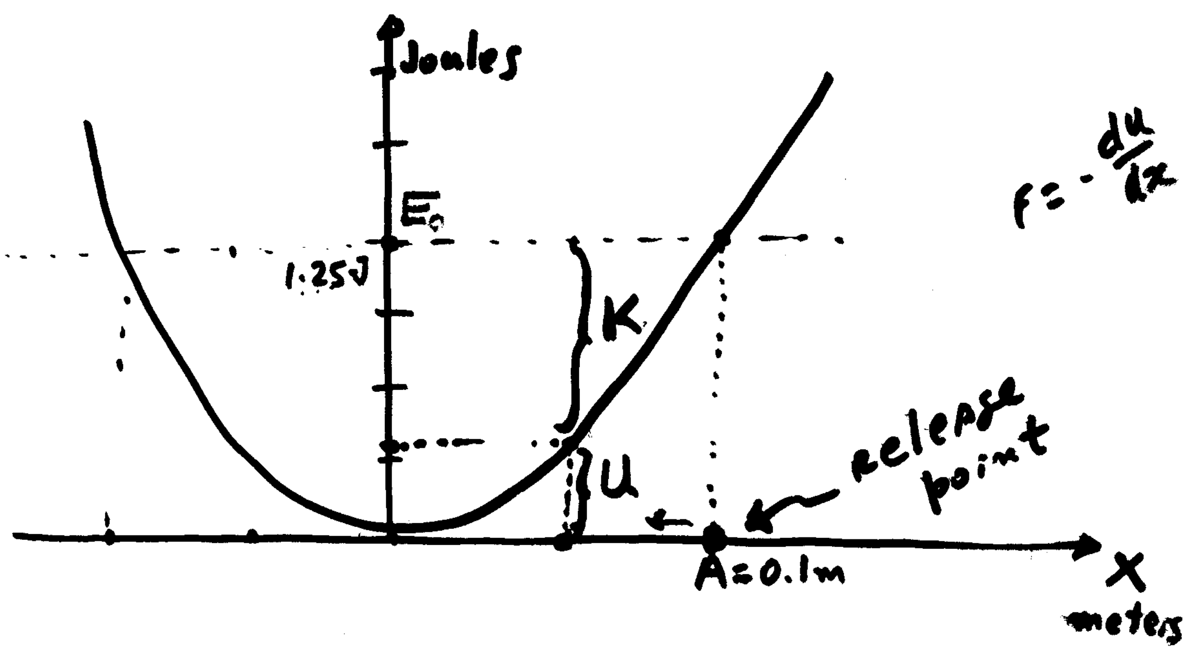
b) Second, we find the mechanical energy of the system.

Since $E_0$ remains constant throughout the motion, we can figure out its value by evaluating the mechanical energy at the point of its release

$$E_0 = \frac{1}{2} m (0)^2 + \frac{1}{2}\left(2.5 \frac{N}{0.01 m}\right) A^2$$

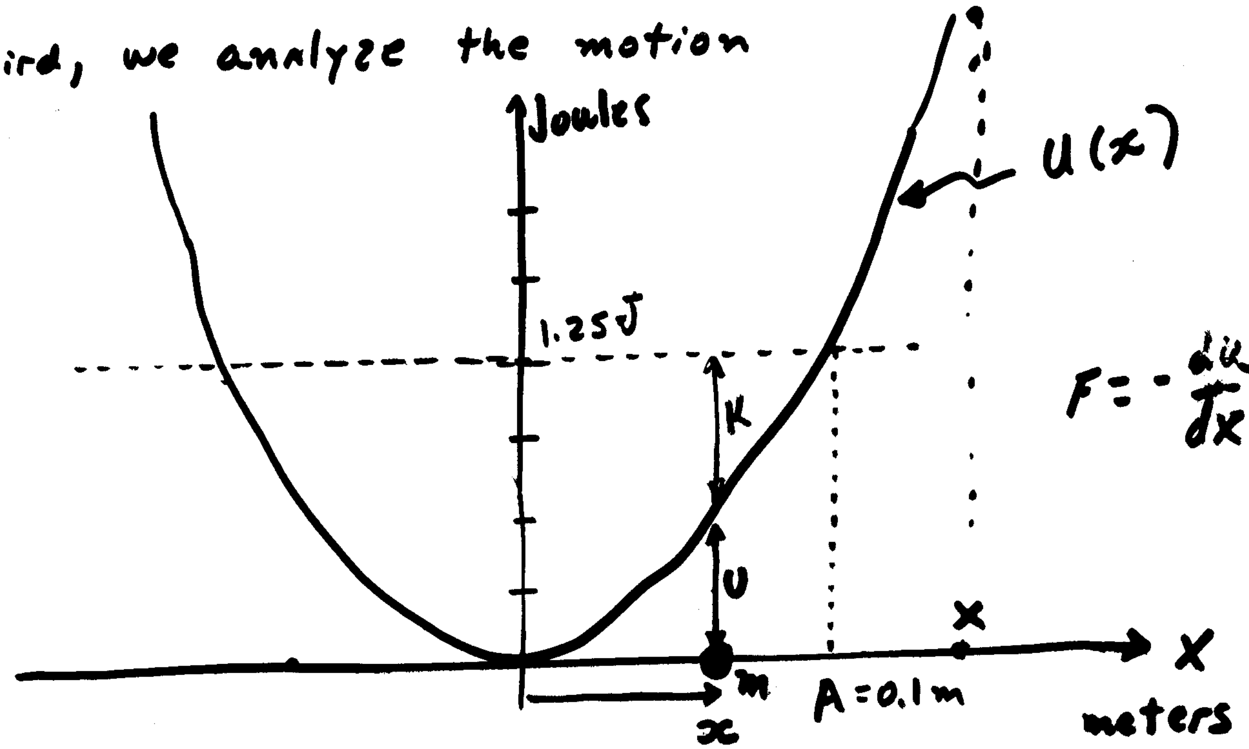
initial elongation

$$= \frac{1}{2}\left(2.5 \frac{N}{0.01m}\right)(0.10\ m)^2 = 1.25\ J$$




$F = -\frac{du}{dx}$

c) Third, we analyze the motion

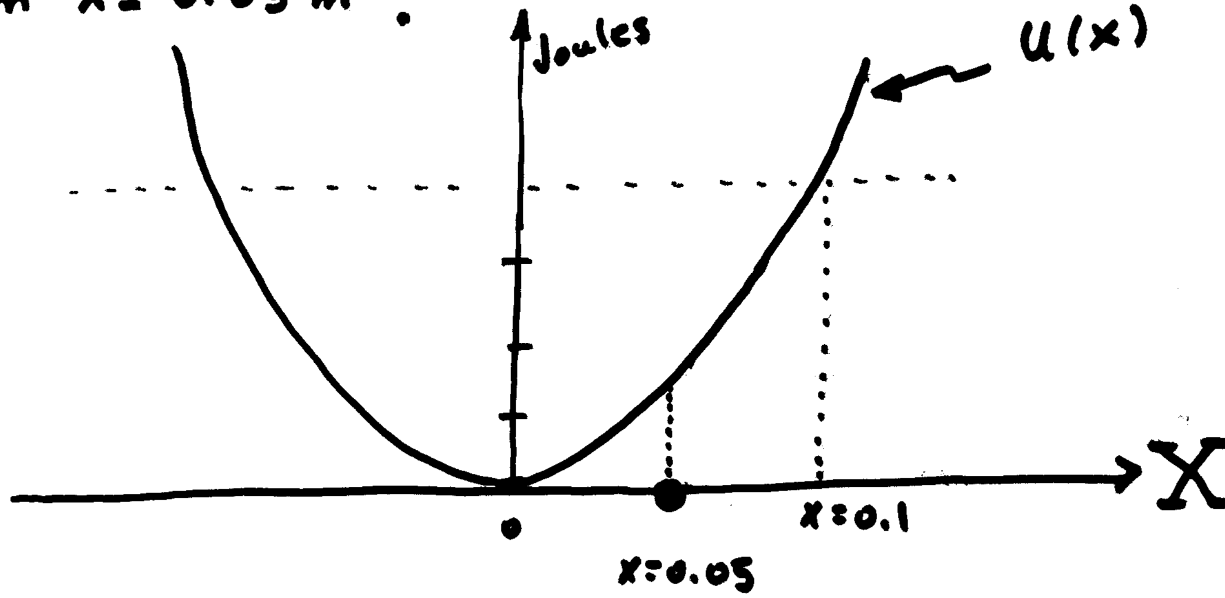



1.25 Joules = U + K

the higher U, the smaller K
the smaller U, the higher K

- Where does the block have maximum K ?
  Answer: At $x =$
- Where does the block have max U ?
  Answer: At $x =$
- Could we ever find the block of mass "m" at $x > 0.1$ meter ?
- Could we ever find the block at $x < -0.1$ meter ?

What is the force acting on the block when $X = 0.05\,m$ ?




$$U(x) = \frac{1}{2} k x^2$$

$$= \frac{1}{2}\left(2.5\frac{N}{cm}\right)x^2 = \frac{1}{2}\left(2.5\times10^2\frac{N}{m}\right)x^2$$

$$F(x) = -\frac{dU}{dx} = -2.5\times10^2\frac{N}{m}\,x$$

When $X = 0.05\,m$ $\quad F_1 = -12.5\,N$ $\quad \overset{F_1}{\longleftarrow}$

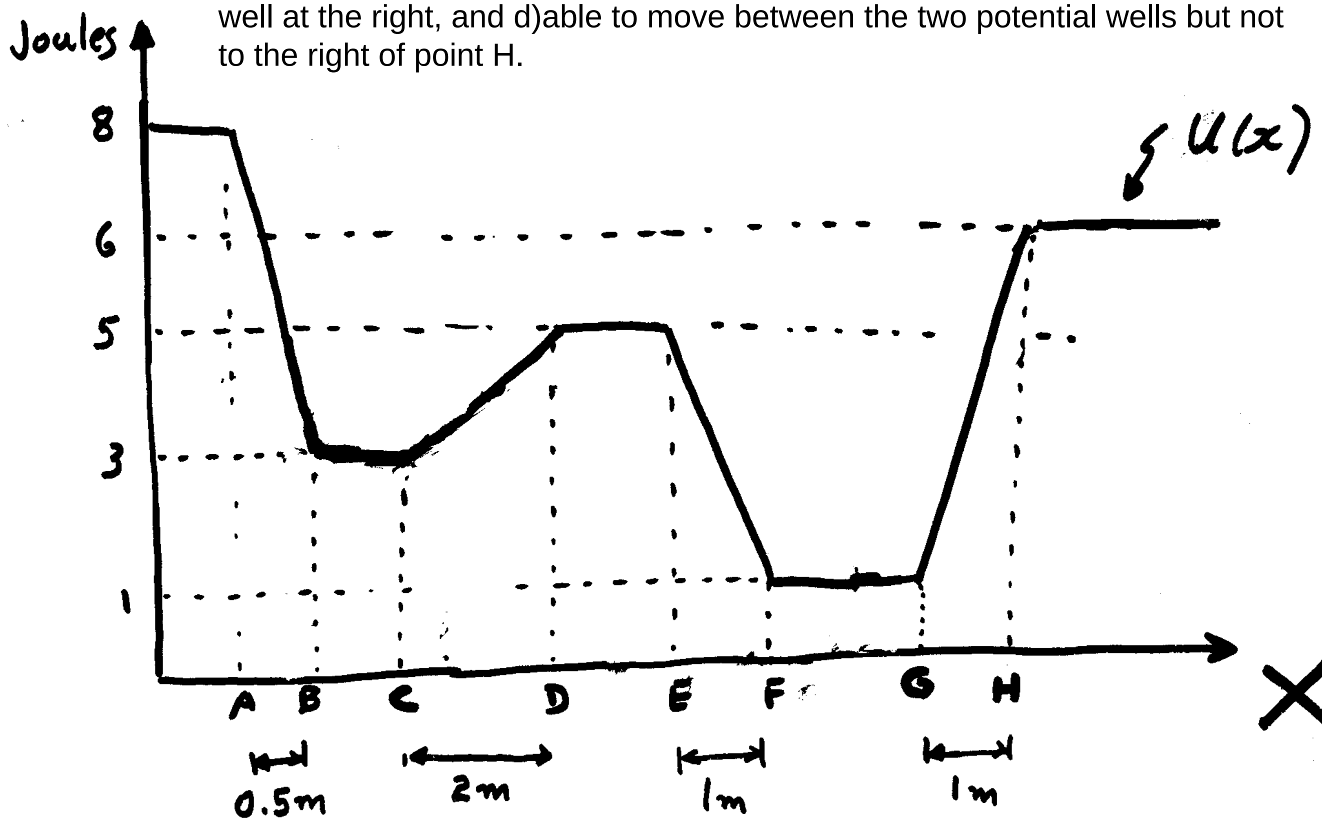
when $= 0$ $\quad F_2 = 0$

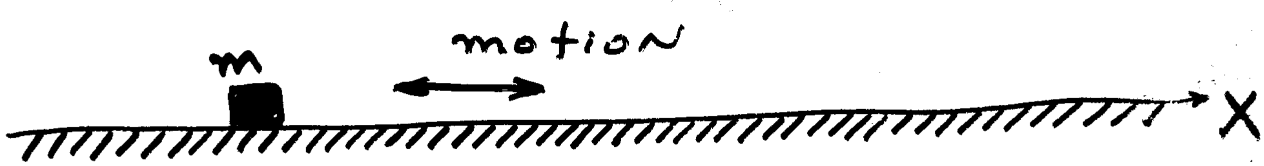
when $= -0.05\,m$ $\quad F_3 = 12.5\,N$ $\quad \underset{F_3}{\longrightarrow}$

**Exercise: Question #4** , Chapter 8, (page 188)

The figure gives the potential-energy function of a particle. What value must be the mechanical energy $E_{mec}$ of the particle not to exceed if the particle is to be b) trapped in the potential well at the left, c) trapped in the potential well at the right, and d)able to move between the two potential wells but not to the right of point H.

6



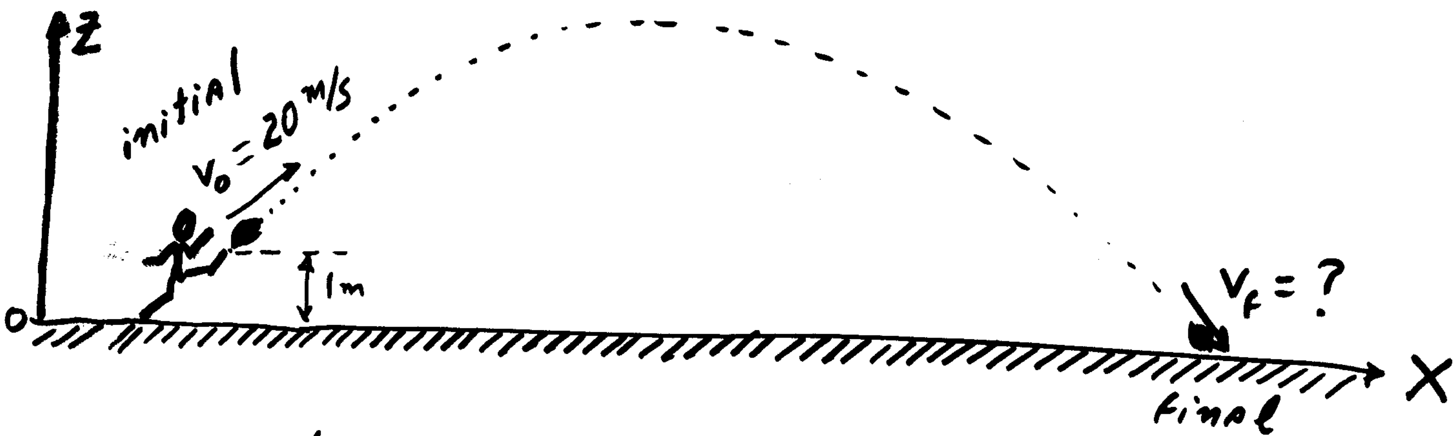




- Rank the regions AB, BC, ... according to the magnitude of the force acting on the block.

- If E = 5.5 Joules, in which region will the particle have the greatest kinetic energy?

# Example

(conservation of energy)




initial mechanical energy $E_0 = E_f$ final mechanical energy

$$\frac{1}{2} m v_0^2 + m g z_0 = \frac{1}{2} m v_f^2 + m g z_f$$

Notice $z_0 = 1m$

$z_f = 0$

$$\frac{1}{2}(20\ m/s)^2 + (9.8\ m/s^2)(1\ m) = \frac{1}{2} v_f^2 + 0$$

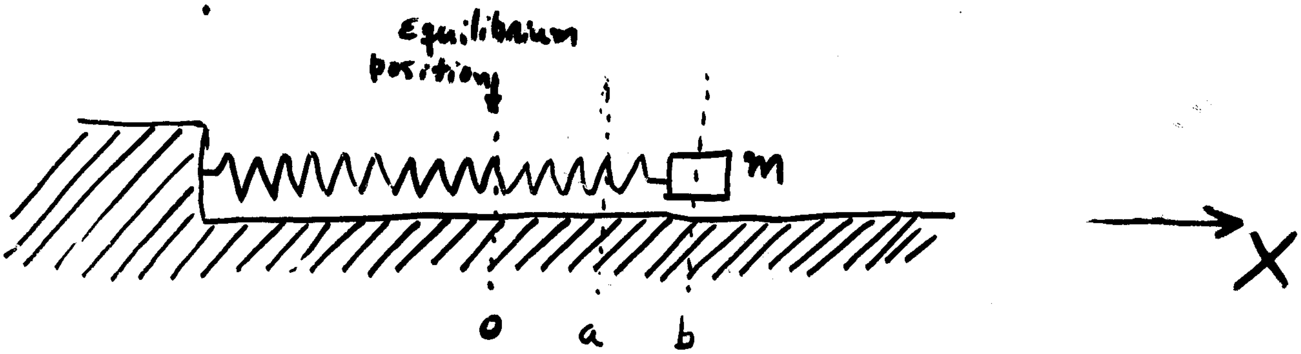
Solving for $v_f$:

$$v_f = \sqrt{420}\ m/s$$

$$\approx 20.5\ m/s$$

# Practice questions:

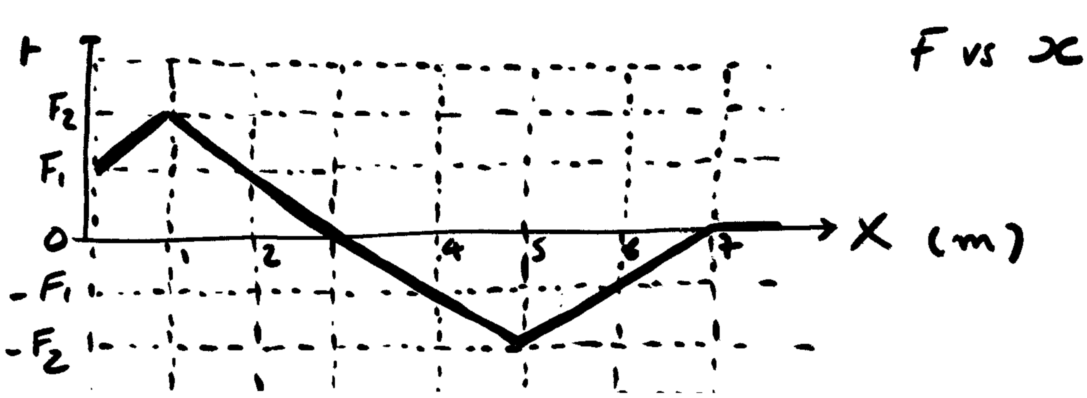
- Does the spring force always do a negative work ?




$W_{(a \to b)} =$ (+) or (−)

$W_{(b \to a)} =$ (+) or (−) ?

- **Exercise: Question #8** , Chapter 7, (page 159)



7"

the graph show the force $F = F(x)$ that will act on a particle. If the particle begins at rest at $x = 0$, what is the particle's coordinate when it has

a) its greatest kinetic energy

b) its " speed

c) zero speed

d) what is the particle's direction of travel after it reaches $x = 6$ m