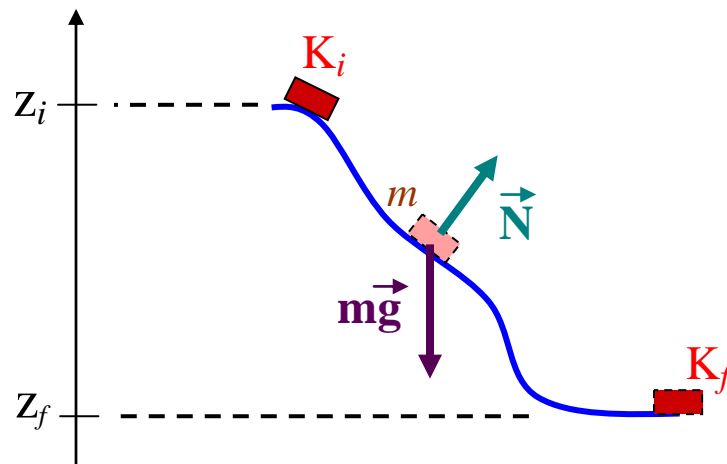


MECHANICAL ENERGY



We know

$$\begin{aligned}
 K_f - K_i &= W_{\text{total}} \\
 &\text{work done by all the forces} \\
 &\text{acting on the mass } m \\
 &= W_{\text{gravity-force}} + W_{\text{normal-force}} \\
 &= -mg(z_f - z_i) + 0
 \end{aligned}$$

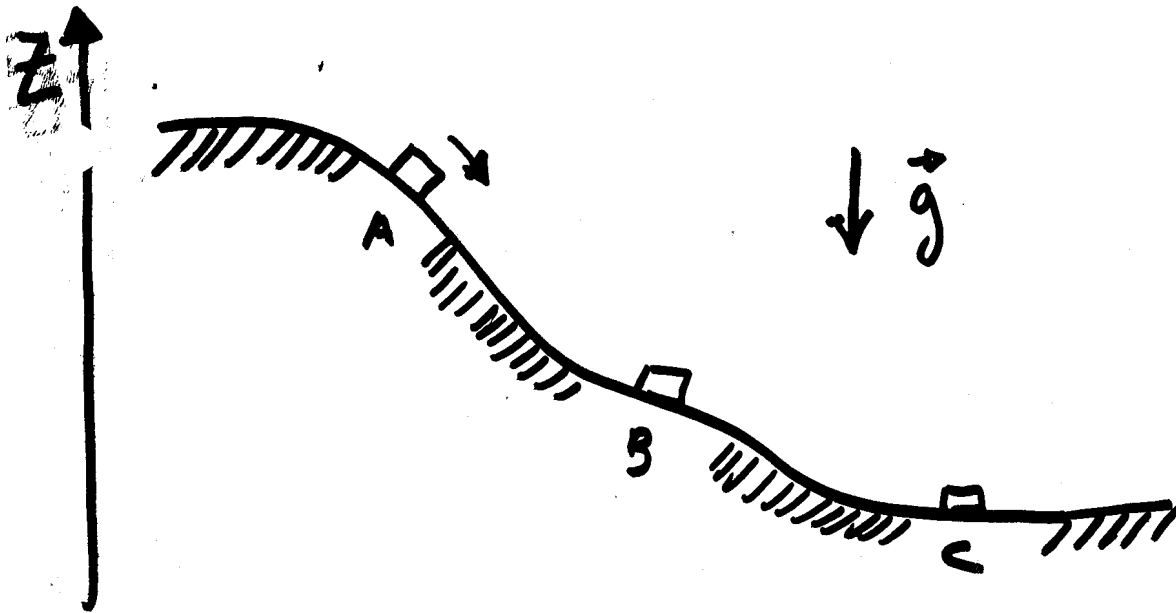
That is,

$$K_f - K_i = -mg(z_f - z_i)$$

or

$$K_f + mg z_f = K_i + mg z_i$$

This result indicates that,



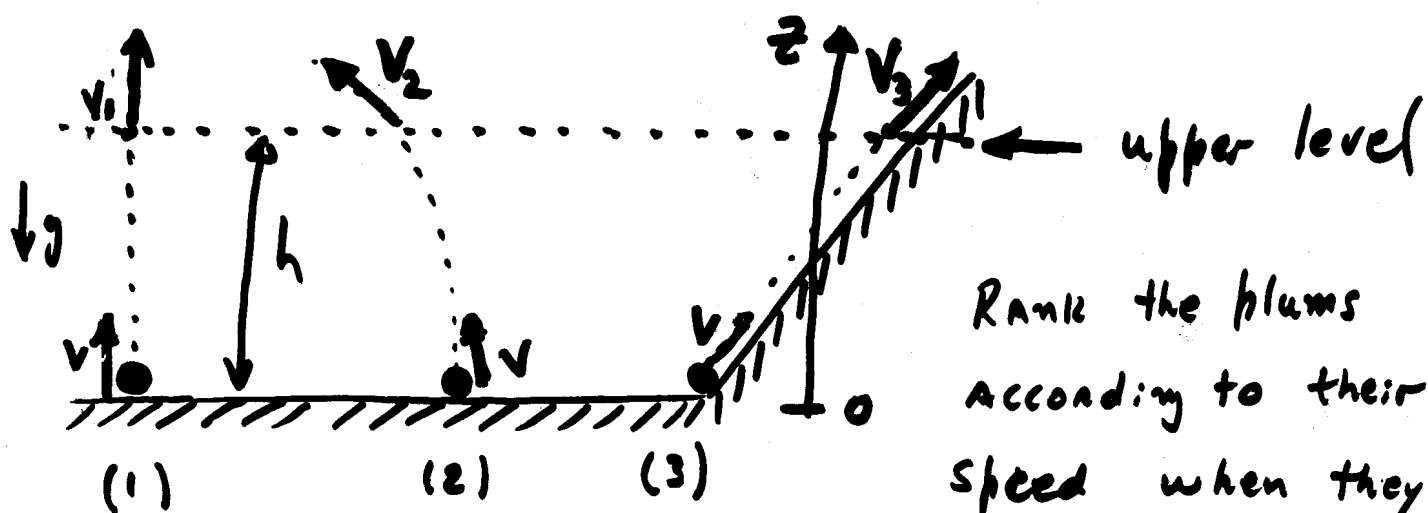
$$E_A = E_B = E_C$$

What we have obtained is:

If we know the mechanical energy of the system (i.e. block and gravitation) at just ONE point

Then

we know its mechanical energy everywhere. It is the SAME!

EXERCISE 1

Rank the plums according to their speed when they reach the upper level

Each plum is launched from the same bottom level, and with the same speed v

$$E = K + U = K + U$$

$$(1) \rightarrow \frac{1}{2} m v^2 + 0 = \frac{1}{2} m v_1^2 + mgh$$

$$(2) \rightarrow \frac{1}{2} m v^2 + 0 = \frac{1}{2} m (v_2)^2 + mgh$$

$$(3) \rightarrow \frac{1}{2} m v^2 + 0 = \frac{1}{2} m (v_3)^2 + mgh$$

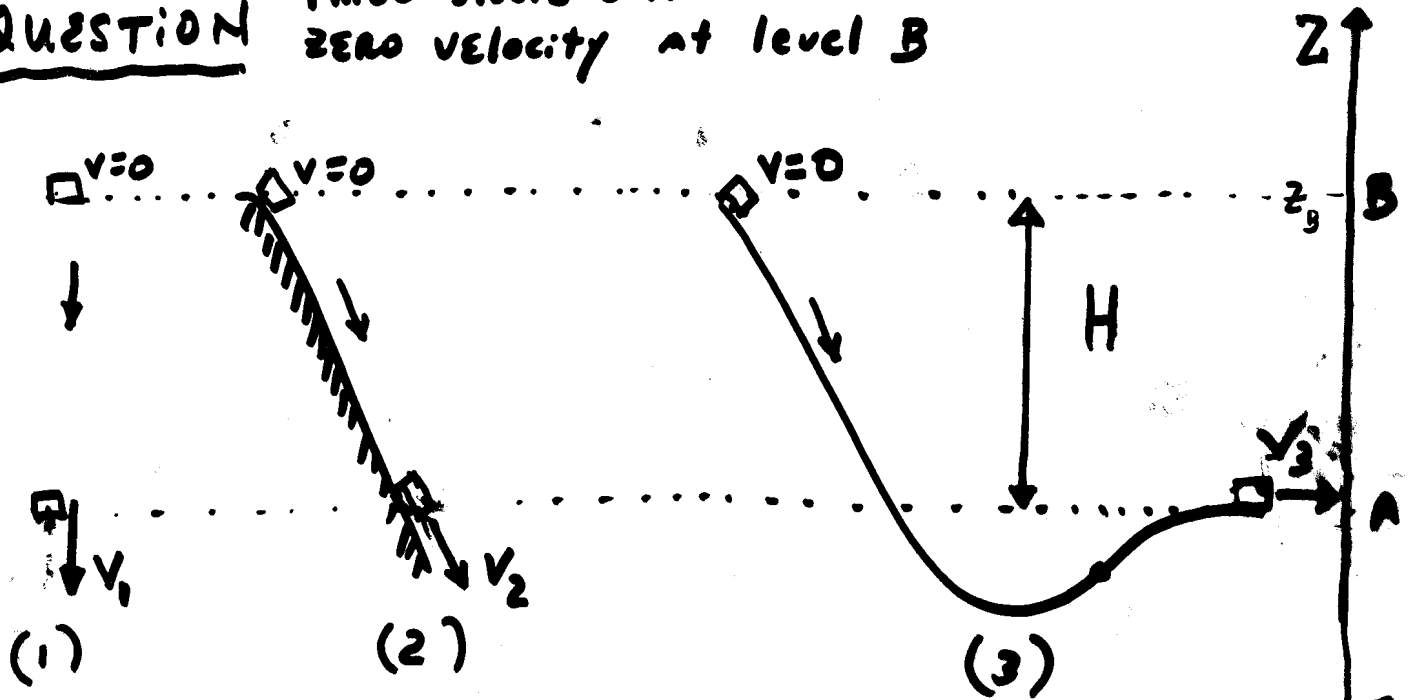
bottom

level $z=0$

upper level

$z=h$

QUESTION three blocks start their motion with ZERO velocity at level B



- DRAW the vector velocity the level A in each case

In which case is the speed at the level A higher?

$$E = \text{KINETIC ENERGY} + \text{POTENTIAL ENERGY}$$

CASE (1) Energy at B: $0 + mgz_B = E_B$

Energy at A: $\frac{1}{2}mv_1^2 + mgz_A = E_A$

conservation of mechanical energy implies

$$0 + mgz_B = \frac{1}{2}mv_1^2 + mgz_A \Rightarrow \boxed{mgH = \frac{1}{2}mv_1^2}$$

case (2) Energy at B : $0 + mg z_B$

Energy at A : $\frac{1}{2} m v_2^2 + mg z_A$

Conservation of mechanical energy implies,

$$0 + mg z_B = \frac{1}{2} m v_2^2 + mg z_A$$

$$mg(z_B - z_A) = \frac{1}{2} m v_2^2$$

$$mg H = \frac{1}{2} m v_2^2$$

(2)

case (3)

Energy at B : $0 + mg z_B$

Energy at A : $\frac{1}{2} m v_3^2 + mg z_A$

Conservation of mechanical energy implies

$$0 + mg z_B = \frac{1}{2} m v_3^2 + mg z_A$$

$$mg(z_B - z_A) = \frac{1}{2} m v_3^2$$

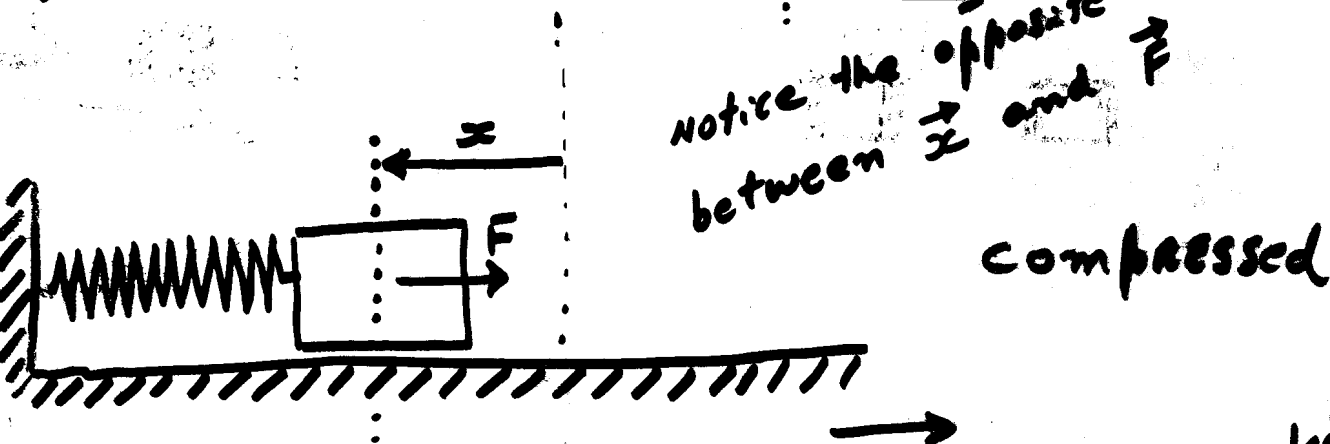
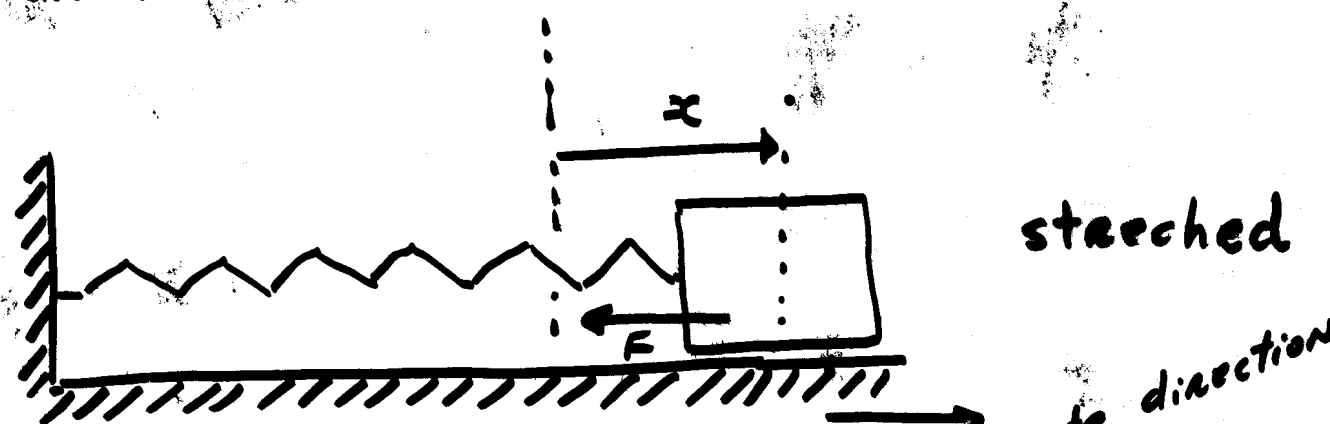
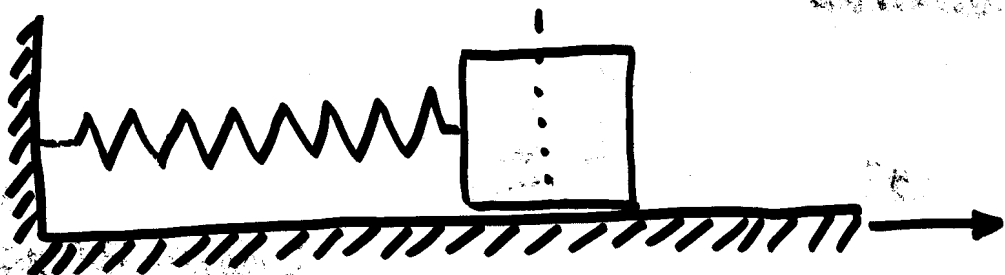
$$mg H = \frac{1}{2} m v_3^2$$

(3)

From (1), (2) and (3) we conclude :

$$v_1 \quad v_2 \quad v_3$$

WORK DONE BY A SPRING FORCE



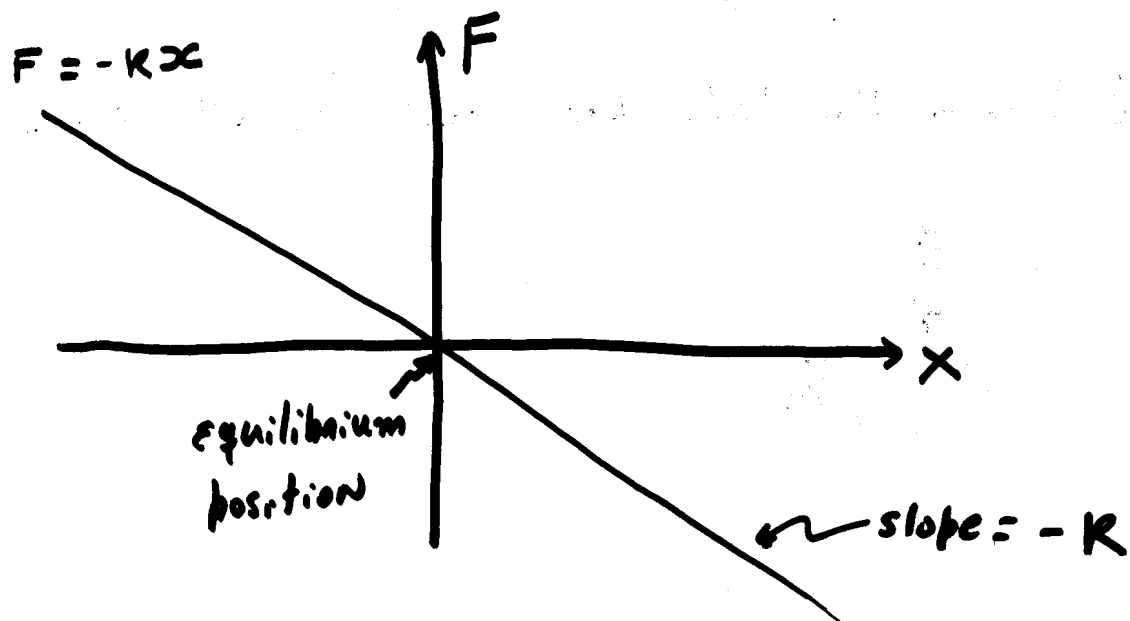
notice the opposite direction between \vec{x} and \vec{F}

Experimental fact:

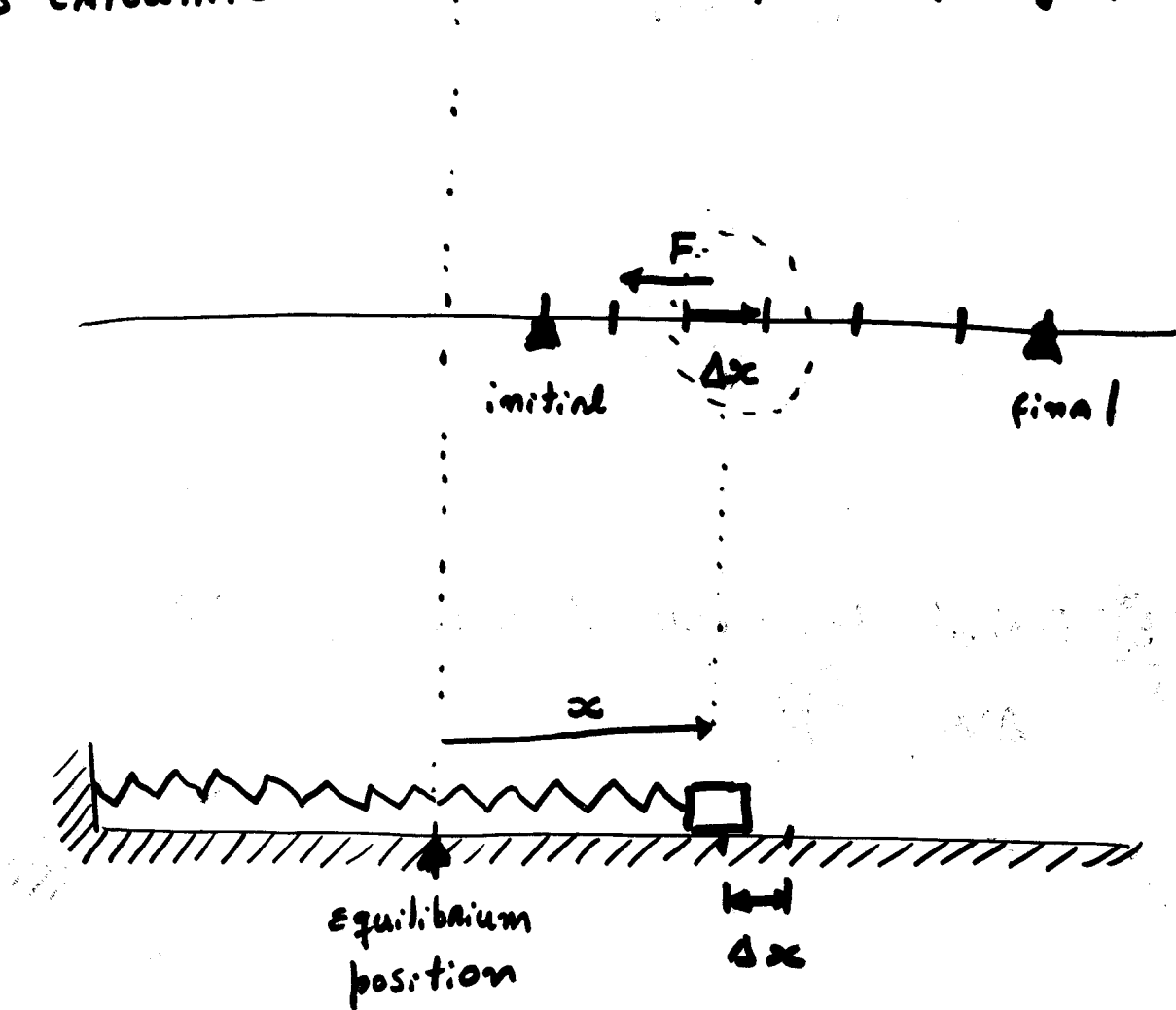
$$F = -kx$$

spring constant
(Newtons/meter)

valid provided we do not stretch the spring too much.



Let's calculate the work done by a spring force



$$W_{i \rightarrow f} = \sum_i^f \vec{F} \cdot \Delta \vec{x}$$

$$= \sum_i^f \underbrace{-kx}_{\text{force}} \underbrace{\Delta x}_{\text{displacement}}$$

When $|\Delta x| \rightarrow 0$



$$W_{i \rightarrow f} = \int_i^f \underbrace{-kx}_{\text{force}} dx$$

When facing with integrals like $\int F(x) dx$, we ask ourselves:

What is that function $G(x)$ such that

$$\frac{dG(x)}{dx} = F(x) ?$$

Once we find $G(x)$ we establish the following

$$\int_i^f F(x) dx = G(x_f) - G(x_i)$$

What is the function $G(x)$ such that its derivative is $-kx$?

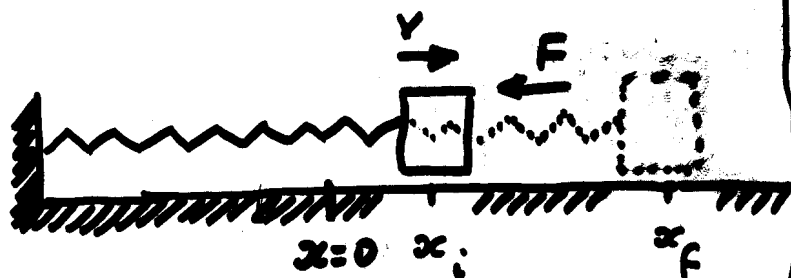
$$G(x) = -\frac{1}{2} kx^2 + C$$

$$W_{i \rightarrow f} = \int_i^f -kx \, dx$$

$$= \left(-\frac{1}{2} k x_f^2 \right) - \left(-\frac{1}{2} k x_i^2 \right)$$

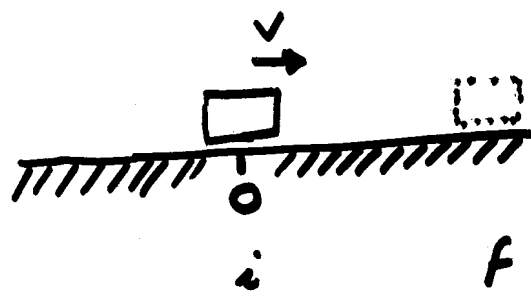
$$W_{i \rightarrow f} = -\frac{1}{2} k (x_f^2 - x_i^2)$$

work done by the
spring force on
the block when such
block moves from
 $x = x_i$ to $x = x_f$

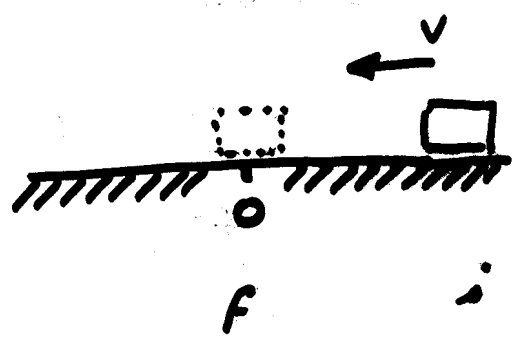
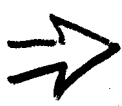


Spring neither
stretched or
compressed

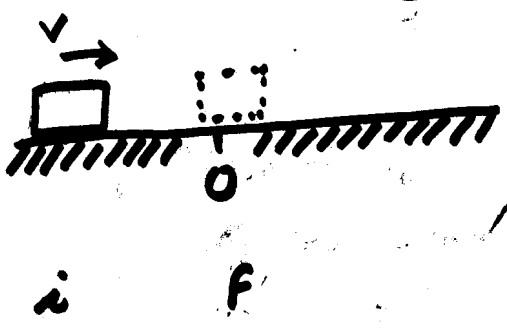
Observation:



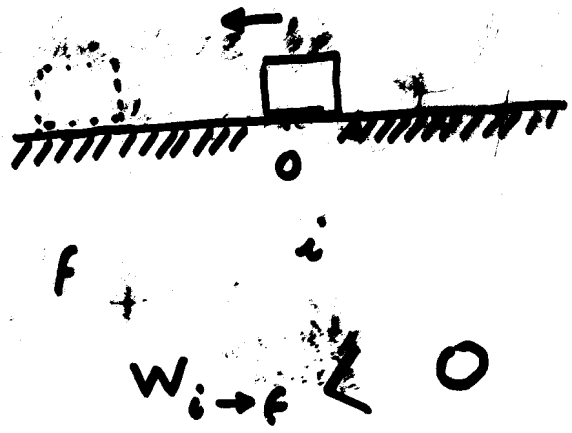
$$W_{i \rightarrow f} < 0$$



$$W_{i \rightarrow f} > 0$$



$$W_{i \rightarrow f} > 0$$



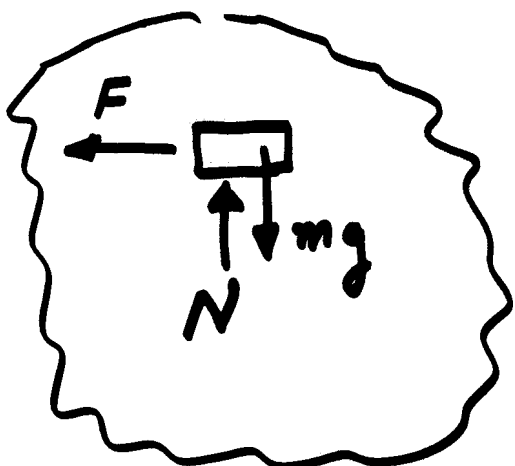
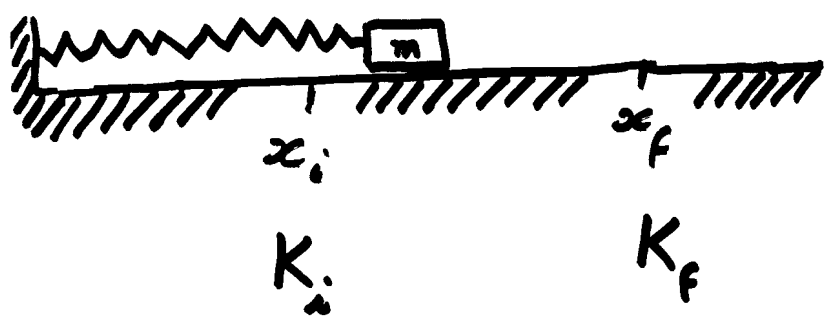
$$W_{i \rightarrow f} < 0$$

Notice also:

The result $W_{i \rightarrow f} = -\frac{1}{2}k(x_f^2 - x_i^2)$

implies that the work done ON the block by the spring force in a ROUND TRIP is zero. **SPRING FORCE IS CONSERVATIVE**

Now, let's apply our general result



free-body diagram

$$K_f - K_i = W_{\text{TOTAL}}$$

work done by All the forces

$$= W_{\text{spring}} + W_{\text{normal force}} + W_{\text{gravity force}}$$

$$= -\frac{1}{2}k(x_f^2 - x_i^2)$$

Rearranging terms we obtain

$$K_f + \frac{1}{2}kx_f^2 = K_i + \frac{1}{2}kx_i^2$$

For this case, the mechanical energy is given by

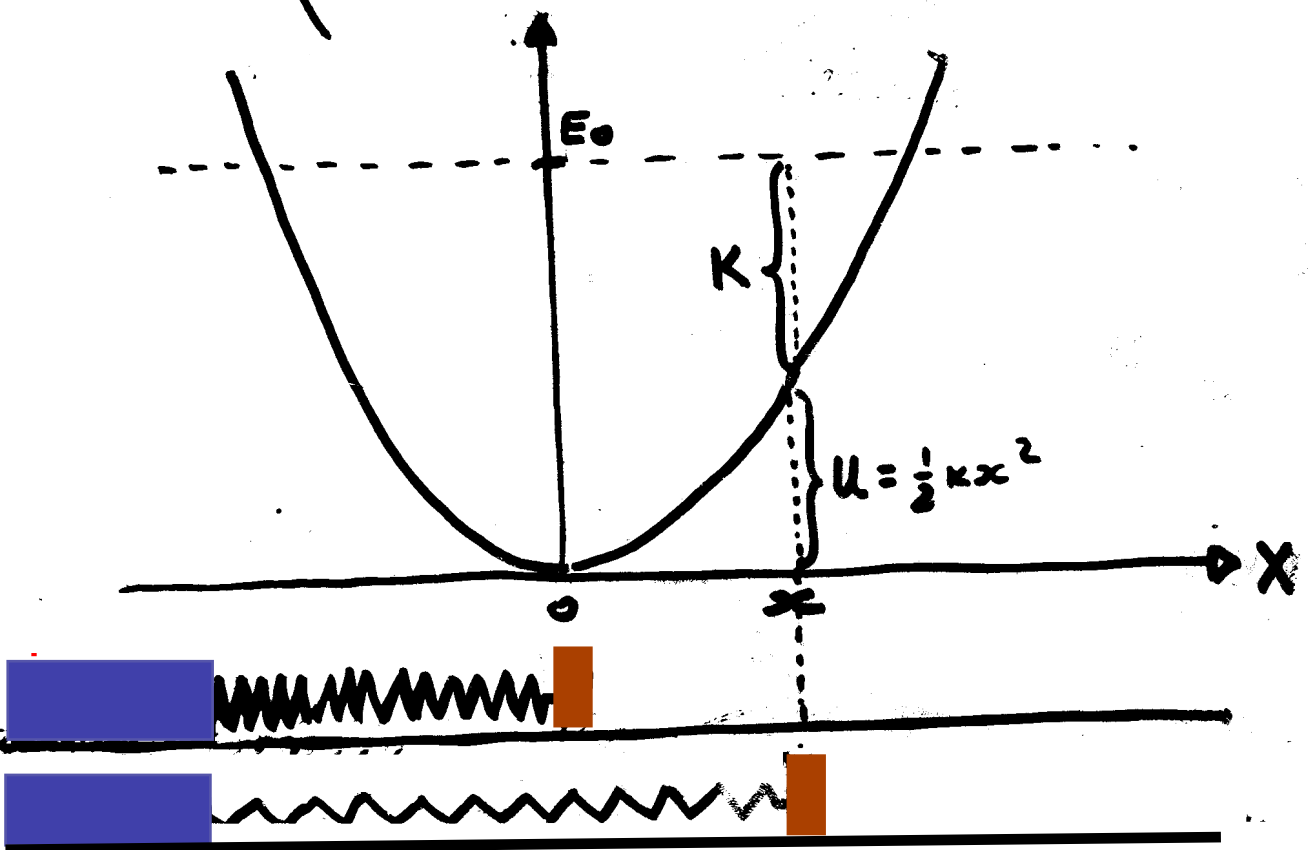
$$E = K + \frac{1}{2} k x^2 = \text{const}$$

MECHANICAL
ENERGY

KINETIC
ENERGY

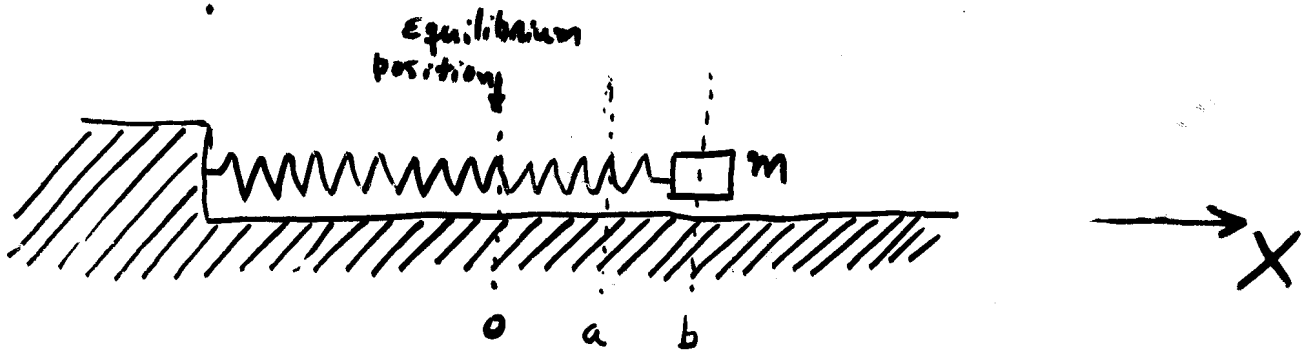
SPRING
potential
ENERGY
U

(smaller + bigger = const = E_0
bigger + smaller = const = E_0)



Practice questions:

- Does the springy force always do a negative work?

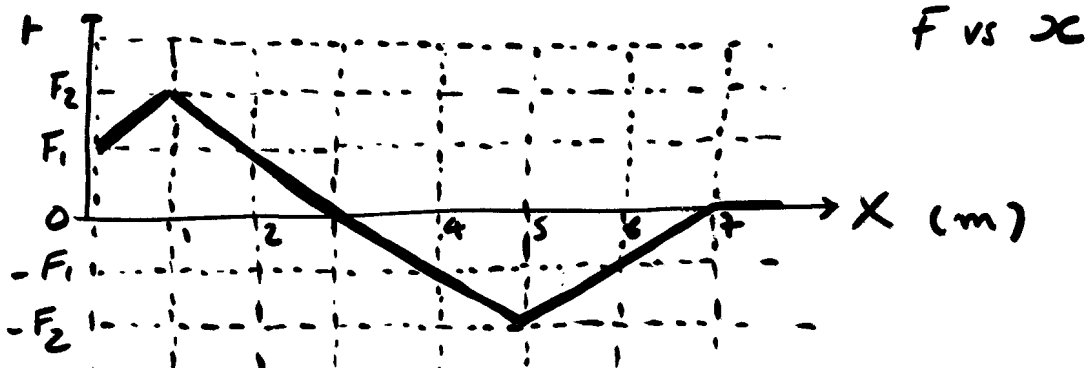


$$W_{(a \rightarrow b)} = (+) \text{ or } (-)$$

$$W_{(b \rightarrow a)} = (+) \text{ or } (-)$$

?

• Exercise: Question #8, Chapter 7, (page 159)



The graph shows the force $F = F(x)$ that will act on a particle. If the particle begins at rest at $x = 0$, what is the particle's coordinate when it has

a) its greatest kinetic energy

b) its " speed

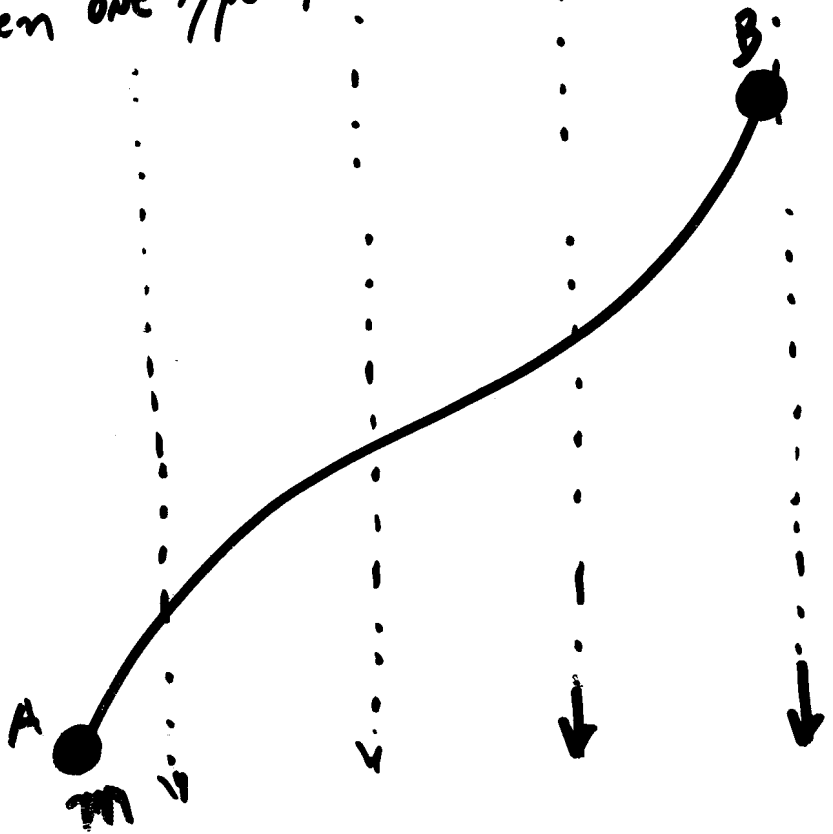
c) zero speed

d) what is the particle's direction of travel after it reaches $x = 6 \text{ m}$

FORMAL DEFINITION OF POTENTIAL ENERGY U

VALID FOR CONSERVATIVE FORCES ONLY

Given one type of conservative force \vec{F}



"field" of a conservative force (gravitational force for example)

$$U_B - U_A = - \int_A^B \vec{F} \cdot d\vec{r}$$

$$\int_A^B \vec{F} \cdot d\vec{r} = \sum \vec{F}_i \cdot d\vec{r}_i$$

We do this, because

$$U_B - U_A = - \underbrace{\int_A^B \vec{F} \cdot d\vec{r}}$$

→ This term is nothing but
the work done by the
force \vec{F} ON the mass "m"

$$= - W_{A \rightarrow B}$$

So, for conservative forces this definition
makes sense, since $W_{A \rightarrow B}$ is indepen-
dent of the path followed by the
mass "m" to go from A to B

$$U_B - U_A \equiv -W (A \rightarrow B)$$

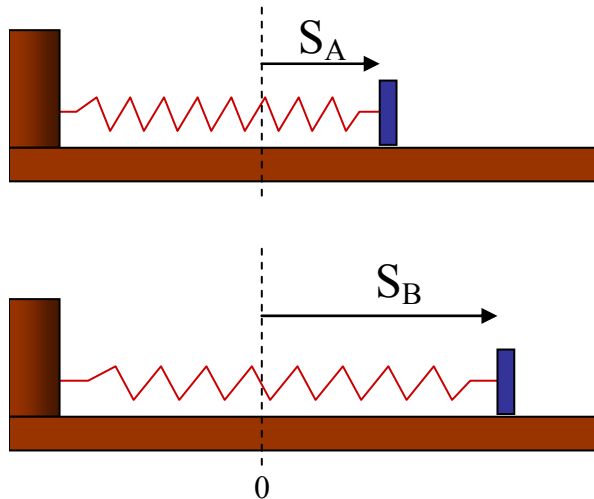
For a given conservative force \mathbf{F} , a potential energy U is defined and associated it.

The change in potential energy $U_B - U_A$ is defined as the negative value of the work W done by the conservative force

That such definition makes sense indeed for the case of conservative forces is illustrated below.

In the case of the spring force, for example, we found

$$W (A \rightarrow B) = - (1/2) k [(s_B)^2 - (s_A)^2]$$

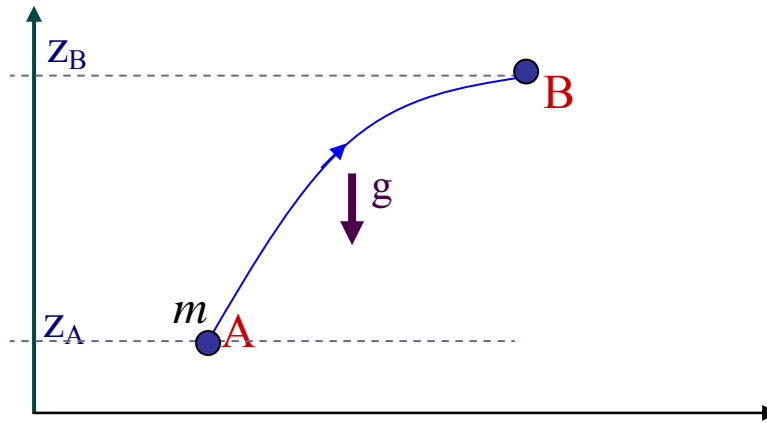


Accordingly, the expression $U_B - U_A \equiv -W (A \rightarrow B)$ implies

$$U_B - U_A \equiv + (1/2) k [(s_B)^2 - (s_A)^2]$$

This quantity depends only on the initial and final positions of the particle

Similarly, in the case of the gravitational force. Previously we found, $W (A \rightarrow B) = - mg [z_B - z_A]$, a value independent of the particular trajectory followed by the object



Accordingly, the expression $U_B - U_A \equiv - W_{(A \rightarrow B)}$ implies

$$U_B - U_A \equiv + mg [z_B - z_A]$$

This quantity depends only on the initial and final positions of the particle

In the definition of the potential energy $U_B - U_A \equiv - W_{(A \rightarrow B)}$, the negative sign in front of W is quite convenient. In effect, as we know, from the work / kinetic-energy theorem

$$W_{(A \rightarrow B)} = K_B - K_A$$

One obtains

$$U_B - U_A \equiv - W_{(A \rightarrow B)} = - (K_B - K_A)$$

which leads to

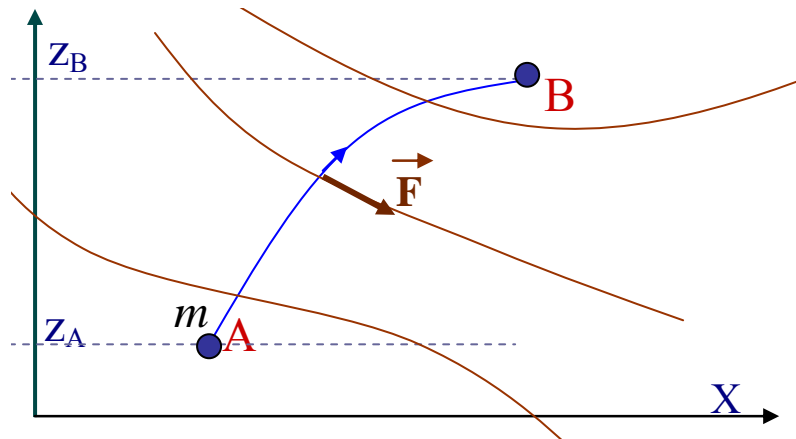
$$K_B + U_B = K_A + U_A = \text{const}$$

[Notice, had we chosen $V_B - V_A \equiv + W_{(A \rightarrow B)}$ we would have ended up with something like $K_B - V_B = K_A - V_A$]

$K + U = E$ is defined as the **mechanical energy** of the particle

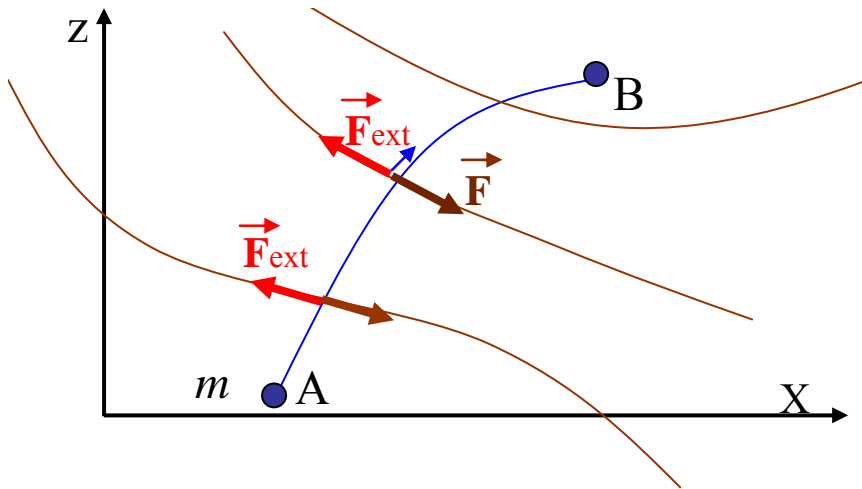
Since A and B are two arbitrary points along the trajectory of the particle, the result in the previous page indicates that the **mechanical energy** conserves through the motion

Alternative definition of the Potential Energy



The graph above shows the force lines of a conservative force \vec{F} . Let's assume that the particle of mass m moves from A to B along the indicated trajectory. As we know, the particle will pick up some kinetic energy, and some potential energy along the way.

Question: What does an external agent will have to do in order to move the particle from A to B, along the same trajectory, but at constant velocity (velocity ~ 0)?



That is

$$\vec{F}_{\text{ext}} = -\vec{F}$$

which implies $W_{\text{ext}}(A \rightarrow B) = -W(A \rightarrow B)$

Then

$$\begin{aligned} U_B - U_A &\equiv -W(A \rightarrow B) \\ &= W_{\text{ext}}(A \rightarrow B) \end{aligned}$$

That is

$$U_B - U_A \equiv W_{\text{ext}}(A \rightarrow B)$$

Work done by the **external force**

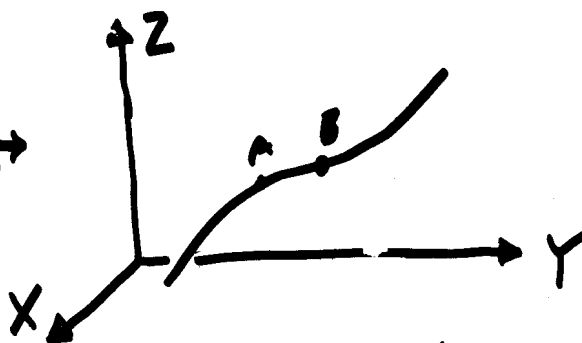
$$U_B - U_A \equiv -W(A \rightarrow B)$$

Work done by the **conservative force**

RELATIONSHIP between POTENTIAL ENERGY and FORCE

From the definition of potential energy:

$$U_B - U_A = - \int_A^B \vec{F} \cdot d\vec{r}$$



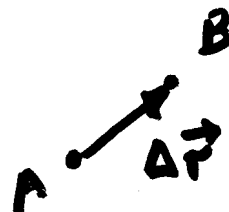
When A and B are infinitesimally close we obtain

$$U_B - U_A \longrightarrow \Delta U$$

$$\int_A^B \vec{F} \cdot d\vec{r} \longrightarrow \vec{F} \cdot \Delta \vec{r}$$

So

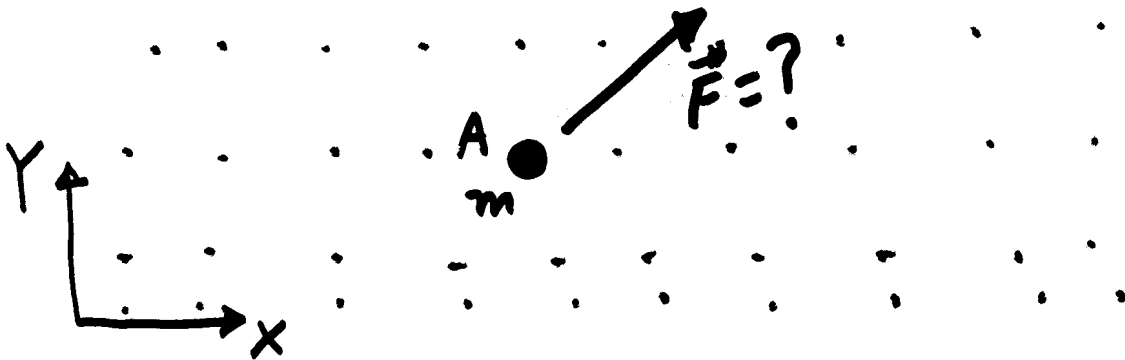
$$\boxed{\Delta U = -\vec{F} \cdot \Delta \vec{r}}$$



$$\Delta \vec{r} = \Delta x \hat{i} + \Delta y \hat{j} + \Delta z \hat{k}$$

$$\vec{F} = F_x \hat{i} + F_y \hat{j} + F_z \hat{k}$$

$$\boxed{\Delta U = -F_x \Delta x - F_y \Delta y - F_z \Delta z}$$



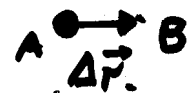
Assuming we know $U = U(\vec{r})$ at every point (x, y, z) ,
 How to find the force \vec{F} acting on the
 mass " m " located at position A ?

We'll do it component by component.

FINDING F_x

Procedure:

- Choose a horizontal path passing by A
- Find the ΔU between A and a neighbor point B
- Notice, the displacement vector from A to B is $\Delta \vec{r} = \Delta x \hat{i}$



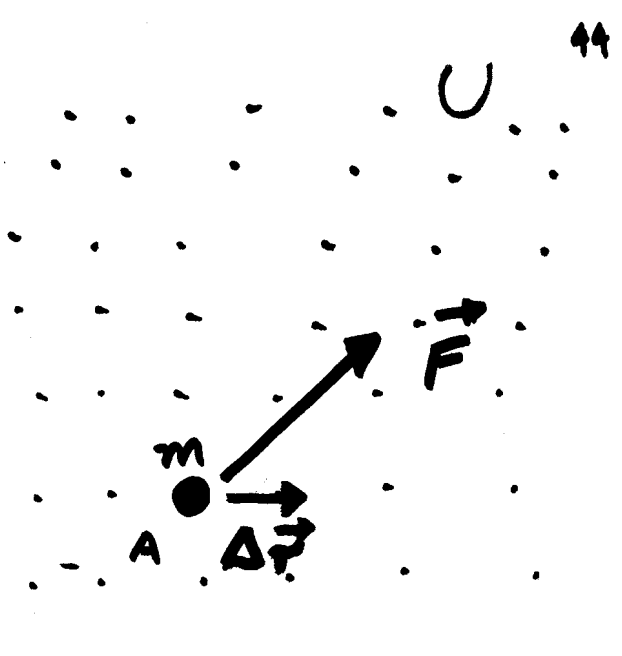
$$\Delta U = U_B - U_A$$

- $\Delta U = - \vec{F} \cdot d\vec{r}$
becomes, for this case

$$\Delta U = - \vec{F} \cdot (\Delta x \hat{i})$$
$$= - F_x \Delta x$$

or

$$F_x = - \frac{\Delta U}{\Delta x}$$



The x-component
of this force \vec{F}
is given by

Similarly, we obtain

$$F_y = - \frac{\Delta U}{\Delta y}$$

$$F_z = - \frac{\Delta U}{\Delta z}$$