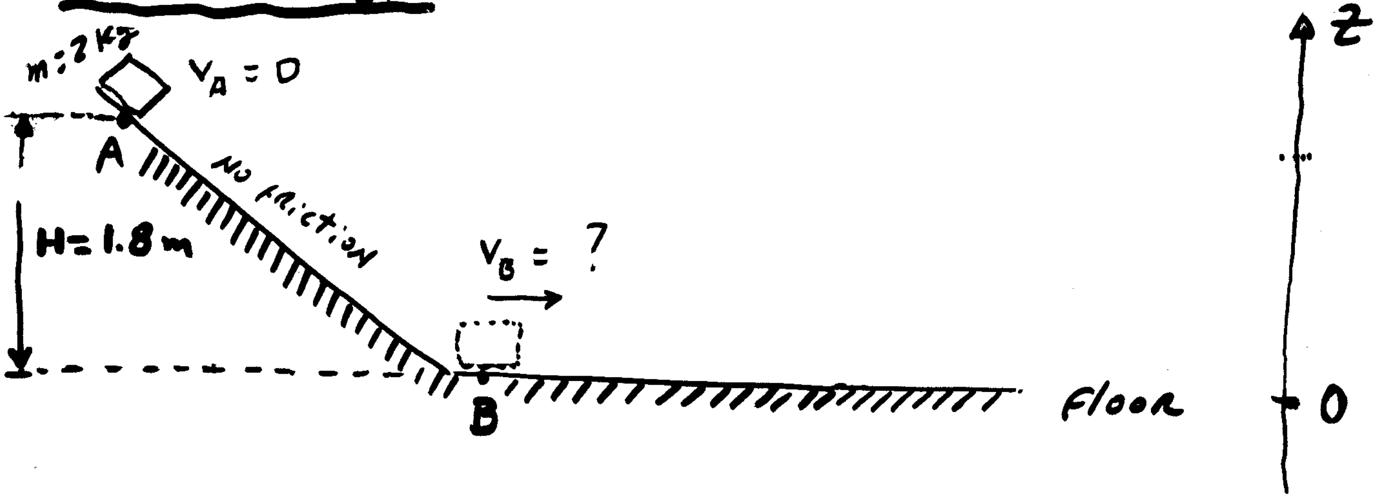
Work done by NON-CONSERVATIVE FORCES

Failure of the conservation of mechanical energy



mechanical energy at A $E_A = E_B$ mechanical energy at B

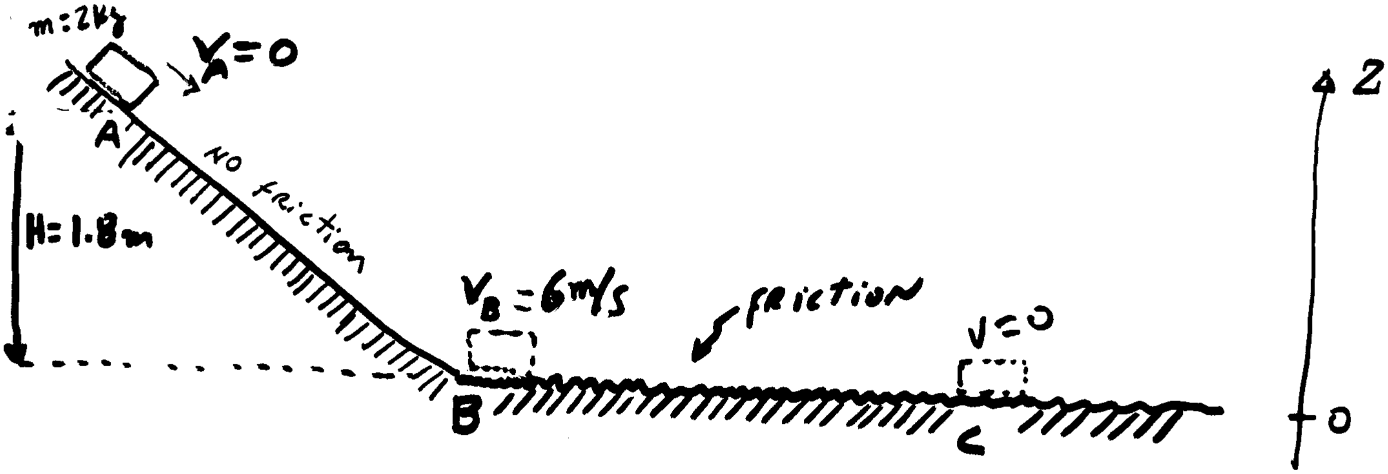
$$\frac{1}{2} m v_A^2 + m g z_A = \frac{1}{2} m v_B^2 + m g z_B$$

Notice, the expression above is valid regardless of where we choose the origin of our system of coordinates. For convenience, we choose the floor level as $z = 0$. This makes $z_A = 1.8\,m$ $z_B = 0$

$$\frac{1}{2} m v_A^2 + m g \times (1.8 \text{ meter}) = \frac{1}{2} m V_B^2 + 0$$

$$0 + (9.8\,m/s^2)(1.8\,m) = \frac{1}{2} V_B^2$$

$$\Rightarrow V_B = 6\,m/s$$



Trajectory: A ⟶ B ⟶ C

Choosing positions A and B:

$$\frac{1}{2}m\underbrace{V_A^2}_{0} + mg\underbrace{z_A}_{1.8\,m} = \frac{1}{2}m\underbrace{V_B^2}_{6\,m/s} + mg\underbrace{z_B}_{0}$$

we verify this equality holds.
i.e. mechanical energy conserves

Choosing positions B and C

$$\frac{1}{2}m\underbrace{V_B^2}_{6\,m/s} + mg\underbrace{z_B}_{0} = \frac{1}{2}m\underbrace{V_c^2}_{0} + mg\underbrace{z_c}_{0}$$

$$\frac{1}{2}\times 36\,\frac{m^2}{s^2} = 0 \quad ? \quad X$$

mechanical energy does **NOT** conserve

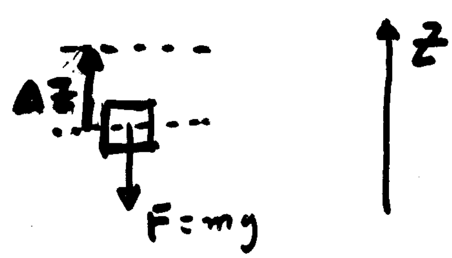
We know the reason why the mechanical energy did not conserve when the block moved from B to C: there was a non conservative force acting on the block

Whenever we see something that does not seem to be right, we go back to the basics:

$$\Delta K = W_{\text{total}}$$

$$= W_{\substack{\text{conserv}\\ \text{forces}}} + W_{\substack{\text{non conserv.}\\ \text{forces}}}$$

block moving up



$U = mgz$

$W = -F\Delta z$

$= -mg|\Delta z|$

$\Delta U = mg\Delta z$

$$W_{\substack{\text{conserv}\\ \text{force}}} = -\Delta U$$

A conservative force F always ends up with a definition of the corresponding potential energy U

$$\Delta K = W_{\substack{\text{constrv}\\ \text{forces}}} + W_{\substack{\text{non constrv.}\\ \text{forces}}}$$

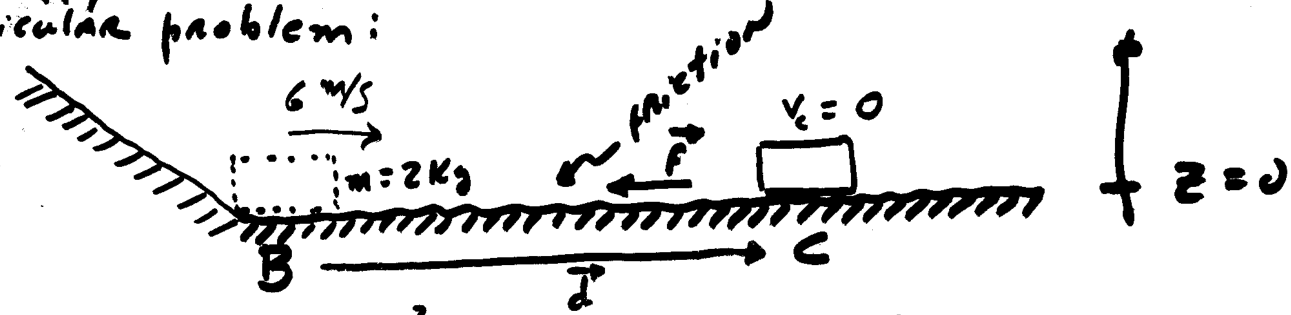
$$= -\Delta U + W_{\substack{\text{non-constrv}\\ \text{forces}}}$$

$$\Delta(\underbrace{K+U}_{\text{mechanical energy}}) = W_{\substack{\text{non-constrv}\\ \text{forces}}}$$

$$\boxed{\Delta E = W_{\substack{\text{non-conserv.}\\ \text{forces}}}}$$

the change in the mechanical energy of the block, is equal to the work done <u>on</u> the block by the non-conservative forces.

Let's apply this concept to our particular problem:



$E_B = \frac{1}{2} m v_B^2$
$= 36 J$

$E_c = 0$

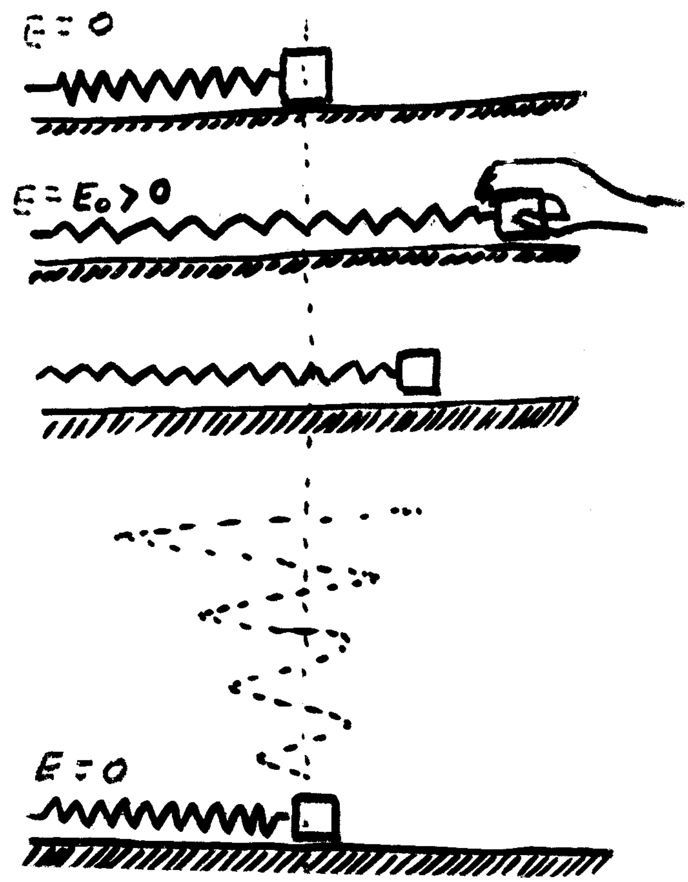
$\Delta E = E_c - E_B = 0 - 36 J = -36 J$

$W_{\text{non-constrv}} = \vec{F} \cdot \vec{d}$ = has to be equal to $-36 J$

MAIN CONCLUSION :

The mechanical energy $E = K + U$
(kinetic energy + potential energy)
does **NOT ALWAYS** remains
constant throughout the motion.
(In some cases it does, in others don't)

We are right to wonder, what ever happens to the initial mechanical energy E_0 a block may have.
Where does that energy go?



$$\underbrace{E}_{\text{mechanical energy}} = \tfrac{1}{2}mv^2 + mgz$$

We realize there exists some INTERNAL ENERGY not counted by the mechanical energy

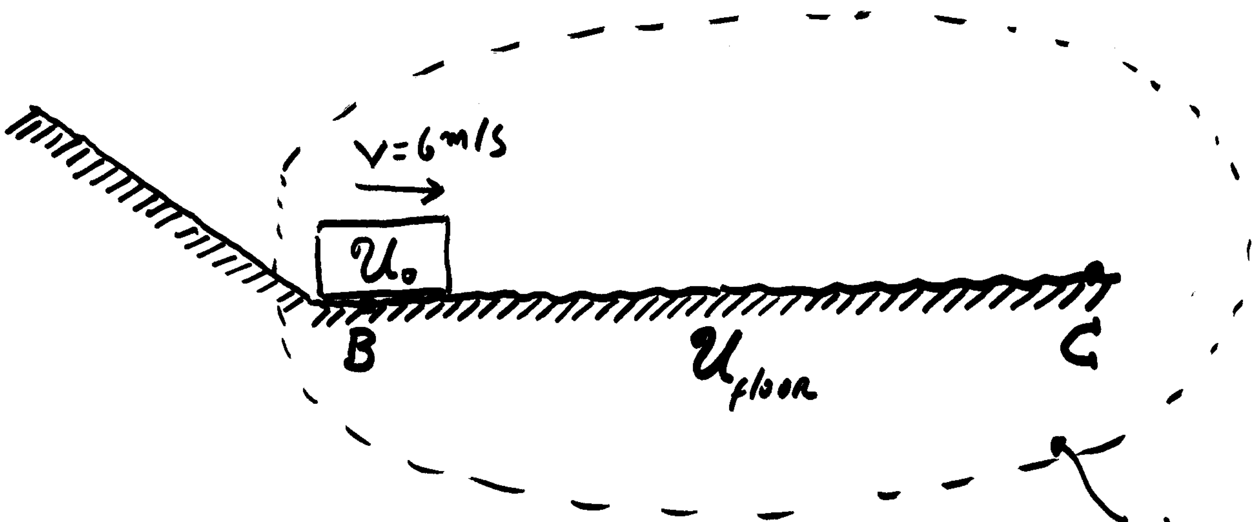
INTERNAL ENERGY $\mathcal{U}$:
electrons moving around, ions vibrating around their equilibrium position, etc.

So,

$$\boxed{\text{TOTAL ENERGY} = \underbrace{E}_{\text{mechanical energy}} + \underbrace{\mathcal{U}}_{\text{internal energy}}}$$

$\Uparrow$ MACROSCOPIC (under E)

$\Uparrow$ microscopic (under $\mathcal{U}$)



TOTAL ENERGY here must conserve

$$\left.\text{Total energy} = \tfrac{1}{2}mv^2 + \mathcal{U}_o + \mathcal{U}_{floor}\ \right\}\ \text{when the block is at position "B"}$$

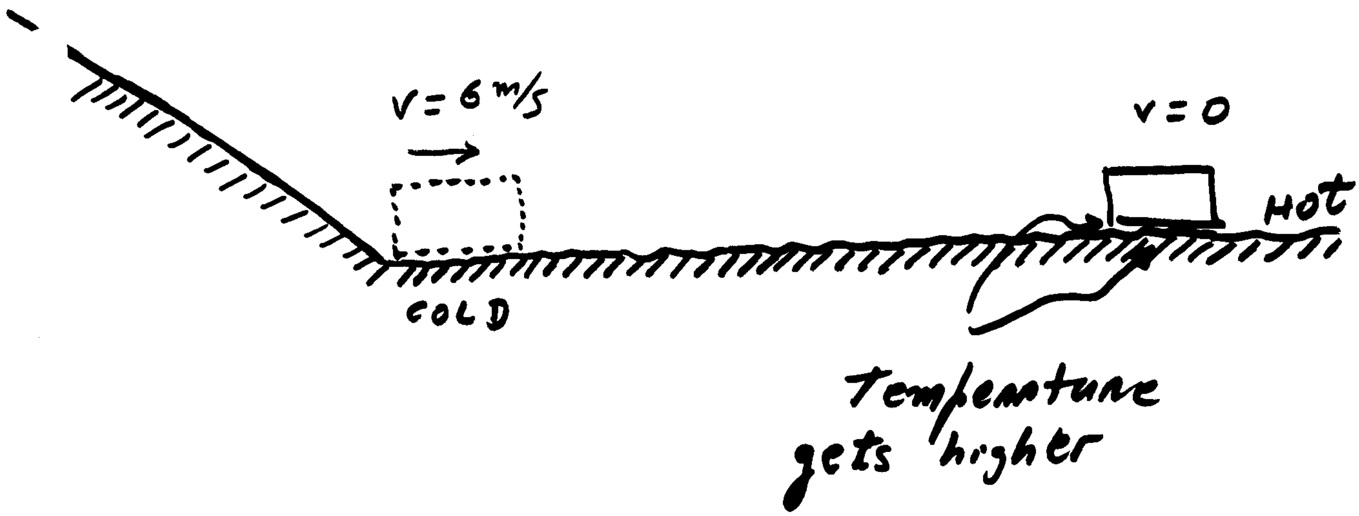
$$\left.= 0 + \mathcal{U}_o' + \mathcal{U}_{floor}'\ \right\}\ \text{when the block is at position "C"}$$

From these 2 expressions we obtain

$$\underbrace{\tfrac{1}{2}mv^2}\ = \underbrace{(\mathcal{U}_o' - \mathcal{U}_o)} + \underbrace{(\mathcal{U}_{floor}' - \mathcal{U}_{floor})}$$

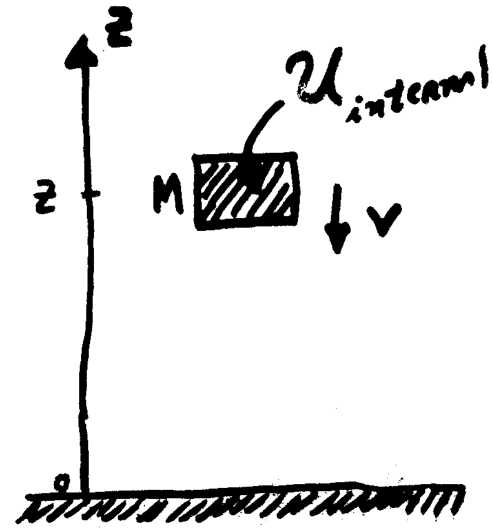
Macroscopic mechanical energy converted into internal energy of the block

As a consequence of this interchange of energy:



CONCLUSION

The mechanical energy $E = K + U$ does not always conserve (during the motion of the block)



It is the TOTAL ENERGY

$$E_{TOTAL} = E_{mechan} + \mathcal{U}_{internal}$$

that is conserved

$$E_{mech} = \frac{1}{2} M v^2 + M g z$$

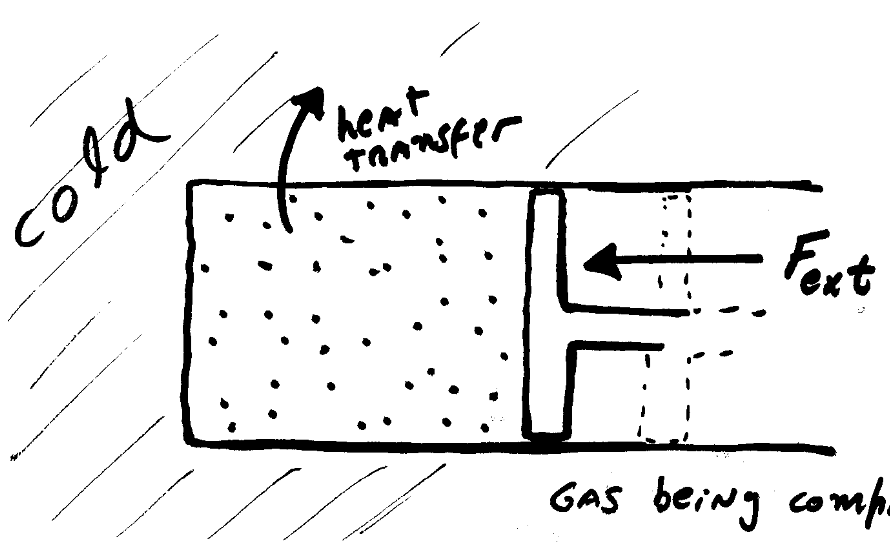
$\mathcal{U}_{internal}$ = harder to quantify

(thermodynamics)

Notice, in the previous analysis we considered the whole system block + floor.

If we were interested in making statements of conservation of total energy in the block only, we would state:

$$\underbrace{\Delta E}_{\text{mechanical energy}} + \underbrace{\Delta U}_{\text{internal energy}} = W_{\text{work done on the block}} + \underbrace{Q}_{\text{heat transfer}}$$



GAS being compressed

We can increase the TOTAL ENERGY of a system via doing WORK or via heat transfer. When the system receives energy, we can make a distinction between the (macroscopic in character) mechanical energy and the (microscopic in character) internal energy. But once we have a system with a determined total energy, we can not say wheter the system acquired its energy via work or heat transfer.

In this sense:

$E_{mechanical}$ and $U_{internal}$ are physical quantities that characterize the "state" of a system. Work and heat transfer are quantities that do not characterize the system

(we are calling "system" to the block in motion in our previous examples)

ANALOGY:

$$\Delta E_{mechan} + \Delta U_{internal} = W_{\text{done on the block}} + Q_{\text{heat transfer}}$$

checking account — SAVING account — deposits by checks — deposits by cash.

$$\underbrace{\text{checking account} \quad \text{SAVING account}}_{\text{Amount of money you have in your bank account}}$$

You can verify how much is in the checking account and how much in the saving account

But having the total you may not remember (or care) if the money got there through checks or through cash.