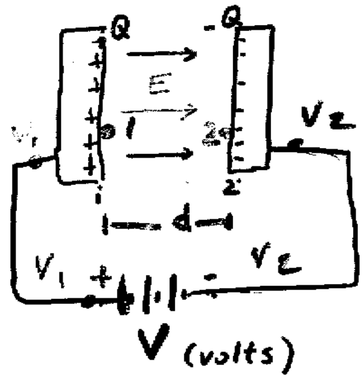
CAPACITANCE

Parallel-Plate capacitor

page 591



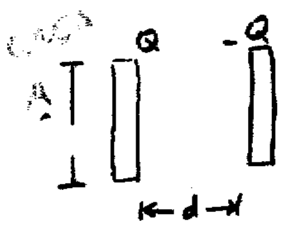
$$V_2 - V_1 = -\int_1^2 E\,dx$$

E is uniform between the parallel plates

$$= -E\int_1^2 dx = -E\,d$$

$$\boxed{V_2 - V_1 = -E\,d}$$

this expression indicates that $V_1 > V_2$



A: area of the plate

On the other hand, we already know E:

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q/A}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

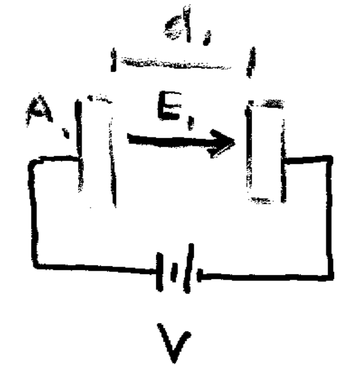
$$\underbrace{V_1 - V_2}_{V\text{ (battery)}} = E\,d$$

$$V = E\,d = \frac{Q}{\epsilon_0 A}\,d$$

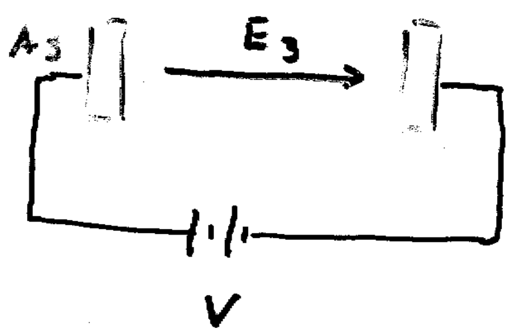
$$\boxed{\underbrace{\frac{\epsilon_0 A}{d}}_{\text{capacitance}} V = Q}$$

$Q = C\,V$

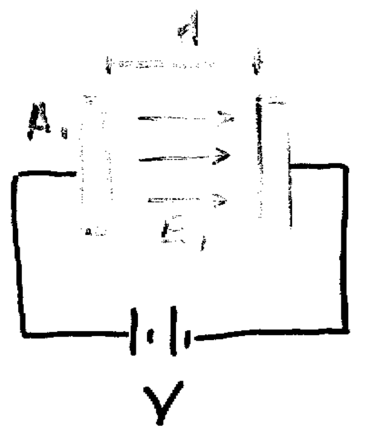
capacitance



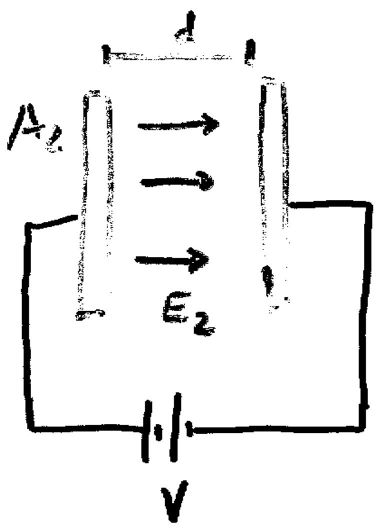
E_1 E_3



$\boxed{A_1 = A_3}$



E_1 E_2



$\boxed{A_2 > A_1}$

$$Q = C\,V$$

storaged charge — capacitance (value depends on the geometry of the electrodes) — electric potential difference

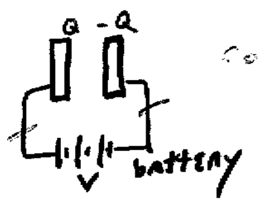
For a parallel-plate capacitor: $\boxed{C = \dfrac{\epsilon_0 A}{d}}$

<u>unit</u> $1\ \text{farad} = 1\text{F} = \dfrac{\text{coulomb}}{\text{volt}}$

We use capacitors to store charge:

step 1  Each plate is called "an electrode"

frontal view of 2 parallel plates

step 2 

connection to a battery

step 3 

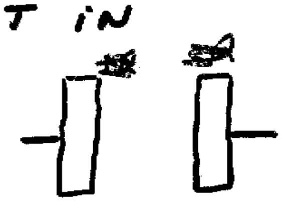
after removing the battery, plates remained charged.

Notice: the higher the <u>voltage</u>, the more <u>charge</u> is deposited in the electrodes. $Q = CV$

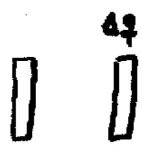
* Exercise: Problems 2E, 5E, page 608
Chapter 26

capacitors also store energy (electrical energy)

The question we are going to answer is
"HOW MUCH ENERGY IS SPENT IN CHARGING A CAPACITOR"

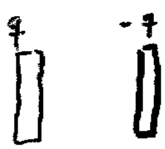


Step 1



unchanged plates
$C = \epsilon_0 A/d$

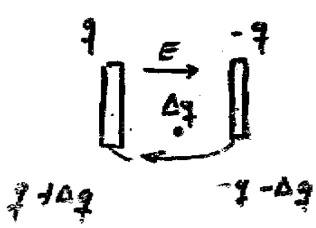
step 2



Assume, at a given time, an amount of charge q has been transferred from the right plate to the left one.

the potential difference between the plates is $V = \frac{q}{C}$

step 3



$\Delta W_{ext} = F_{ext} \cdot d = \underbrace{(\Delta q)E}\, d = (\Delta q)\, V$

{ external work required to transport a small charge Δq from the right plate to the left plate.

$q = CV$

$$\Delta W_{ext} = V\,\Delta q = \frac{q}{C}\,\Delta q$$

$$W_{ext} = \int_0^Q \frac{q}{C}\,dq = \frac{1}{C}\,\frac{q^2}{2}\Bigg|_0^Q$$

$$\boxed{W_{ext} = \frac{1}{2}\,\frac{Q^2}{C}}$$

EXTERNAL ENERGY required to charge a capacitor.

And, since $Q = CV$

$$\boxed{W_{ext} = \frac{1}{2} C V^2}$$

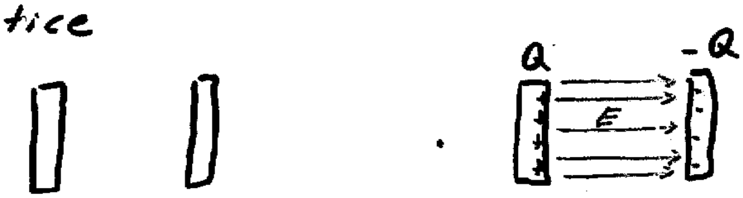
More common notation

$$\boxed{U = \frac{1}{2}\,\frac{Q^2}{C} = \frac{1}{2} C V^2}$$

U : POTENTIAL Energy stored in the capacitor.

QUESTION: Where is this energy U located?

Notice



before

after

We may say that the energy U is distributed inside the volume of the plate capacitor.

We can further say that the energy U is stored in the electric field E

Some hints :

$$U = \frac{1}{2} C V^2 = \frac{1}{2} C (E d)^2 = \frac{1}{2} \epsilon_0 \frac{A}{d} \times (E d)^2$$

$$= \frac{1}{2} \epsilon_0 \underline{Ad}\, E^2$$

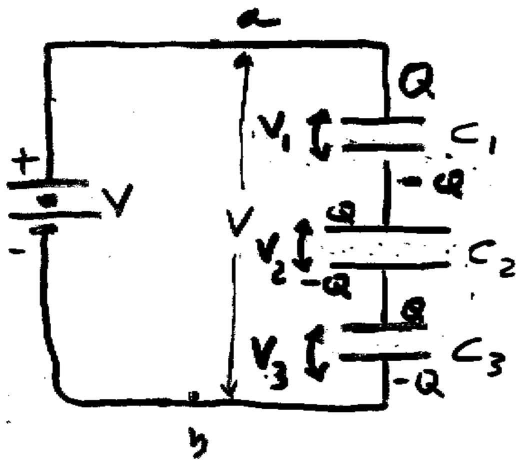
$$\frac{U}{\text{volume}} = \frac{U}{Ad} = \frac{1}{2} \epsilon_0 E^2 \; !$$

wherever there exist an electric field E, there exist a <u>potential energy density</u> $\underline{u}$

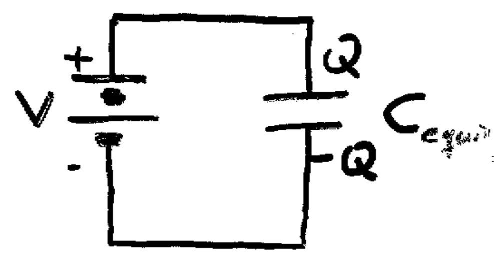
$$\boxed{u = \frac{1}{2} \epsilon_0 E^2}$$ potential energy per unit volume

"the electrical potential energy stored in the capacitor is considered to be stored in the electric field"

SEVERAL CAPACITORS CONNECTED IN SERIES :-



Equivalent to $\longrightarrow$



$C_{equiv} = ?$

$$Q = C_{equiv} V$$

$$V = V_1 + V_2 + V_3$$

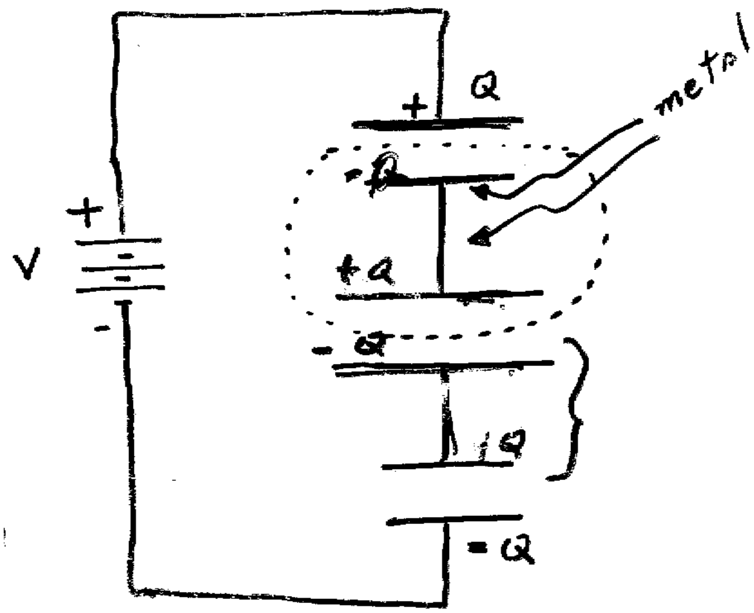
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}$$

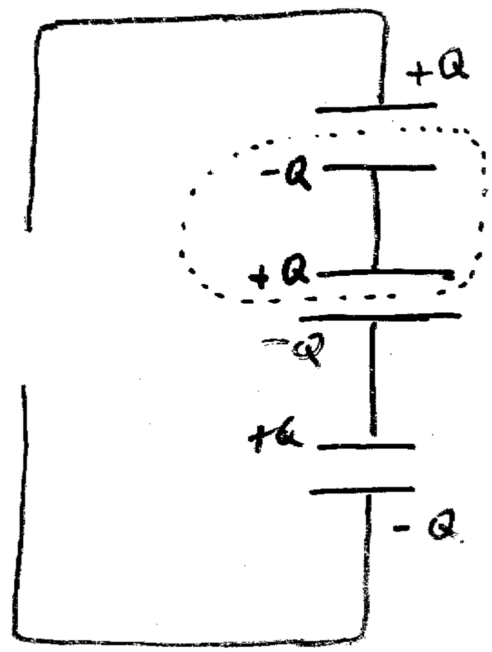
$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)$$

$$\Rightarrow \underbrace{\frac{1}{\left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)}}_{C_{equiv}} V = Q$$

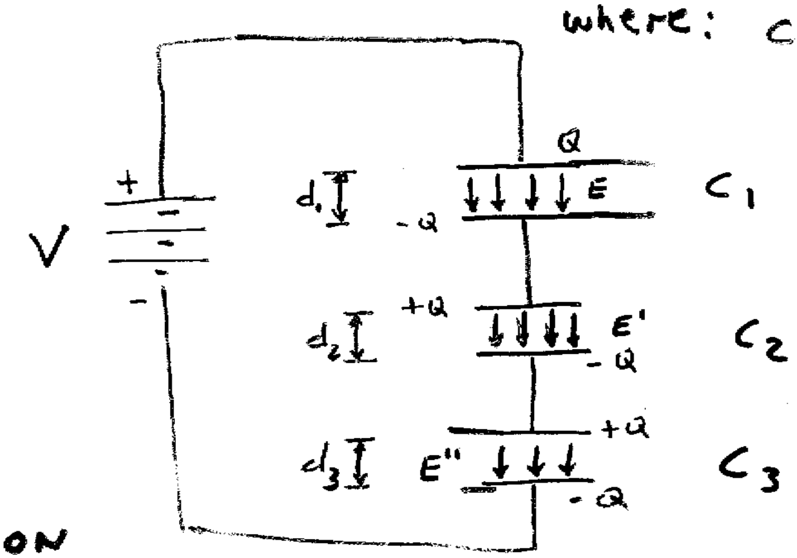
$$C_{equiv} = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\right)^{-1}$$

$$\boxed{\frac{1}{C_{equiv}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}}$$





where: $C_1 \neq C_2 \neq C_3$



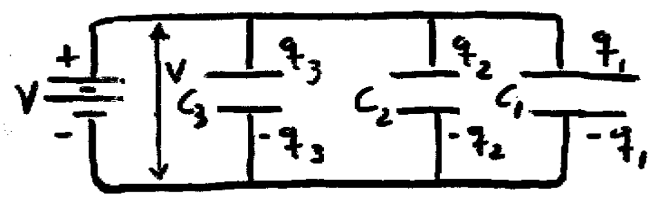
QUESTION

IS $E = E'$?

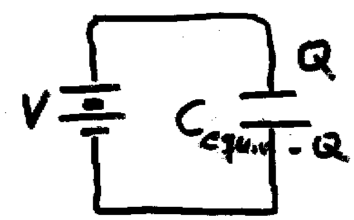
$E' = E''$?

SEVERAL CAPACITORS CONNECTED IN PARALLEL.-

page 594



Equivalent to



$q_3 = C_3 V$ $\quad$ $q_2 = C_2 V$ $\quad$ $q_1 = C_1 V$

$Q = C_{equiv} V$

For this circuit to be equivalent to the one in the left requires that

$$Q = q_1 + q_2 + q_3$$

Therefore:

$$Q = q_1 + q_2 + q_3 = C_1 V + C_2 V + C_3 V$$

On the other hand $= V(C_1 + C_2 + C_3)$

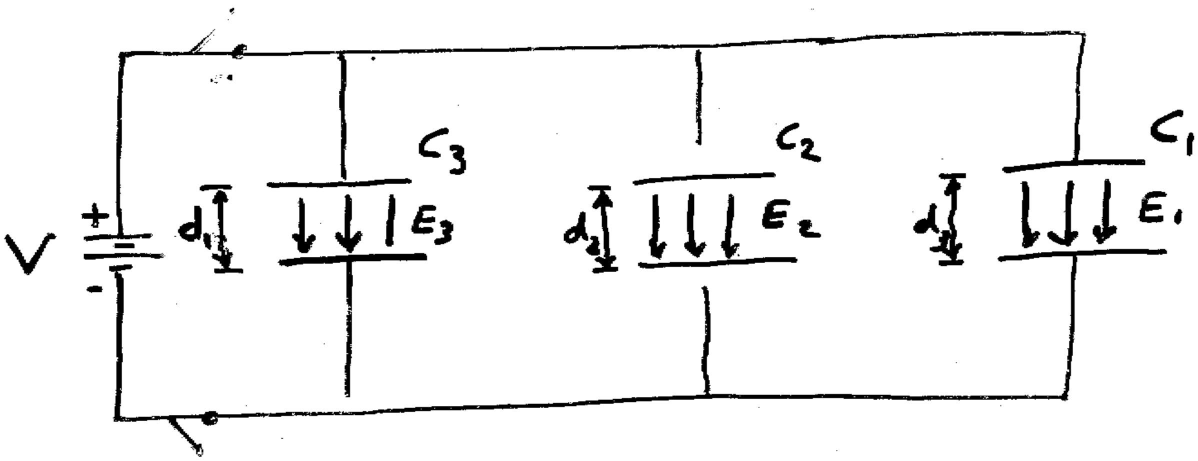
$Q = C_{equ} V$

Which implies

$$\boxed{C_{equiv} = C_1 + C_2 + C_3}$$

C is the equivalent capacitance

10



QUESTION: In the particular case that $d_1 = d_2 = d_3$, what can we say about E_1, E_2 and E_3?

$E_3 = E_2$?

$E_2 = E_1$?

(assume $C_1 \neq C_2 \neq C_3$)

SEVERAL CAPACITOR CONNECTED IN SERIES and PARALLEL.-

Sample Problem 26-2.
(we'll do it on the blackboard)

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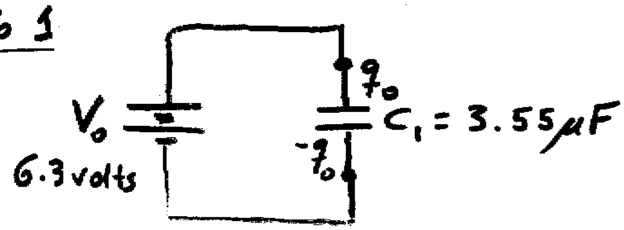
* Sample problem 26-3, page 597

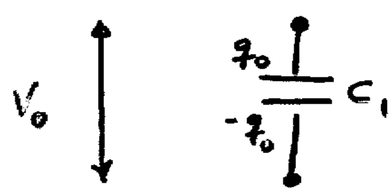
Step 1



find how much charge is stored in the capacitor

$q_0 = ? = C_1 V_0$

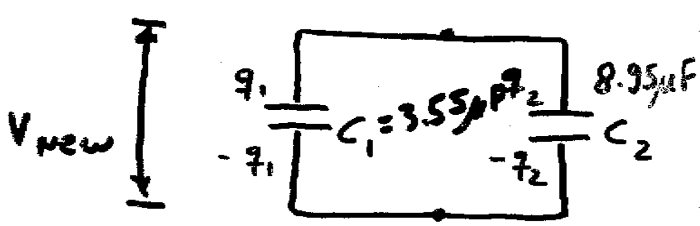
Step 2 Battery removed



What is the potential difference across the capacitor

$V_0 = ?$ 6.3 Volt

STEP 3



What is the potential difference across the capacitors?

charge conservation indicates

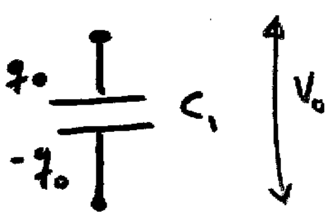
$$\underbrace{q_0}_{\text{before}} = \underbrace{q_1 + q_2}_{\text{after}}$$

$$C_1 V_0 = C_1 V_{new} + C_2 V_{new}$$

$$\boxed{\frac{C_1 V_0}{C_1 + C_2} = V_{new}}$$

$V_{new} = 1.79\ V$

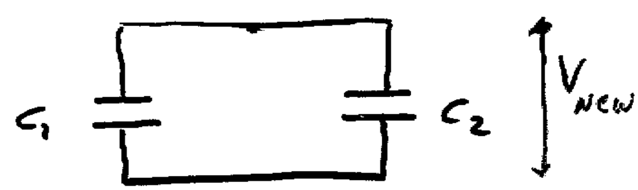
From step 2



POTENTIAL ENERGY STORED

$U_i =$

step 3



POTENTIAL ENERGY STORED

$U_f =$

Q: Is there a violation of the conservation of energy in this example?

$$U_i = \frac{1}{2}\frac{q_0^2}{C_1} = \frac{1}{2}\frac{(22.4\,\mu C)^2}{3.55\,\mu F} = 70.45\,\mu J$$

$$U_f = \frac{1}{2}\frac{q_1^2}{C_1} + \frac{1}{2}\frac{q_2^2}{C_2}$$

$$= \frac{1}{2}\frac{(C_1 V_{new})^2}{C_1} + \frac{1}{2}\frac{(C_2 V_{new})^2}{C_2}$$

$$= \frac{1}{2}\frac{(6.35\,\mu C)^2}{C_1} + \frac{1}{2}\frac{(16\,\mu C)^2}{C_2}$$

$$= 5.69\,\mu J + 14.34\,\mu J$$

Notice

$$U_f < U_i$$

final initial

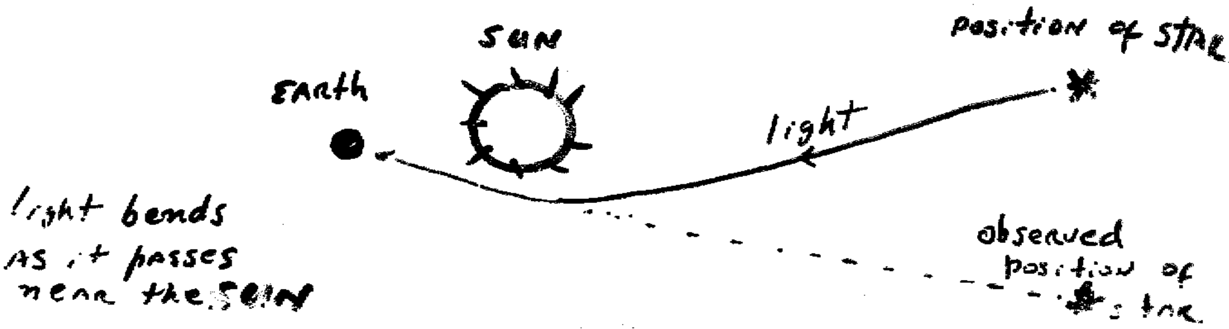
Extra curricular reading:

- Feynman, "Lectures on Physics", vol II
  "The ambiguity of the field energy"
  page 27-6

  - No unique way to define u
    neither where "exactly" is located.

  - Posible solution: theory of gravitation

    Theory of gravity says:
    "all energy is source of
    gravitational attraction"
    therefore, this theory will require that
    the energy density of electricity
    must be located properly is we
    are to know in which direction
    the gravity force acts.



Extra curricular reading:

- Feynman Lectures, Vol-II, chapter 28
  "ELECTROMAGNETIC MASS"

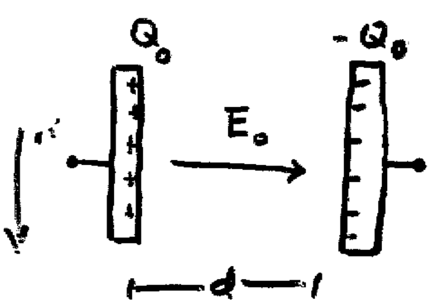
- Also, pages 19-11 to 19-14.
  (about energy inside a capacitor)

* EXERCISE : Problems 23E and 24E, chapter 26, page 609.

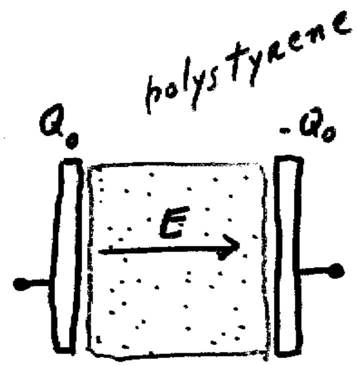
* Reading assignment : sample problem 26-2
" " 26-3

* Practice: Questions 4 and 9 (page 607)

CAPACITORS WITH A DIELECTRIC.-



$\Rightarrow$ inserting a dielectric



$$C_0 = \frac{\epsilon_0 A}{d}$$

$$V_0 = E_0 d$$

(capacitor was initially charged using battery. Then the battery was removed.)

Experimentally it is found that <u>the potential difference</u> between the plates <u>decreases</u>

$$\boxed{V = \frac{V_0}{K}}$$

K : dielectric constant (material property)

- what is the new electric field E?

Since $V = E d$ we obtain

$$\frac{V_0}{K} = E d$$

$$E_0 \quad \frac{V_0}{K d} = E \Rightarrow \boxed{E = \frac{E_0}{K}}$$

- what is the new capacitance

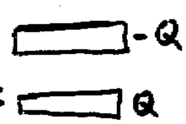
$$C = \frac{\text{charge}}{\text{voltage}}$$

$$= \frac{Q_0}{V_0/K} = K \frac{Q_0}{V_0} \qquad \boxed{C = K C_0}$$

* Discussion of sample problem 26-5 ; page 602

$: 13.5\ pF$

$= 13.5 \times 10^{-12} F$



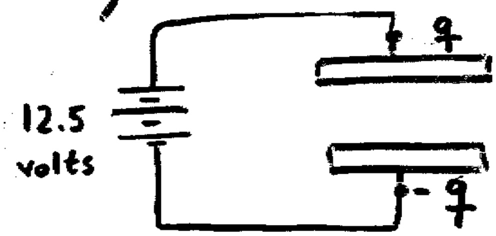
$K = 6.5$

$V = 12.5$ volts.

$$U_i = \frac{1}{2}\frac{Q^2}{C} = 1100\ pJ \qquad U_f = \frac{1}{2}\frac{Q^2}{KC} = 160\ pJ$$

Example : Discussion of sample problem 26-5
(p. 602)

a)

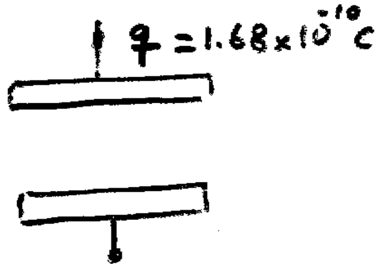


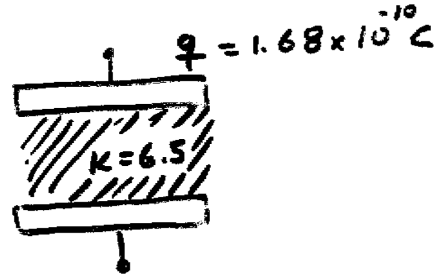
$C = 13.5\ pF$

A parallel-plate capacitor is INITIALLY connected to a battery

$$q = CV = 13.5 \times 10^{-12}\, F \times 12.5 \text{ volts} = 1.68 \times 10^{-10}\, C$$

b) The battery is disconnected and a dielecteic $(K = 6.5)$ is slipped between the plates





What are the energies stored is the capacitor before and after introducing the dielectric ?

BEFORE

$$u_i = \frac{1}{2}\frac{q^2}{C} = \frac{1}{2}\frac{(1.68 \times 10^{-10} C)^2}{13.5 \times 10^{-12} F}$$

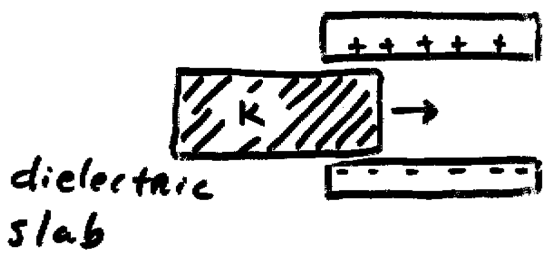
$$= 1055 \times 10^{-12}\, J$$

AFTER

$$u_f = \frac{1}{2}\frac{q^2}{KC}$$

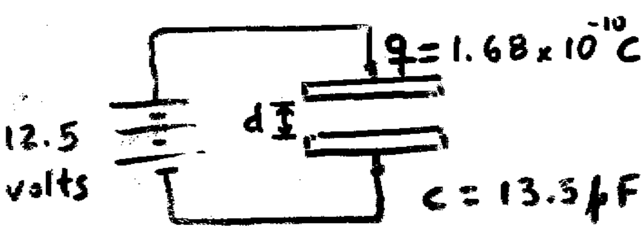
$$= 162 \times 10^{-12}\, J$$

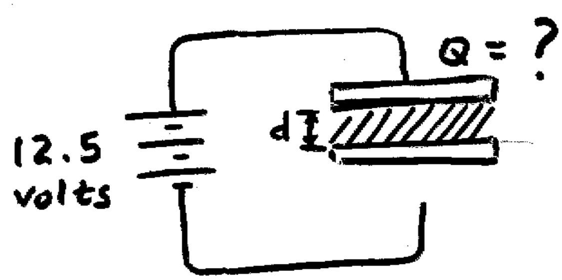
Notice, the electrical energy DECREASES by a factor K = 6.5 after the dielectric is introduced.



" the capacitor exerts a tiny tug on the slab "

Example: consider the previous problem but keeping the battery CONNECTED

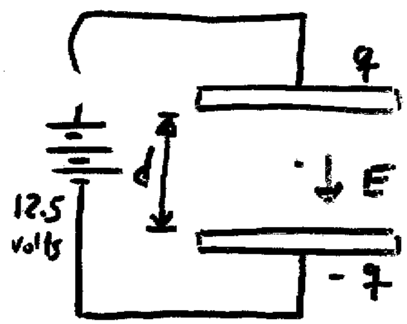


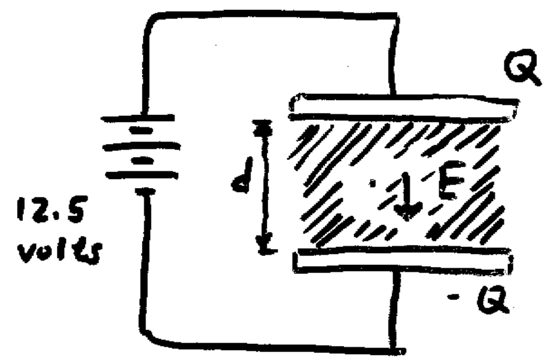


Is $Q = q$

$Q < q$?

$Q > q$





12.5 volts = E d   12.5 volts = E d

So, the electric field has to be the same in both cases.

We also know a relationship between the electric field and the amount of charge in the plates

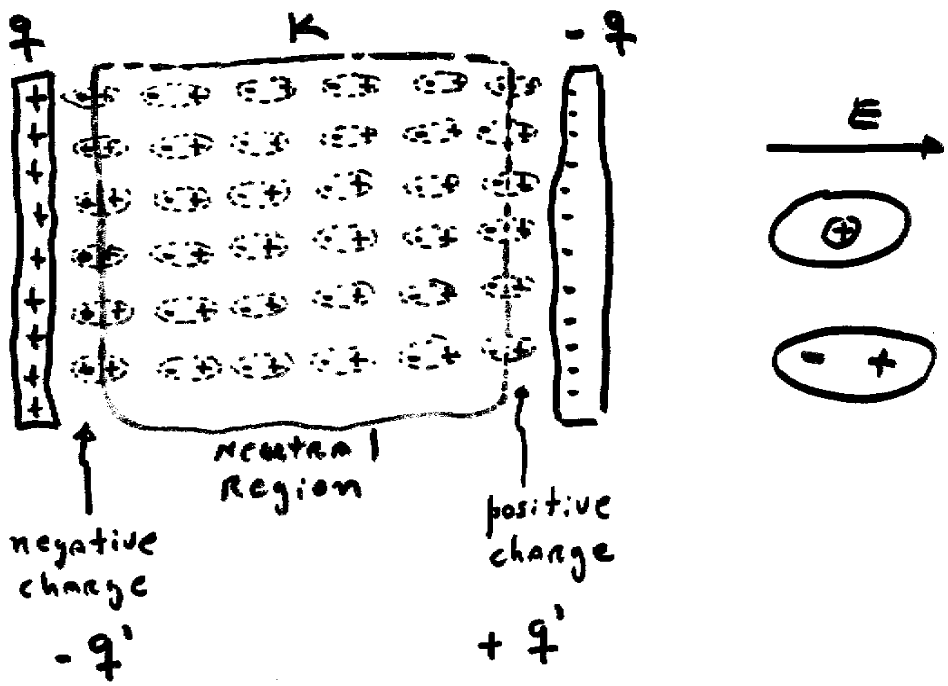
$$E = \frac{q/A}{\epsilon_0} \qquad\qquad E = \frac{Q/A}{K\,\epsilon_0}$$

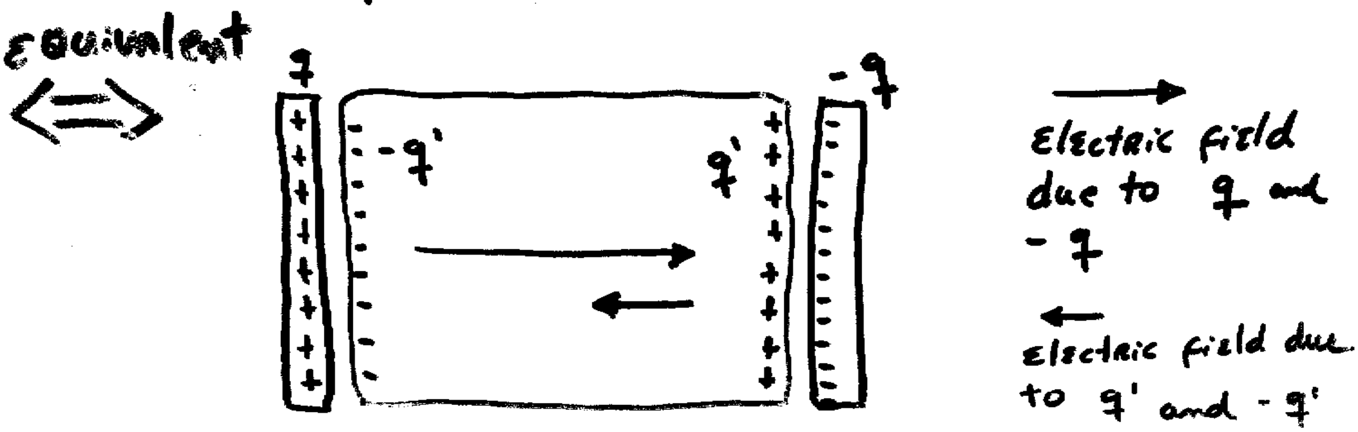
therefore:

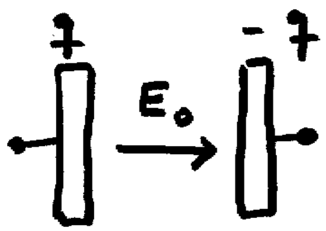
$$q = \frac{Q}{K}$$

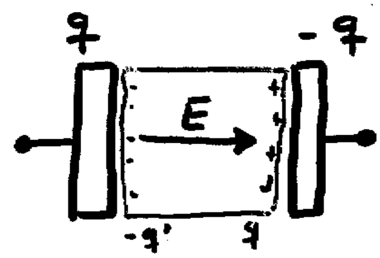
or $\boxed{Q = K\,q}$

DIELECTRICS: a microscopic view



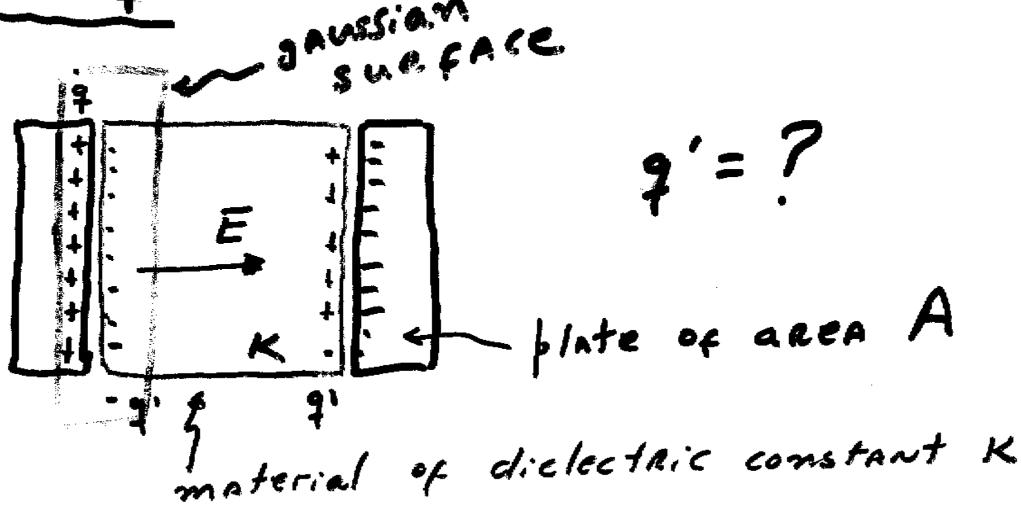






$$\underrightarrow{E} = \underrightarrow{E_0} + \leftarrow$$

Let's use Gauss' law to find q'



$q' = ?$

$$\epsilon_0 \int \vec{E} \cdot d\vec{s} = \text{charge inside} = q - q' \qquad (1)$$

$$\epsilon_0 E A = q - q'$$

electric field

$$\epsilon_0 \frac{E_0}{K} A = q - q' \qquad (2)$$

But, let's remember that

$$E_0 = \frac{\sigma}{\epsilon_0} = \frac{q}{A\,\epsilon_0} \quad (3)$$

value of the electric field without dielectric inside the plates.

So, replacing (3) in (2) we obtain

$$\frac{q}{K} = q - q' \qquad (4)$$

Or

$$\boxed{q' = \frac{K-1}{K}\, q} \qquad (5)$$

bound charge

free charge

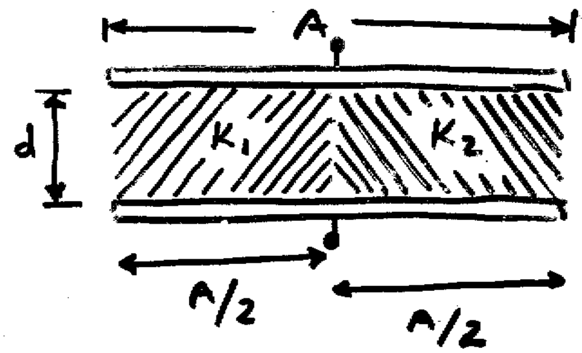
Notice also that, replacing (4) in (1) we obtain

$$\boxed{\int_S \epsilon_0 K E \cdot d\vec{S} = q}$$

GAUSS' LAW WITH DIELECTRIC

free charge only

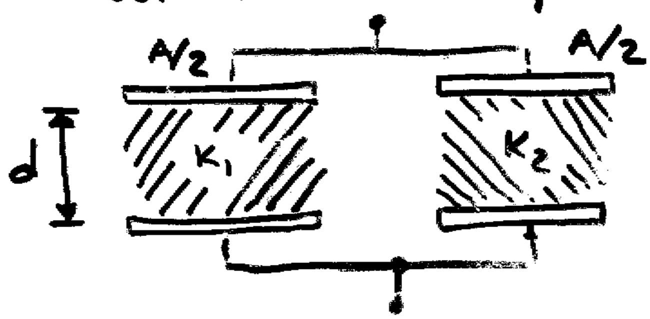
Example



Parallel-plate capacitor filled with 2 dielectrics

Find the capacitance of the device.

Solution: the device looks like two capacitors connected in parallel.



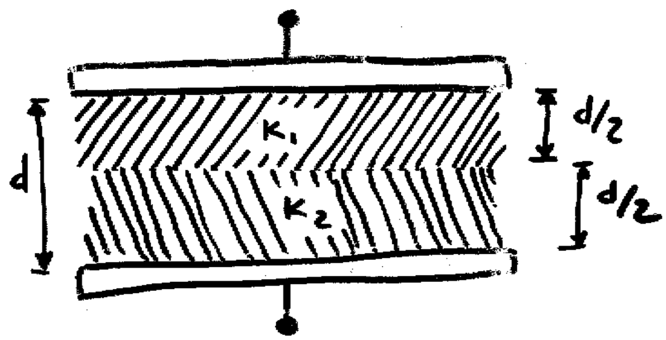
$$C_1 = K_1 \frac{(A/2)\,\epsilon_0}{d} \qquad C_2 = K_2 \frac{(A/2)\,\epsilon_0}{d}$$

$C = C_1 + C_2$ (for parallel connection)

$$= \frac{(A/2)\epsilon_0}{d}(K_1 + K_2)$$

$$\boxed{C = \frac{A\epsilon_0}{d} \cdot \frac{K_1 + K_2}{2}}$$

Example

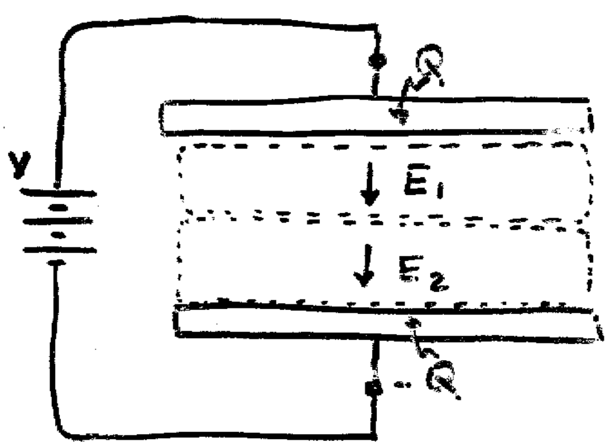


Parallel-plate capacitors filled with 2 dielectrics.

Find the capacitance of the device

Solution: It is a little less obvious in this case that we have the equivalent of 2 capacitors connected in series.

So, let's follow a more secure method to solve this problem.



$$V = E_1 \cdot \frac{d}{2} + E_2 \cdot \frac{d}{2}$$

$$= \frac{E_0}{K_1} \frac{d}{2} + \frac{E_0}{K_2} \cdot \frac{d}{2}$$

$$= \frac{E_0 d}{2} \left(\frac{1}{K_1} + \frac{1}{K_2} \right)$$

but $E_0 = \frac{\sigma}{\epsilon_0} = \frac{Q}{A\epsilon_0}$

$$= \frac{Q d}{A \epsilon_0 2} \left(\frac{1}{K_1} + \frac{1}{K_2} \right)$$

$$Q = \underbrace{\frac{A\epsilon_0}{d}\,\frac{2k_1k_2}{k_1k_2}}_{C} V$$

Example Find the charge distribution in the plates.

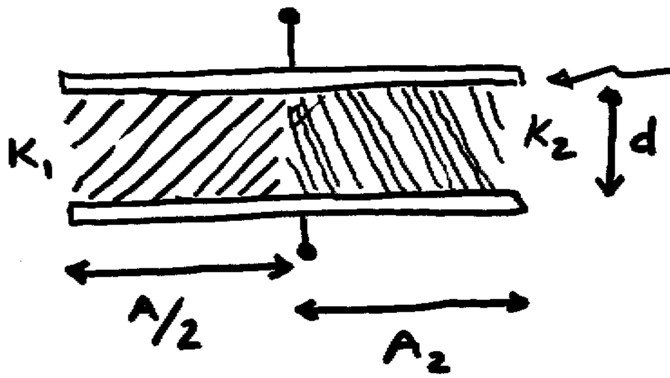
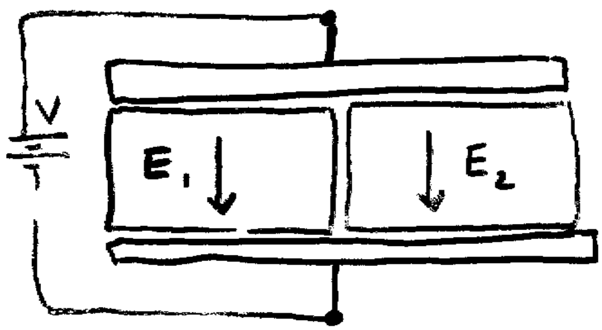


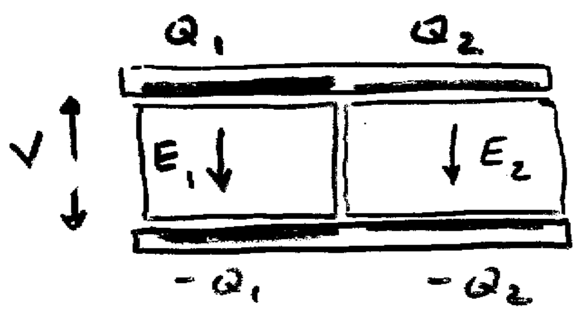
plate of area A

Find the capacitance.



$$\left.\begin{array}{l} V = E_1\, d \\ V = E_2\, d \end{array}\right\} \Rightarrow E_1 = E_2$$

Since $K_1 \neq K_2$ and $E_1 = E_2$ then we suspect that the charge distribution is not uniform over the plates

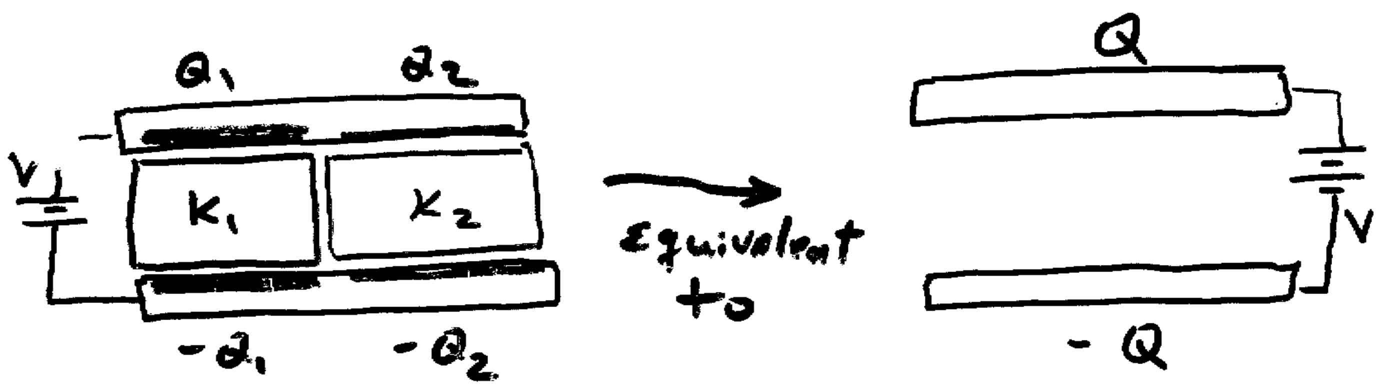


What is the relationship between the charges?

$$E_1 = \frac{\text{Field without dielectric}}{K_1} = \frac{\frac{Q_1}{(A/2)\epsilon_0}}{K_1} = \frac{2Q_1}{A\epsilon_0 K_1}$$

$$E_2 = \frac{\text{field without dielectric}}{K_2} = \frac{2Q_2}{A\epsilon_0 K_2}$$

$$E_1 = E_2 \Rightarrow \quad \frac{Q_1}{K_1} = \frac{Q_2}{K_2} \qquad (1)$$



$$Q = CV$$

$$Q = (Q_1 + Q_2) = Q_1 + \frac{K_2}{K_1} Q_1$$

$$= Q_1 \left(1 + \frac{K_2}{K_1}\right) \qquad (2)$$

But $Q_1 = C_1 V$ where $C_1 = \frac{(A/2)\epsilon_0}{d} K_1$.

$$Q_1 = \frac{A\epsilon_0}{2d} K_1 V \quad (3)$$

So, replacing (3) in (2) we obtain

$$Q = \underbrace{\frac{A\epsilon_0 K_1}{2d}\left(1 + \frac{K_2}{K_1}\right)} V$$

$$C = \frac{A\epsilon_0}{2d}(K_1 + K_2)$$

Ground: A very large conductor that can supply an unlimited amount of charge

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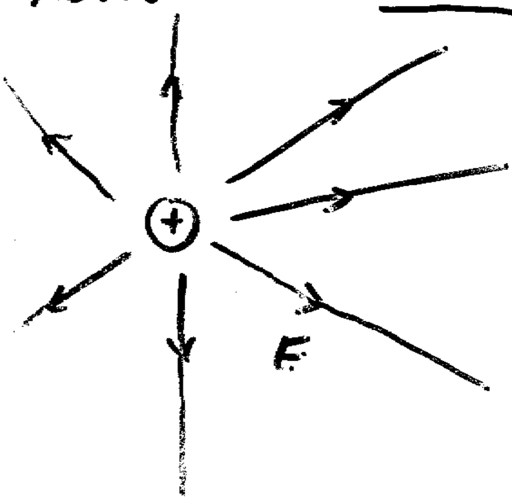
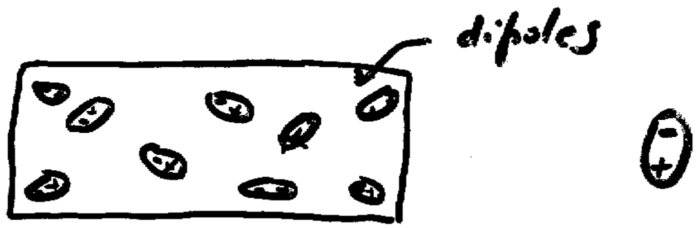
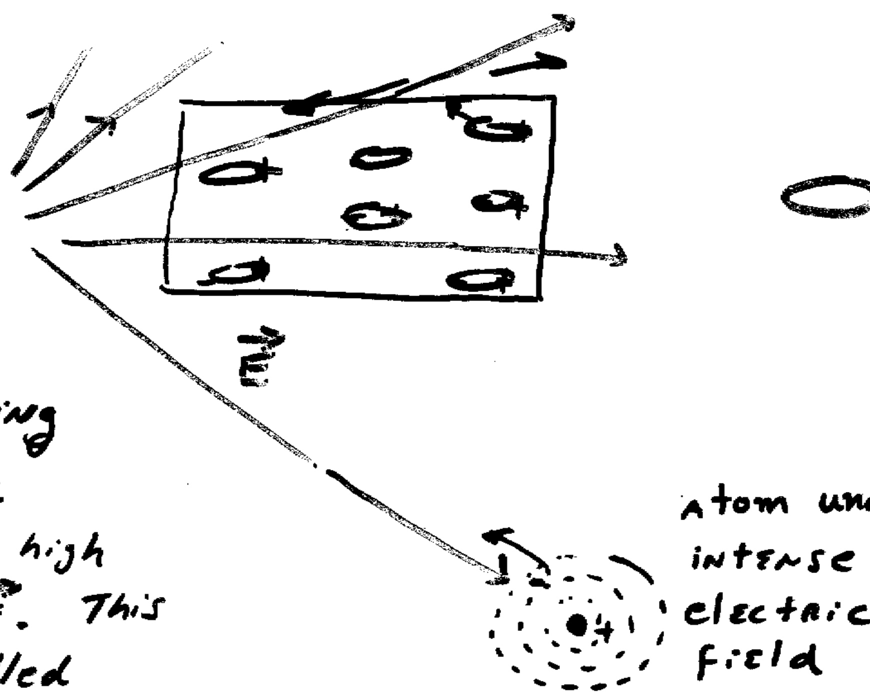
Review on <u>DIELECTRIC BREAKDOWN</u>: sample problem 23-6, p. 565





Dielectric material
(air, for example)



Atom under intense electric field

Many nonconducting materials become ionized in very high electric fields $\vec{E}$. This phenomenon is called "ELECTRIC BREAKDOWN.

Reading assignment:

- Ink-Jet Printing, page 533
- sample problem 23-8, page 568

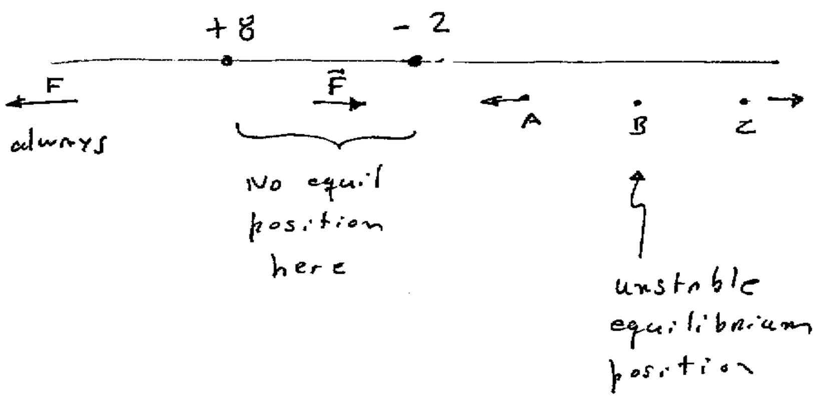
! - what is "electrical breakdown"?, read sample - problem 23-6, page 565

! - Problem 37 P

PRACTICE PROBLEMS

(1) CH-22 QUESTION #3 (p. 517) Ans. a) & b)

(2) Sample problem 22-2 (p. 512)

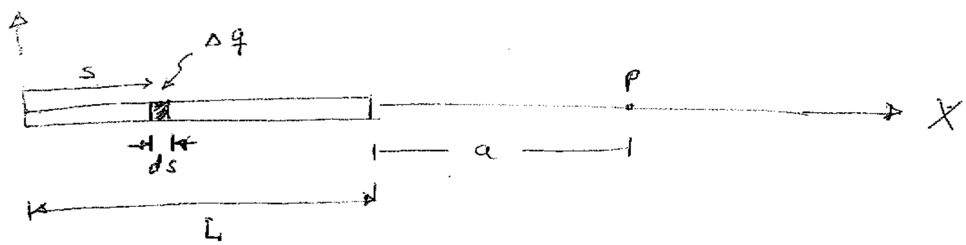


(3) Sample problem 22-1 (p. 511)

Part b) Highlight that eventhough particle 3 is in between, still $F_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2}$

(6) Sample problem 23-3 (p. 529)

Problem 23P (Chapter - 23)



a) $\Delta q = \left(-\frac{q}{L}\right)\Delta s$

b) $\Delta E = \frac{1}{4\pi\epsilon_0}\frac{(\Delta q)}{(L+a-s)^2} = \frac{1}{4\pi\epsilon_0}\frac{\left(-\frac{q}{L}\right)\Delta s}{(L+a-s)^2}$

$$E = -\frac{1}{4\pi\epsilon_0}\frac{q}{L}\underbrace{\int\frac{ds}{(L+a-s)^2}} = -\frac{1}{4\pi\epsilon_0}\frac{q}{L}\left.\frac{1}{(L+a-s)}\right|_0^L$$

NOTICE

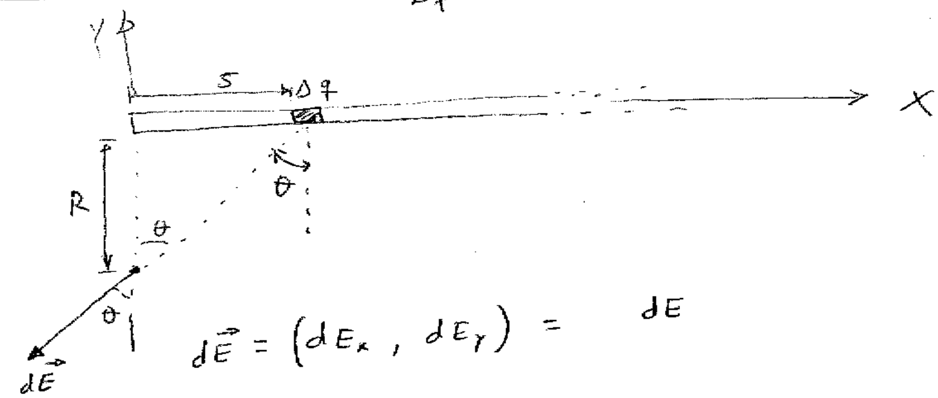
$$\frac{d}{ds}\left\{\frac{1}{(L+a-s)}\right\} = (-1)\frac{1}{(L+a-s)^2}(-1) = \frac{1}{(L+a-s)^2}$$

$$= -\frac{1}{4\pi\epsilon_0}\frac{q}{L}\left[\frac{1}{a} - \frac{1}{L+a}\right] = -\frac{q}{4\pi\epsilon_0}\frac{1}{a(L+a)}$$

c) $E = -\frac{q}{4\pi\epsilon_0}\frac{1}{a(L+a)} \xrightarrow[a\to\infty]{} -\frac{q}{4\pi\epsilon_0}\frac{1}{a^2}$

Problem #25 CH-23

$\Delta q = \lambda\, ds$



$$d\vec{E} = (dE_x,\ dE_y) = dE$$

- $$dE_x = -dE \sin\theta = -\left(\frac{1}{4\pi\epsilon_0}\frac{\Delta q}{R^2+s^2}\right)\frac{s}{(R^2+s^2)^{1/2}}$$

$$E_x = -\frac{1}{4\pi\epsilon_0}\int \frac{(\Delta q)\, s}{(R^2+s^2)^{3/2}} = -\frac{\lambda}{4\pi\epsilon_0}\int_0^\infty \frac{s\, ds}{(R^2+s^2)^{3/2}}$$

Notice: $$\frac{d}{ds}\left(\frac{-1}{(R^2+s^2)^{1/2}}\right) = -1\left(-\frac{1}{2}\right)(R^2+s^2)^{-3/2}\, 2s = \frac{s}{(R^2+s^2)^{3/2}}$$

$$E_x = +\frac{\lambda}{4\pi\epsilon_0}\frac{1}{(R^2+s^2)^{1/2}}\Bigg|_0^\infty = \boxed{-\frac{\lambda}{4\pi\epsilon_0}\frac{1}{R} = E_x}$$

- $$dE_y = -dE\cos\theta = -\left(\frac{1}{4\pi\epsilon_0}\frac{\Delta q}{R^2+s^2}\right)\frac{R}{(R^2+s^2)^{1/2}}$$

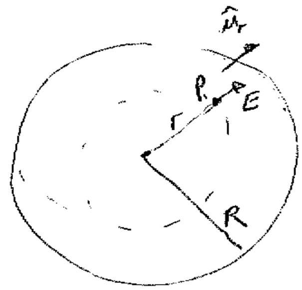
$$E_y = -\frac{1}{4\pi\epsilon_0}R\lambda\int\frac{ds}{(R^2+s^2)^{3/2}} = -\frac{1}{4\pi\epsilon_0}R\lambda\,\frac{s}{(R^2+s^2)^{1/2}R^2}\Bigg|_0^\infty$$

$$= \frac{-\lambda}{4\pi\epsilon_0 R}(1-0) \qquad \boxed{E_y = \frac{-\lambda}{4\pi\epsilon_0 R}}$$

Problem 46 P   CH - 24

a)

$$E\,\cancel{4\pi} r^2 = \rho \frac{\cancel{4}}{3}\cancel{\pi} r^3 \Big/ \epsilon_0$$



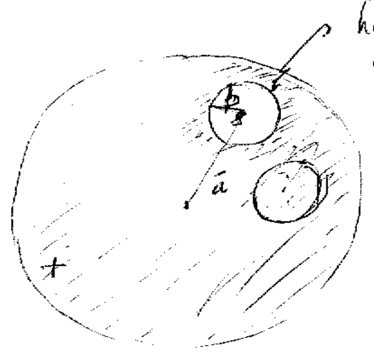
$$\boxed{E = \frac{\rho}{3\epsilon_0} r}$$

$$\vec{E} = \frac{\rho}{3\epsilon_0} \underbrace{r\,\hat{u}_r} \iff \boxed{\vec{E} = \frac{\rho}{3\epsilon_0}\vec{r}}$$

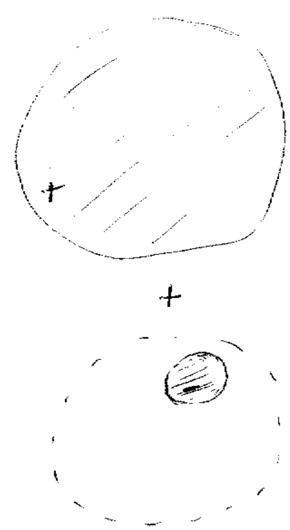
It does not depend on R

b)

hollow sphere of radius b



$\iff$

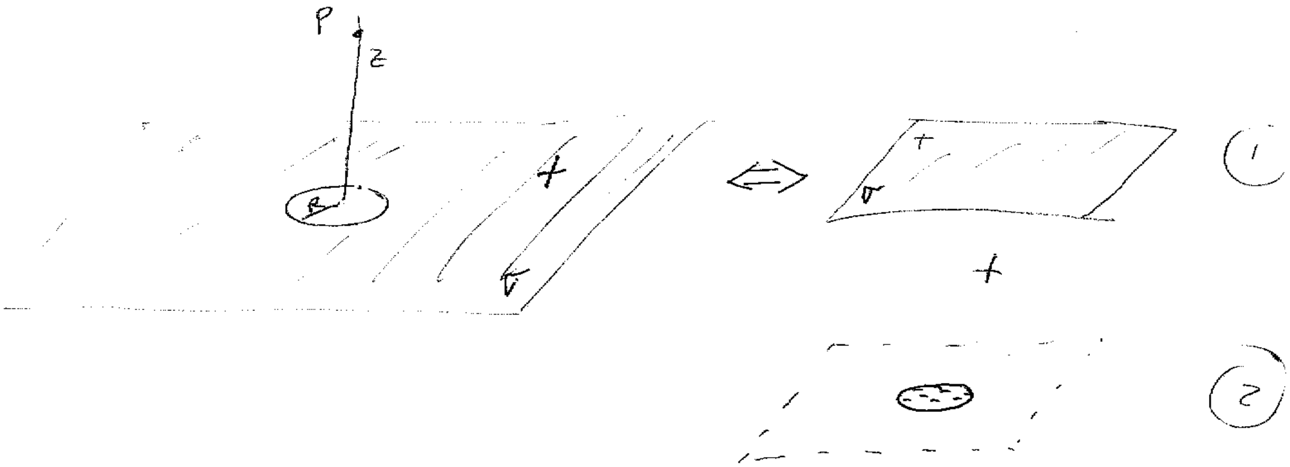


$\vec{E}$ (at P) = $\vec{E}$ due to the big sphere (positively charged) + $\vec{E}$ due to the small sphere (negatively charged)

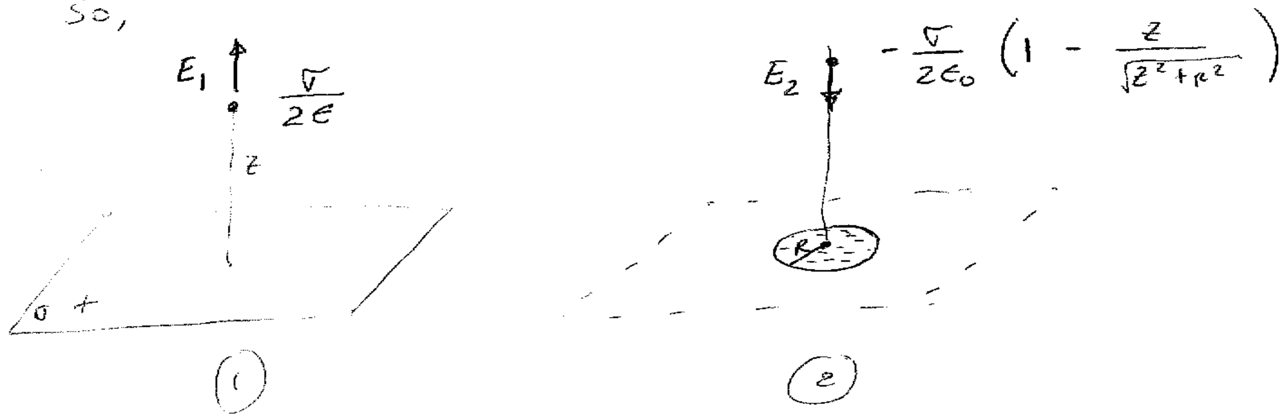
$$= \frac{\rho}{3\epsilon_0}(\vec{a} + \vec{s}) + \frac{(-\rho)}{3\epsilon_0}\vec{s}$$

$$\boxed{\vec{E}\,(\text{at } P) = \frac{\rho}{3\epsilon_0}\vec{a}}$$

Problem # 28E CH-24



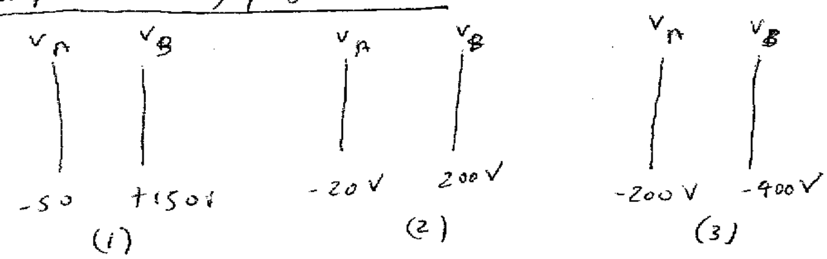
So,



$$E(z) = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2+R^2}}\right) = \frac{\sigma}{2\epsilon_0}\frac{z}{\sqrt{z^2+R^2}}$$

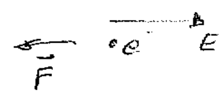
Practice Problems

checkpoint #6, page 578

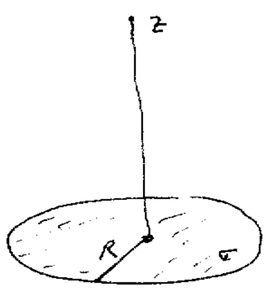


$$E = -\frac{\Delta V}{\Delta x} = -\frac{V_B - V_A}{d}$$

$\leftarrow$ $\frac{200V}{d}$ $\quad$ $\leftarrow$ $\frac{220}{d}$ $\quad$ $\rightarrow$ $\frac{200}{d}$



sample proble 25-5



$$V(0,0,z) = \frac{\sigma}{2\epsilon_0}\left(\sqrt{z^2+R^2} - z\right)$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\sigma}{2\epsilon_0}\left(\frac{1}{2}\frac{2z}{(z^2+R^2)^{1/2}} - 1\right)$$

$$= \frac{\sigma}{2\epsilon_0}\left(1 - \frac{z}{(z^2+R^2)^{1/2}}\right)$$