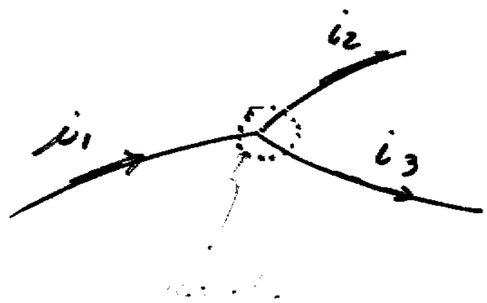
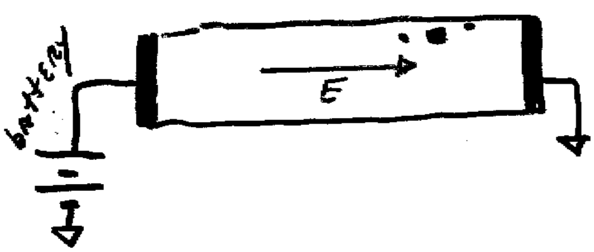
Lecture

STEADY CURRENTS of conduction electrons moving through metallic conductors

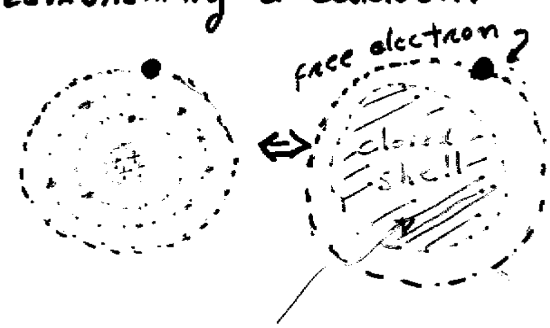


CHARGE CONSERVATION implies:

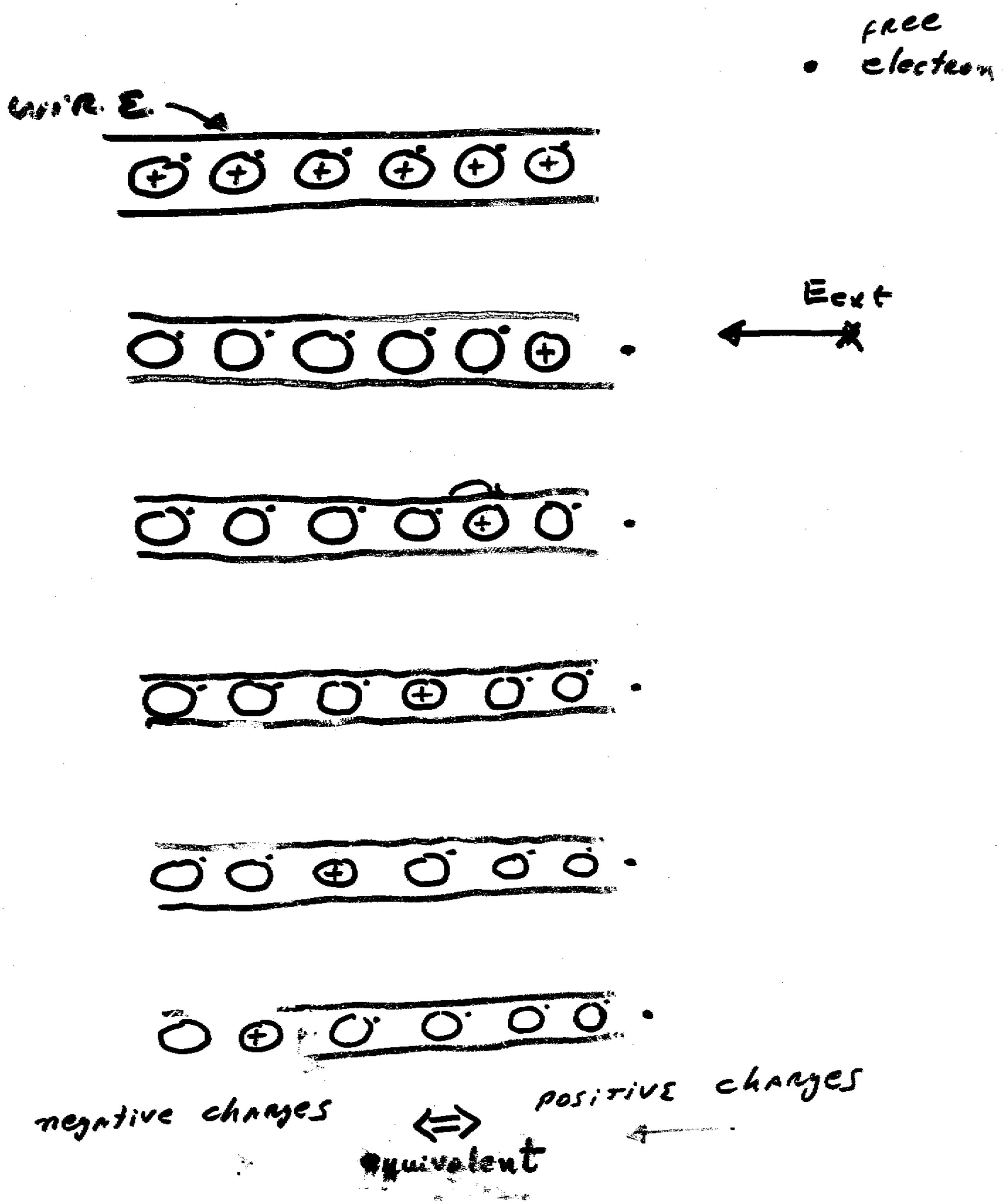
$$\boxed{i_1 = i_2 + i_3}$$



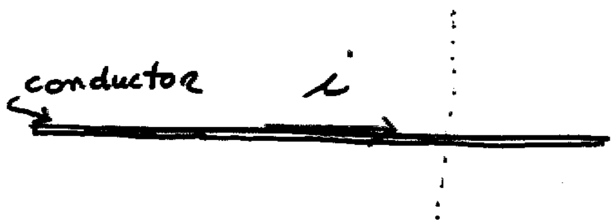
Electric fields $\vec{E}$ act inside the material, exerting forces on the conduction electrons, causing them to move, and thus establishing a current



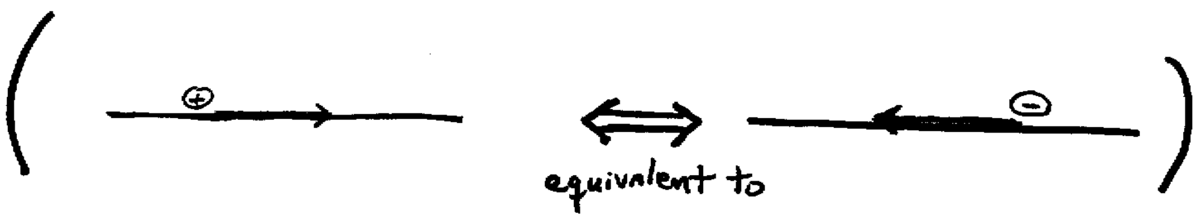
Electrons in the closed shell do not participate in the +


free electron
wire
Eext
negative charges
positive charges
equivalent

Definition of current



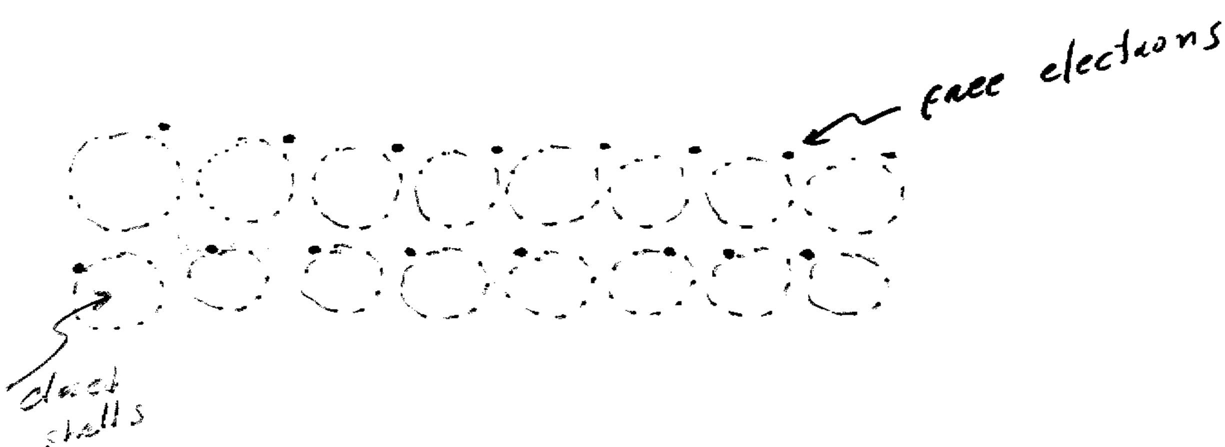
current arrow drawn in the direction in which positive charge carrier **would** move.



$$i = \frac{dq}{dt}$$

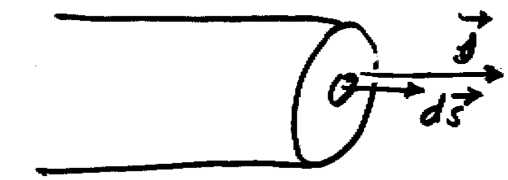
unit of i : ampere

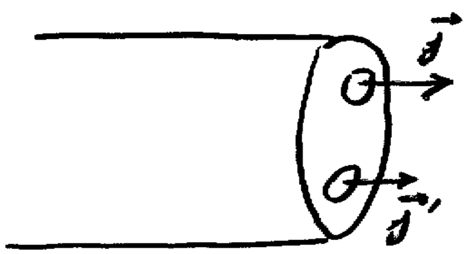
$$\text{ampere} = \frac{\text{coulomb}}{\text{sec}}$$



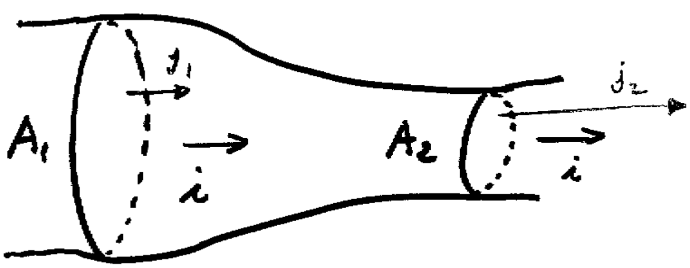
Each atom contributes with	1	free electron	Cu, Au
	2	"	Zn, Cd

Current Density. – $\vec{j}$





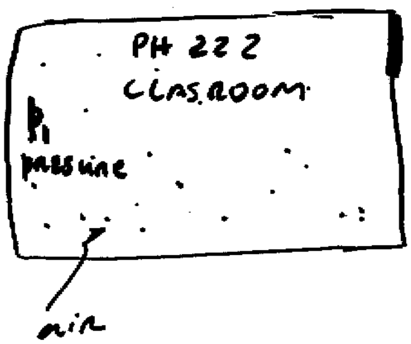
$$i = \int \vec{j} \cdot d\vec{s}$$



$$j_1 = \frac{i}{A_1} \qquad j_2 = \frac{i}{A_2}$$

DRIFT SPEED

Comparison

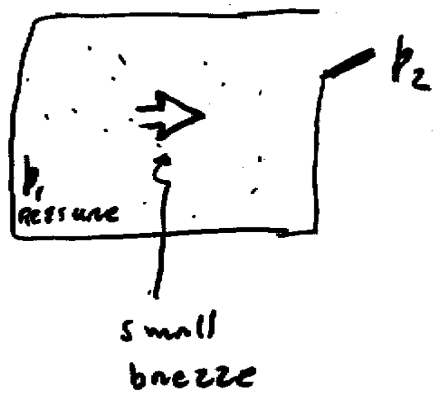


each molecule moves at ~ 400 m/s

But, average velocity

$\langle v \rangle = 0$

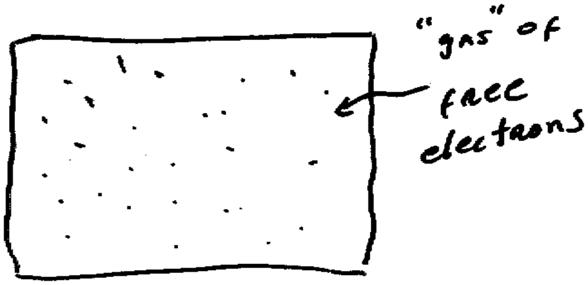
under a pressure difference $p_2 - p_1$



$\langle v \rangle \neq 0$

air molecules acquire a small drift velocity, superimposed to their 400 m/s instantaneous speed.

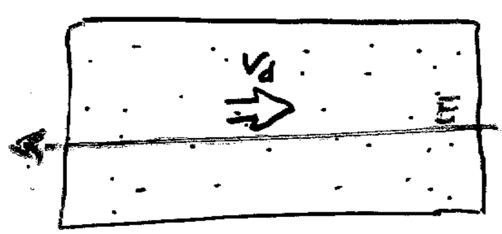
chunk of metal



At room temperature, electrons move with speed ~ 10^7 m/s

But $\langle \vec{v} \rangle = 0$ (Because e^- move randomly)

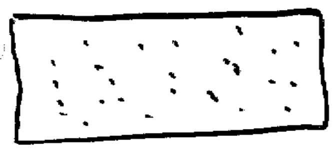
under an electric field E



the free electrons acquire a drift speed v_D

$v_D \simeq 5 \times 10^{-7}$ m/s

Evaluating the drift speed V_d



Copper

- $M = 63.5\ gr/mol$

  1 mol of Cu atoms ——— 63.5 gr

  6.023×10^{23} atoms of Cu ——— 63.5 gr

- density $= 8.93\ gr/cm^3$

  6.023×10^{23} atoms of Cu ——— 63.5 gr

  $X =$ # of atoms in $1\ cm^3$ ——— 8.93 gr

$$\frac{8.93}{63.5} \times 6.023 \times 10^{23} \text{ atoms}/cm^3$$

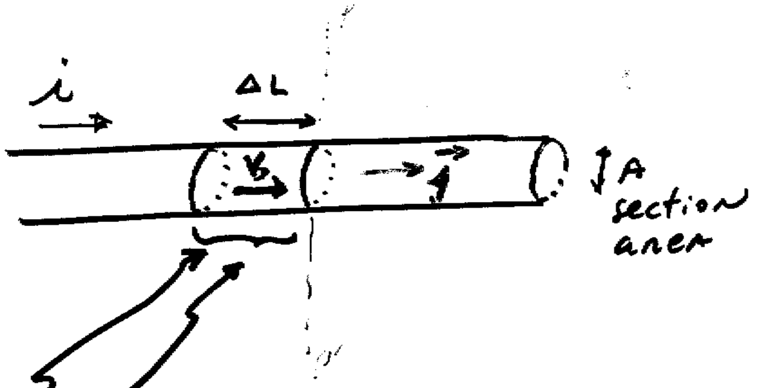
- 1 free electron per atom

$$\Rightarrow \frac{8.93}{63.5} \times 6.023 \times 10^{23} \text{ free-electrons}/cm^3$$

$= n$ number of carriers per unit volume

$= 8.5 \times 10^{22}$ free-electrons$/cm^3$

$= 8.5 \times 10^{28}\ /m^3$



$e = 1.6 \times 10^{-19}\,C$
electron charge

Amount of charge in this volume $\Delta q = ne \times \text{volume}$

$$= ne \times \Delta L \times A = \Delta q$$

All this charge will cross the plane PP' in a time Δt

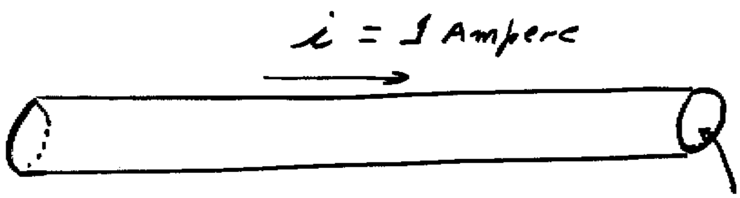
$$\Delta t = \frac{\Delta L}{V_D}$$

$$i = \frac{\Delta q}{\Delta t} = n e V_D A$$

velocity of the carriers
amount of charge in each carrier
# of carriers per volume

$$j = \frac{i}{A} = n e V_D$$

$$\boxed{j = n e V_D}$$



$A = \pi r^2$

$j = n e v_D$

$$\frac{i}{A} = n e v_D \Rightarrow \boxed{\frac{i}{n e A} = v_D}$$

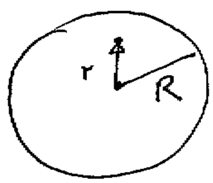
$$\frac{1\ C/s}{(8.5 \times 10^{28}/m^3)(1.6 \times 10^{-19} C)\ \pi\ (0.8 \times 10^{-3} m)^2} = v_D$$

$$3.5 \times 10^{-5}\ m/s = v_D$$

EXCERSISE From Paul A. Tipler; "Physics"; vol-2; 4th Ed.
How long would it take for an electron to drift from your car battery to the starter motor, a distance of about 1 m, if it drift speed is 3.5×10^{-5} m? Answer: ~ 8 hours

Example (similar to Problem 10 CH-27)

A cylinder carries a current, which is established by a current density $j = j(r)$.



Consider the following current densities:

a) $j(r) = j_0$

b) $j(r) = j_0 \frac{r}{R}$

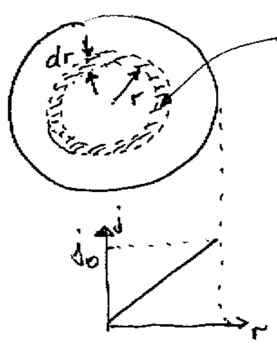
c) $j(r) = j_0 \left(1 - \frac{r}{R}\right)$

Calculate the total current flowing through the cylinder

Solution

a) $I = j_0 \times \text{area} = \boxed{j_0 \times \pi R^2 = I}$

b)



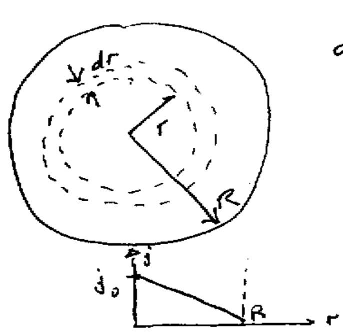
$dA = 2\pi r\, dr$

$dI = j\, dA = \underbrace{j}\times 2\pi r\, dr$

$\int dI = \int \left(j_0 \frac{r}{R}\right) 2\pi r\, dr = j_0 \frac{2\pi}{R} \int_0^R r^2 dr = j_0 \frac{2\pi}{R} \frac{R^3}{3}$

$\boxed{I = \frac{2}{3} j_0 \pi R^2}$

c)



$dI = j\, dA = \underbrace{j}\, 2\pi r\, dr$

$I = \int dI = \int j_0 \left(1 - \frac{r}{R}\right) 2\pi r\, dr$

$= j_0\, 2\pi \frac{R^2}{2} - j_0 \frac{2\pi}{R} \frac{R^3}{3} =$

$= j_0 \pi R^2 - j_0 \frac{2}{3} \pi R^2 \Rightarrow \boxed{I = \frac{1}{3} \pi R^2}$

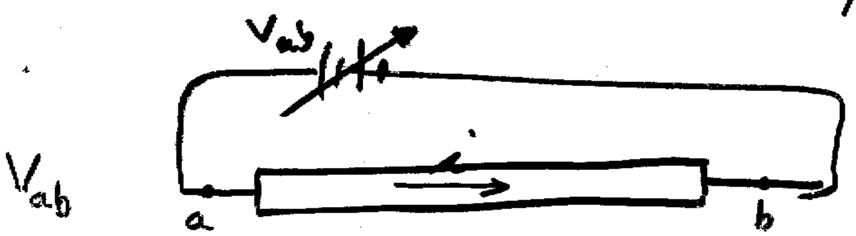
RESISTANCE and RESISTIVITY

R

MACROSCOPIC object PROPERTY

ρ

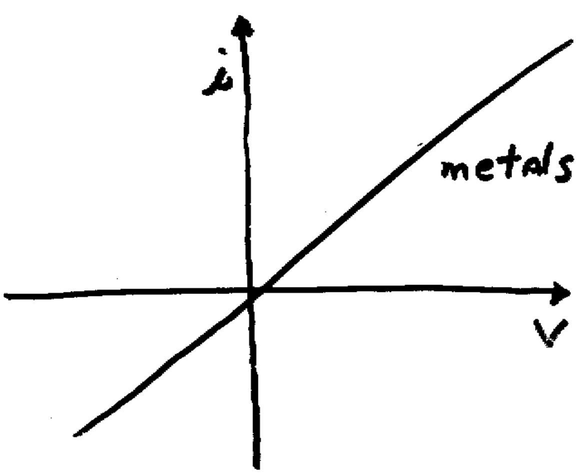
material property



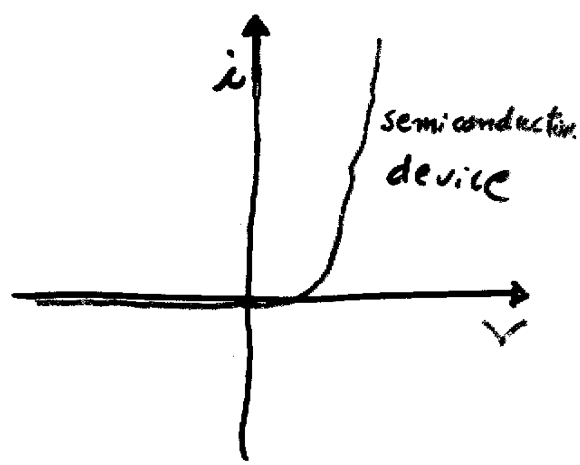
$$R = \frac{V_{ab}}{i}$$

units:

$$\text{Ohm} = \frac{\text{volt}}{\text{Amp}}$$

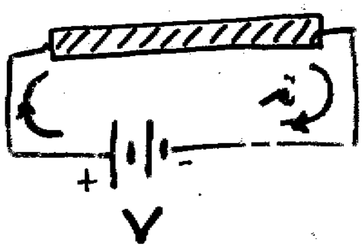


Some materials obey Ohm's "law"



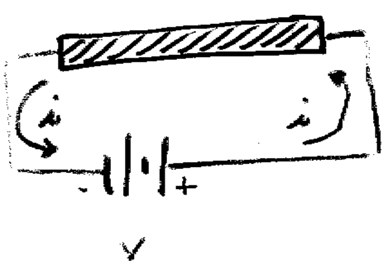
other materials do <u>not</u> follow Ohm's "law"

Material that is said to obey "ohm's law"



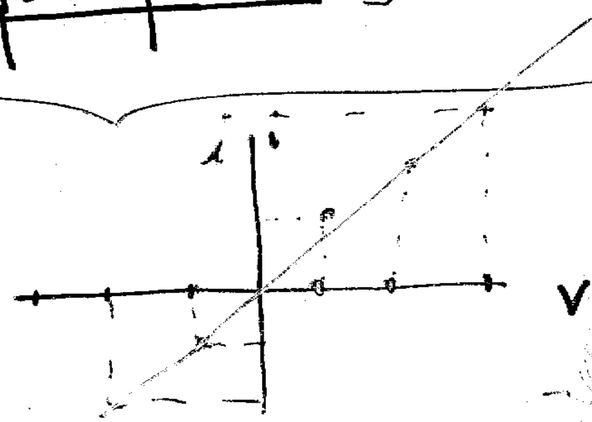
V	i	$R = \frac{V}{i}$
1v	0.2A	
2v	0.4A	
3	0.6A	

if all these number turn out to be the same, then we say that the material is ohmic

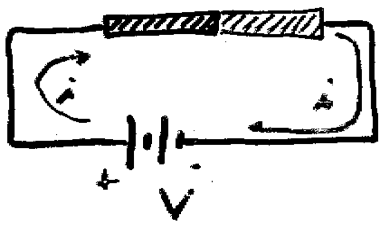


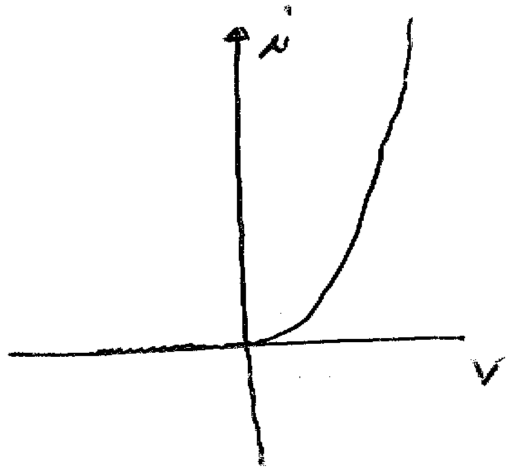
V	i	$R = \frac{V}{i}$
1.5v	0.3A	
2.5v	0.5A	
3.5v	0.7A	

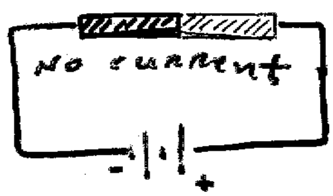
V	i	R
3v	0.6A	
2v	0.4A	
1v	0.2A	
-1.5	-0.3	

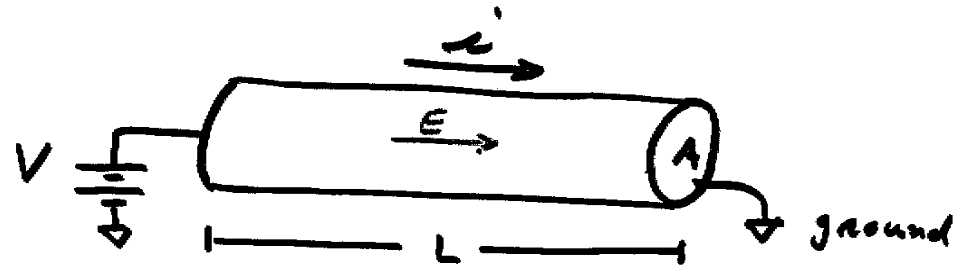


A diode, composed of 2 different semiconductor materials, DOES NOT obey ohm's law







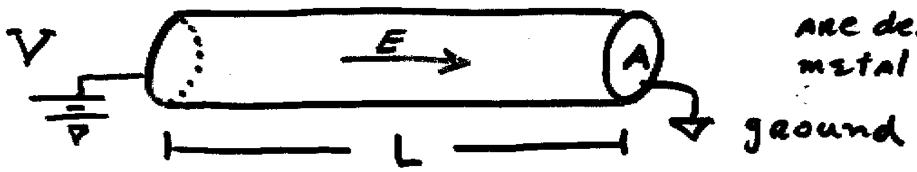


$$V = EL$$
$$= \rho j L$$
$$= \rho \frac{i}{A} L \quad \Rightarrow \quad V = \underbrace{\frac{\rho L}{A}}\, i$$

$$\frac{V}{i} = \boxed{\underbrace{R}_{\text{RESISTANCE}} = \underbrace{\rho}_{\substack{\text{material}\\\text{property}}} \underbrace{\frac{L}{A}}_{\text{geometry}}}$$

units of ρ : ohm-meter
$= \Omega$-m

RESISTIVITY (ρ), CONDUCTIVITY (σ), ELECTRIC FIELD (E)
RESISTANCE (R)



The following analysis is valid whether we are dealing with a metal or a homogeneous semiconductor material

The higher V → the higher $\vec{E}$ (electric field)

the higher $\vec{E}$ → the higher $\vec{J}$ (current density)

$$\boxed{\vec{J} = \sigma \vec{E}}$$

conductivity

Definition of resistivity ρ

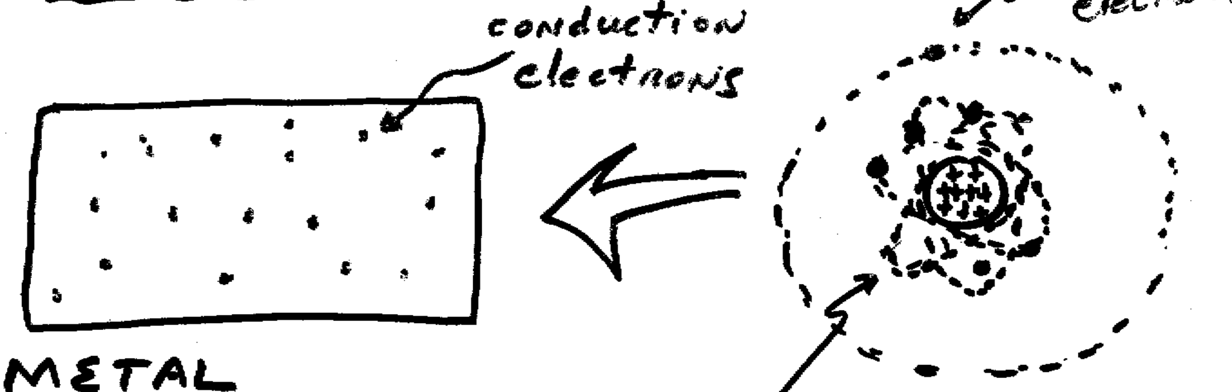
$$\rho = \frac{1}{\sigma}$$

$$\boxed{\vec{J} = \frac{1}{\rho} \vec{E}}$$

RESISTIVITY

SILVER	$1.62 \times \underline{10^{-8}}$ Ω-m	METALS
ALUMINUM	$2.75 \times \underline{10^{-8}}$ Ω-m	
SILICON (pure)	$2.5 \times \underline{10^{3}}$ Ω-m	SEMICONDUCTORS
silicon (n type) (silicon + phosphorous impurities)	$8.7 \times \underline{10^{-4}}$ Ω-m	
silicon (p-type) (silicon + Aluminum impurities)	$2.8 \times \underline{10^{-3}}$ Ω-m	
GLASS	$10^{10} - 10^{14}$ Ω-m	INSULATORS
Fused quartz	10^{16} Ω-m	

Microscopic view

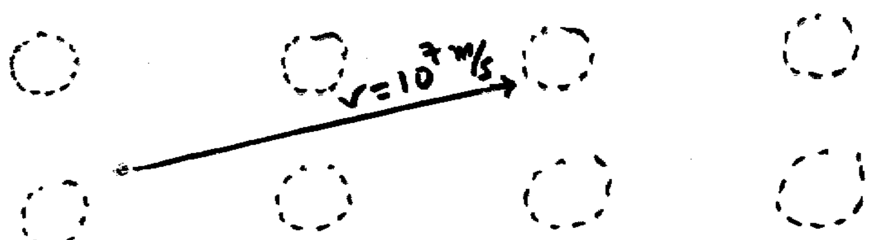


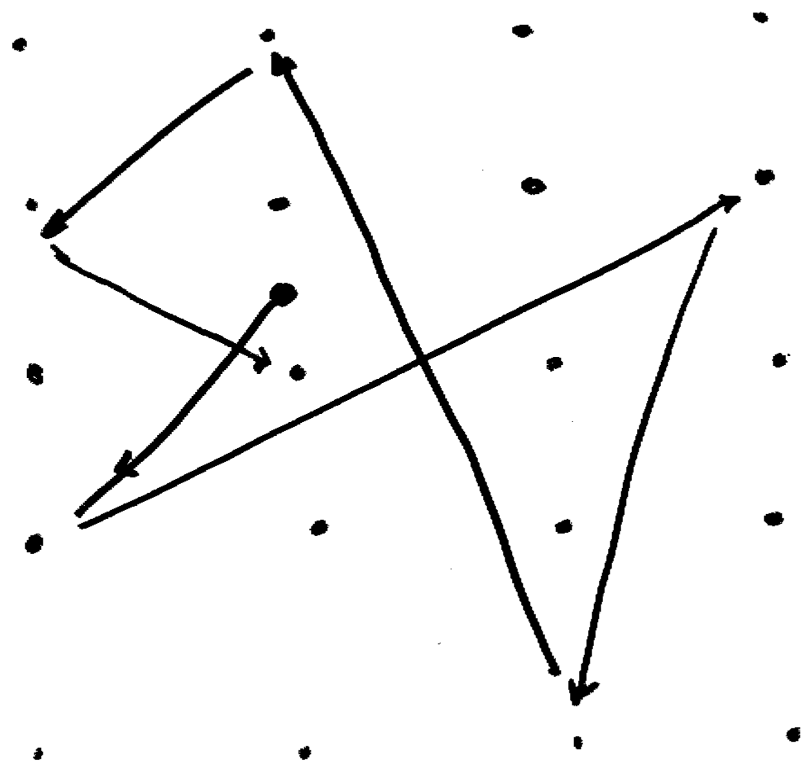
case of:
1 free-electron per atom

electrons tight to the atom

"Free-electron model"

This model assumes that electrons collide not with one another (!) but only with atoms of the metal





In average, the free electrons do not have a net displacement

$$a = \frac{F}{m} = \frac{eE}{m}$$

$$\langle v \rangle = \langle a\,\tau \rangle = \frac{eE}{m}\tau \qquad (1)$$

$$j = n e \langle v \rangle$$

$$\cancel{\langle v \rangle} = \frac{eE}{m}\tau = \frac{j}{ne}$$

$$\left(\frac{e^2 n \tau}{m}\right) E = j$$

$$\boxed{\sigma = \frac{e^2 n}{m}\tau}$$

$$\rho = \frac{m}{e^2 n \tau}$$

<u>Sample Problem 27-5</u>

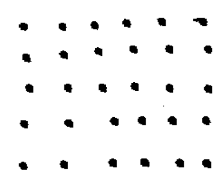
Specially read part (b) of this sample problem

Additional advanced reading.

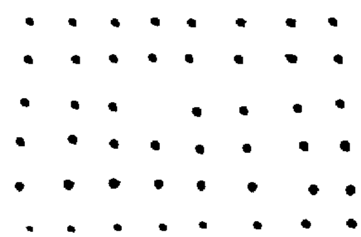
N. Ashcroft and N. Mermin; "SOLID STATE PHYSICS"; Saunder College Ed.

- page 1 to 10
- Last paragraph on page 215, and page 216

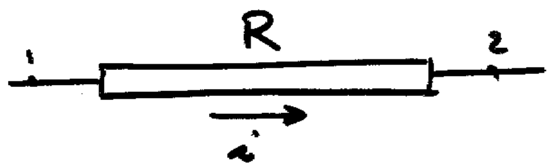
"the conductivity of a perfect periodic crystal is infinite"



"metals have an electrical resistance because no real solid is a perfect crystal: missing atoms
dislocations
impurities
thermal vibration



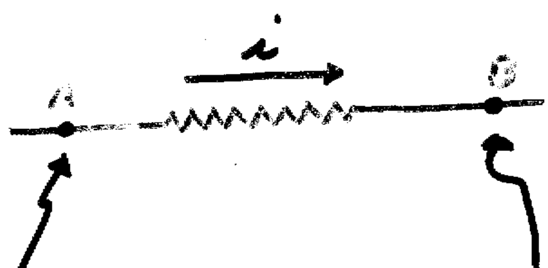
CIRCUITS



$V_{12} = R\,i$

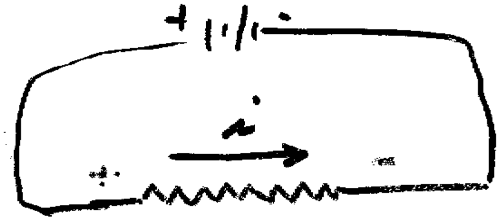
Symbol 

Unit of R $\quad \dfrac{\text{volt}}{\text{Amp}} = \text{Ohm}$

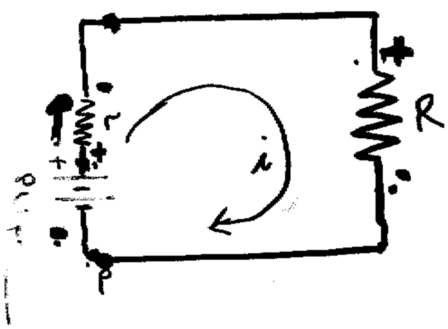


at higher electric potential

lower electric potential



SINGLE-LOOP circuits



$$\cancel{V_p} + \mathcal{E} - ri - Ri = \cancel{V_p}$$

$$\mathcal{E} = (r + R)i$$

$$\boxed{i = \frac{\mathcal{E}}{r + R}}$$

Energy source { battery
electric generator
solar cell!

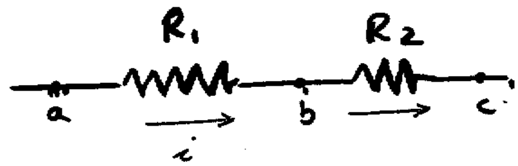
$\mathcal{E}$ is called "electromotive force"
$\underbrace{\text{force"}}_{\text{outdated phrase}}$

$\mathcal{E}$ is __not__ a force

Unit of $\mathcal{E}$ is volts!

Resistors connected in series

$$V_{ac} = V_{ab} + V_{bc}$$

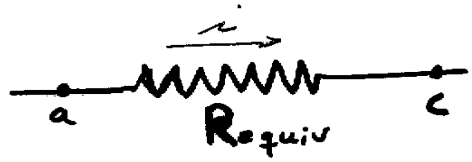


$$= R_1 i + R_2 i$$

$$= \underbrace{(R_1 + R_2)}\, i$$

This expression tell us that the resistors arrangement shown above is equivalent to

$$V_{ac} = R_{eq}\, i$$



provided that

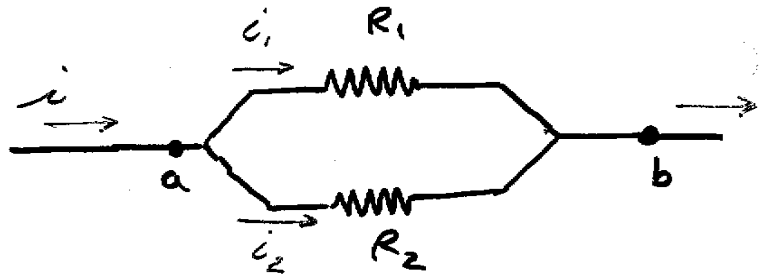
$$R_{equiv} = R_1 + R_2$$

For many resistances connected in series

$$\boxed{R_{equiv} = R_1 + R_2 + R_3 + \cdots}$$

Resistors connected in PARALLEL

$V_{ab} = R_1 i_1$



$V_{ab} = R_2 i_2$

$$i = i_1 + i_2$$

$$= \frac{V_{ab}}{R_1} + \frac{V_{ab}}{R_2}$$

$$i = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V_{ab}$$

This expression tells us that the resistors arrangement shown above is equivalent to

if

$$i\left(\frac{1}{\left(\frac{1}{R_1}+\frac{1}{R_2}\right)}\right) = V_{ab}$$

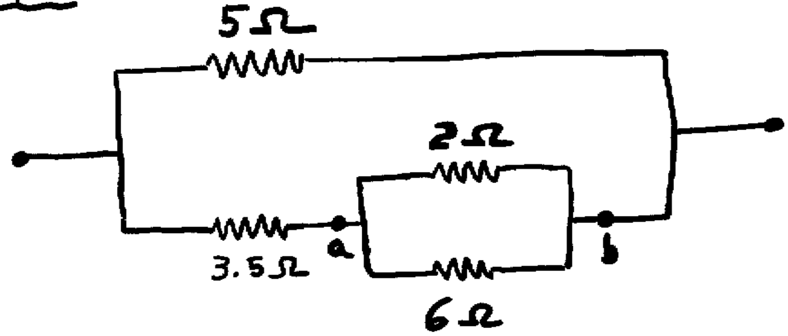
$$i\,R_{eq} = V_{ab}$$

$$R_{eq} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} \longrightarrow \frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2}$$

For many resistors connected in parallel

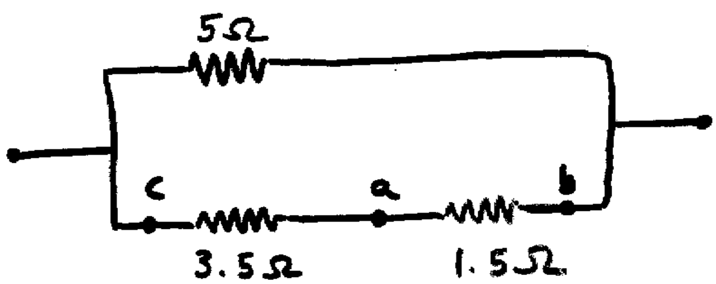
$$\boxed{\frac{1}{R_{equiv}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots}$$

Example



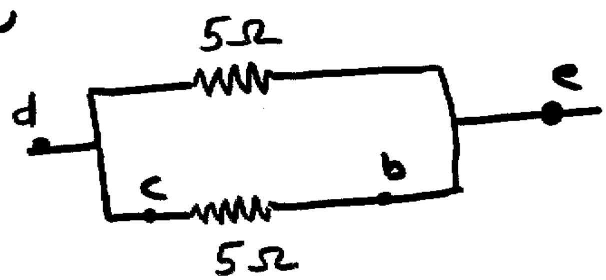
ab $\frac{1}{R_{eq}} = \frac{1}{2\Omega} + \frac{1}{6\Omega} = \frac{4}{6\Omega} \Rightarrow R_{eq} = 1.5\Omega$

PARALLEL CONNECTION



cb $R_{eq} = 1.5\Omega + 3.5\Omega = 5\Omega$

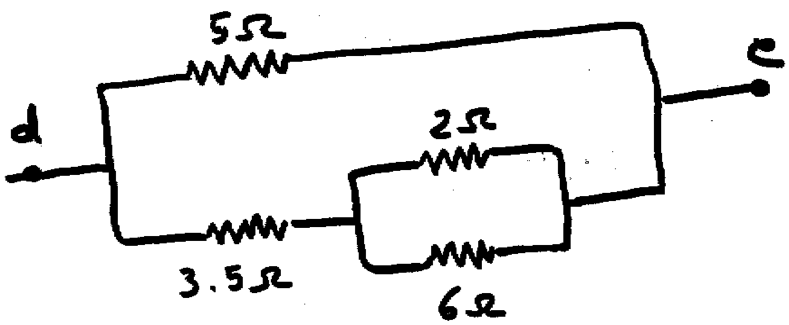
SERIES CONNECTION



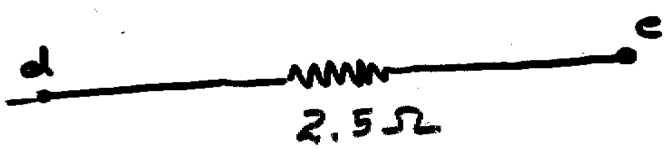
de $\frac{1}{R_{eq}} = \frac{1}{5\Omega} + \frac{1}{5\Omega} = \frac{2}{5\Omega} \Rightarrow R_{eq} = 2.5\Omega$

parallel connection

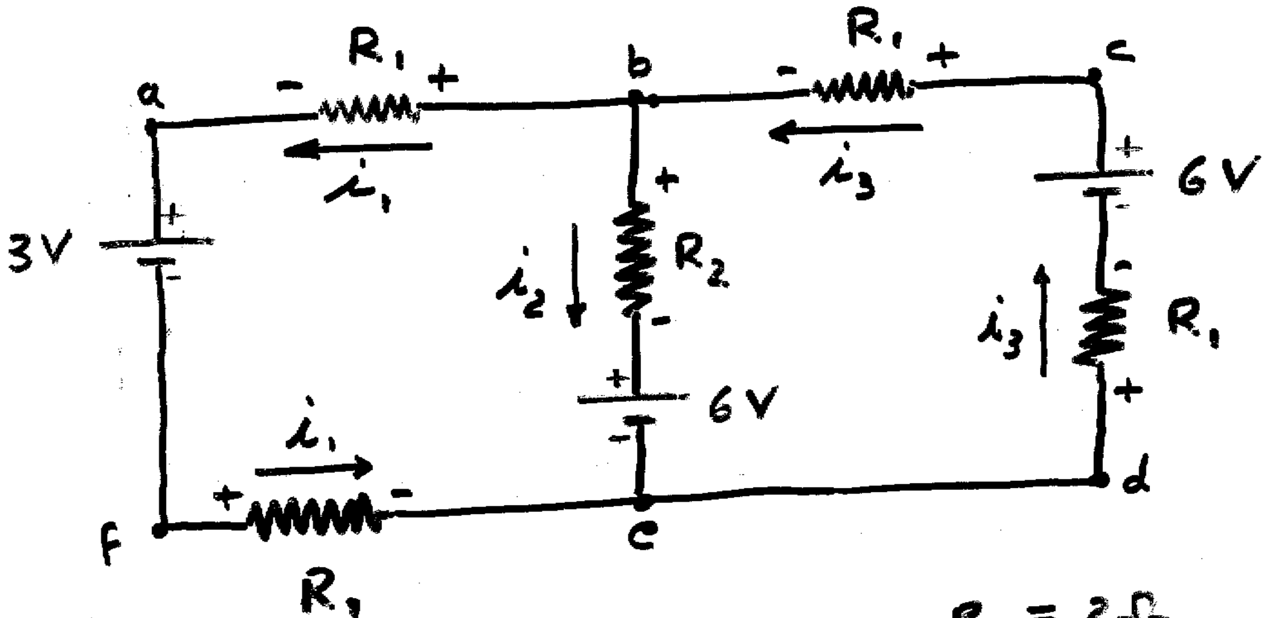
5



this circuit is equivalent to



Multiloop circuit



UNKNOWN : i_1 , i_2 , i_3

$R_1 = 2\Omega$

$R_2 = 4\Omega$

loop bedc

$$\cancel{V_b} - i_2 R_2 - 6V - i_3 R_1 + 6V - i_3 R_1 = \cancel{V_b}$$

$$\Rightarrow \boxed{-i_2 R_2 - 2 i_3 R_1 = 0} \qquad (1)$$

Loop befa

$$\cancel{V_b} - i_2 R_2 - 6V + i_1 R_1 + 3V + i_1 R_1 = \cancel{V_b}$$

$$\Rightarrow \boxed{-i_2 R_2 + 2 i_1 R_1 = 3V} \qquad (2)$$

Node "b"

$$\boxed{i_1 + i_2 = i_3} \qquad (3)$$

We have now 3 equations relating 3 unknown quantities.

Using the given values for R_1 and R_2, we obtain:

From (1): $-i_2 \times 4\Omega - 2 i_3 \times 2\Omega = 0$

$$\Rightarrow \quad i_3 = -i_2 \qquad (4)$$

From (2): $-i_2 \times 4\Omega + 2 i_1 \times 2\Omega = 3\,\text{Volts}$

$$\Rightarrow \quad -i_2 + i_1 = \frac{3}{4}\,\text{Amp.}$$

OR

$$i_1 = \frac{3}{4}\,\text{Amp} + i_2 \qquad (5)$$

Replacing (4) and (5) in (3) we obtain

$$\underbrace{\left(\frac{3}{4}\text{Amp} + i_2\right)}_{i_1} + i_2 = \underbrace{(-i_2)}_{i_3}$$

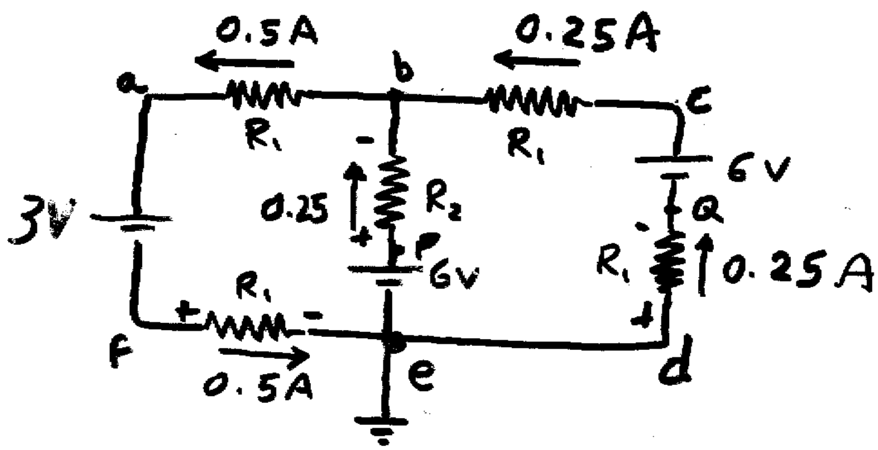
$$\Rightarrow \frac{3}{4}\text{Amp} + 2 i_2 = -i_2 \quad \Rightarrow \quad \boxed{i_2 = -0.25\,\text{Amp}}$$

Also

$$i_1 = \frac{3}{4}A + i_2 = \frac{3}{4}A + (-0.25A) \Rightarrow \boxed{i_1 = 0.5 Amp}$$

And

$$i_3 = -i_2 = -(-0.25 Amp) \Rightarrow \boxed{i_3 = 0.25 Amp}$$



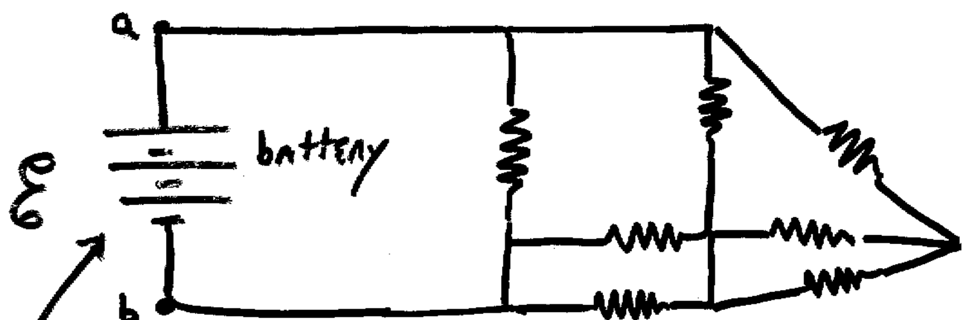
$R_1 = 2\Omega$

$R_2 = 4\Omega$

In the previous circuit we have grounded the node "e". $V_e = 0$

QUESTION: $V_b = ?$; $V_c = ?$; $V_a = ?$

Electric POWER of a battery

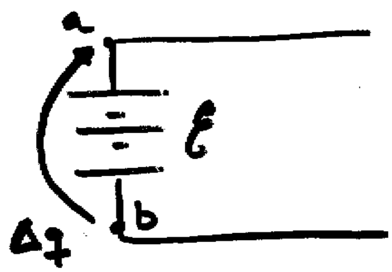


$\mathcal{E}$ is called "electromotive force"

Its job is to maintain the electric potential difference between a and b constant, no matter what

If $\mathcal{E} = 12$ volts for example, the battery will try to keep the potential difference between a and b equal to 12 volts

(in reality, because any real battery has an intrinsic internal resistant, the above statement is followed approximately)



To take an amount of charge Δq from a potential V_b to V_a the battery has to do a work ΔW equal to

$$\Delta W = \Delta q \times \underbrace{(V_a - V_b)}_{= \mathcal{E}}$$

$$\Delta W = \Delta q \times \mathcal{E}$$

Since $i = \frac{\Delta q}{\Delta t}$, we have

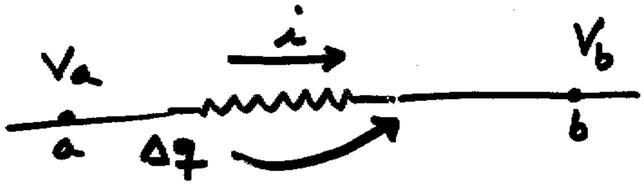
$$\underbrace{\frac{\Delta W}{\Delta t}}_{} = i\,\mathcal{E}$$

work done by the battery per unit time.
We call it power

$$\boxed{P = i\,\mathcal{E}}$$

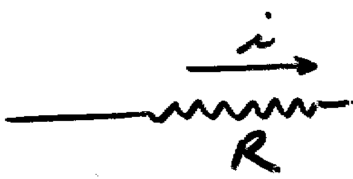
units:
i ampere
$\mathcal{E}$ volts
p watt $= \frac{\text{Joule}}{\text{sec}}$

Power dissipated in a RESISTOR



An amount of charge Δq starts at a high electric-potential V_a and ends up at lower electric potential V_b. The energy lost in electric potential this process is

$$\Delta E_{lost} = \Delta q \times \underbrace{(V_a - V_b)}_{\text{let's call it } V}$$



V: electric potential across the resistance R

$$\Delta E_{lost} = \Delta q \times V$$

Again, since $i = \Delta q / \Delta t$ we obtain

$$\underbrace{\frac{\Delta E_{lost}}{\Delta t}}_{\text{energy dissipated in the resistor per unit time}} = i\,V$$

$$P_{lost} = i\,V$$

For a resistance $V = iR$,

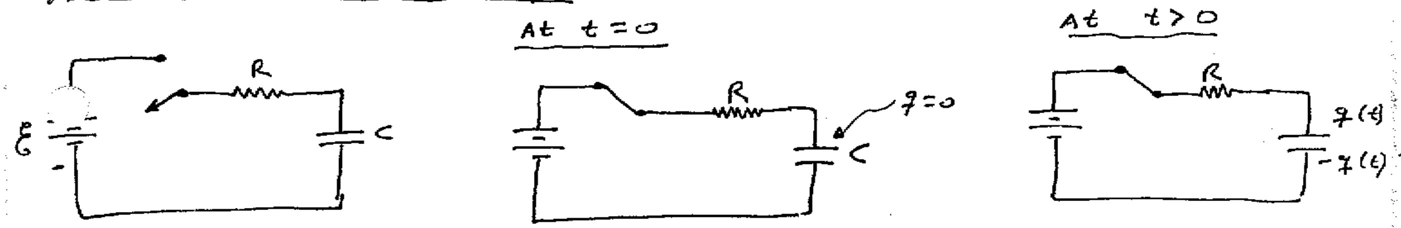
$$P_{lost} = i^2 R$$

i : amperes

R : ohm

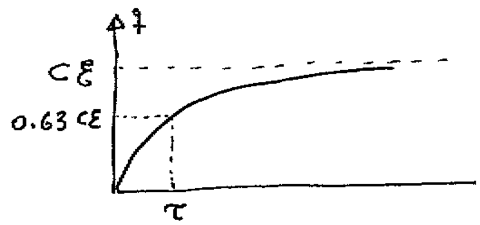
P_{lost} : Watt $= \frac{\text{Joule}}{\text{sec}}$

Charging a capacitor



$$q(t) = C\mathcal{E}\left(1 - e^{-t/RC}\right) \quad \text{for } t \geq 0$$

Defining $\boxed{\tau \equiv RC}$ called "time constant"



$$\boxed{q(t) = C\mathcal{E}\left(1 - e^{-t/\tau}\right)}$$

when $t = \tau$

$q(\tau) = 0.63$

Problem 46E, CH-28 (p. 656)

RC series circuit. $\mathcal{E} = 12$ Volts, $R = 1.4 \times 10^6\,\Omega$, $C = 1.8\,\mu F$

a) $\tau = RC = (1.4 \times 10^6\,\Omega)(1.8\,\mu F) = 2.52$ seconds

b) max charge on the capacitor: $\boxed{q_{max} = C\mathcal{E}} = \boxed{q(t) = q_{max}\left(1 - e^{-t/\tau}\right)}$

$q_{max} = (1.8 \times 10^{-6} F)(12 \text{ volts}) = 21.6\,\mu C$

c) Find the time t for which $q = 16\,\mu C$

$16\,\mu C = q(t) = 21.6\,\mu C\left(1 - e^{-t/\tau}\right)$

$\Rightarrow \quad 0.74 = 1 - e^{-t/\tau}$

$e^{-t/\tau} = 0.26$

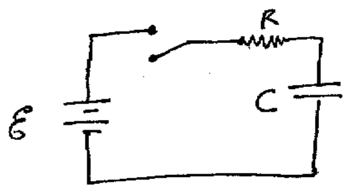
$-\dfrac{t}{\tau} = -1.347$

$t = 1.347\,\tau$

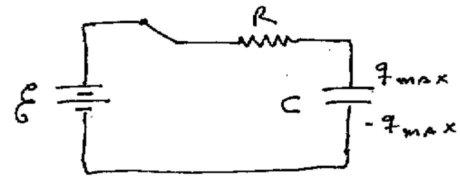
$= 1.347 \times 2.52$ second

$\boxed{t = 3.4 \text{ sec}}$

Problem 50P



At $t=\infty$



- Energy stored in the capacitor:

$$U_c = \frac{1}{2}\frac{q_{max}^2}{C} = \frac{1}{2C}(\mathcal{E}C)^2 = \boxed{\frac{1}{2}C\mathcal{E}^2 = U_c}$$

Energy stored

- Energy supplied by the battery E_{input}

$$\int_{t=0}^{\infty}\frac{\Delta W}{\Delta t}\,dt = \int_0^{\infty} P(t)\,dt = \int_0^{\infty}\mathcal{E}\,i(t)\,dt$$

$$q(t) = q_{max}\left(1-e^{-t/RC}\right) \qquad q_{max} = \mathcal{E}C$$

$$i(t) = \frac{dq}{dt} = q_{max}\times(-1)\,e^{-t/RC}\cdot\left(-\frac{1}{RC}\right)$$

$$i(t) = \frac{q_{max}}{RC}e^{-t/RC} = \frac{\mathcal{E}}{R}e^{-t/RC}$$

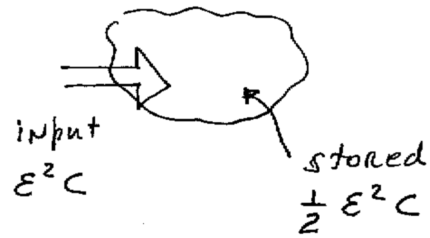
$$E_{input} = \int_0^{\infty}\mathcal{E}\times\frac{\mathcal{E}C}{CR}e^{-t/RC}\,dt = \mathcal{E}^2C\int_0^{\infty}\frac{e^{-t/RC}}{RC}\,dt$$

Notice $\frac{d}{dt}\left(-e^{-t/RC}\right) = +\frac{1}{RC}e^{-t/RC}$

$$E_{input} = \mathcal{E}^2C\left[-e^{-t/RC}\right]_0^{\infty} = \mathcal{E}^2C\,[0+1]$$

$$\boxed{E_{input} = \mathcal{E}^2C}$$ energy provided by the battery

ontinuation Problem 50 P



where did the rest of the energy go ?

Amount of heat dissipated in the resistor.

$$\text{Heat} = \int_0^\infty i^2 R \; dt \quad = \quad \frac{\varepsilon^2}{R^2} e^{-2t/RC}$$

$$= \int_0^\infty \left(\frac{\varepsilon}{R} e^{-t/RC}\right)^2 R\,dt = \int_0^\infty \frac{\varepsilon^2}{R^2} e^{-2t/RC} \times R\,dt$$

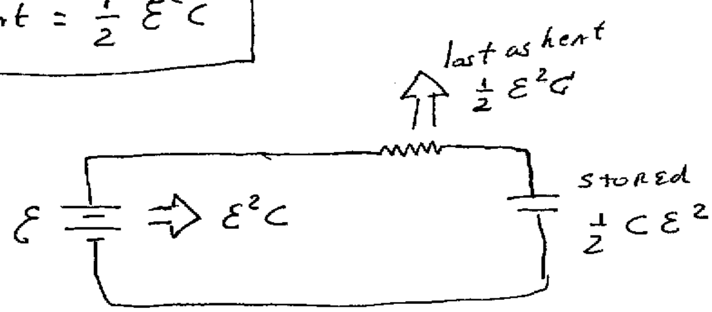
$$= \frac{\varepsilon^2 C}{2} \int_0^\infty \frac{2 \times 1}{RC} \quad e^{-2t/RC}$$

Notice $\dfrac{d}{dt}\left[- e^{-\frac{2}{RC}t}\right] = \dfrac{2}{RC} e^{-2t/RC}$

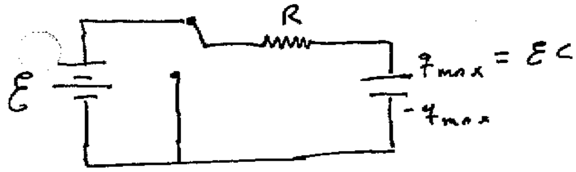
$\cancel{\frac{\varepsilon^2 C}{2}\left[\frac{2}{RC} e\right.} \quad =$

$$\text{Heat} = \frac{\varepsilon^2 C}{2}\left[- e^{-\frac{2}{RC}t}\right]\Bigg|_0^\infty = \frac{\varepsilon^2 C}{2}\left[(-0) - (-1)\right] = \frac{1}{2}\varepsilon^2 C$$

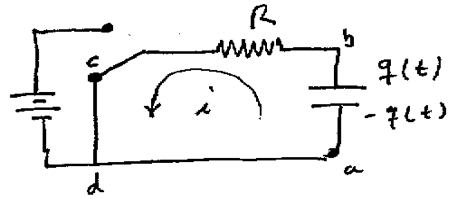
$$\boxed{\text{Heat} = \frac{1}{2}\varepsilon^2 C}$$



Discharging a capacitor



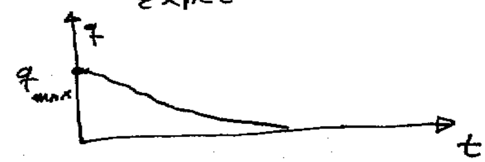
At $t=0$



loop abcd

$$\frac{q}{C} - Ri = 0$$

Expected



Notice:
If we set $i = \frac{dq}{dt}$

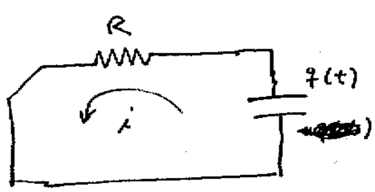
we obtain: $\frac{q}{C} - R\frac{dq}{dt} = 0$

$$\frac{dq}{dt} = \frac{1}{RC}q$$

Solution $q(t) = e^{\frac{1}{RC}t}$ grows as a function of time !

Something must be wrong

What is wrong?



Once we solve this circuits, we anticipate, by physics arguments, that all the quantities in the graph should turn out to be positive

$q(t)$ positive
$i(t)$ positive (because the capacitor is discharging)

But, when we say

$i(t) = \frac{dq}{dt}$ $i(t)$ would turn out to be negative

So, $i = \frac{dq}{dt}$ is <u>not</u> exactly <u>true</u>.

the correct expression ~~is~~ in this case is: $i = -\frac{dq}{dt}$

So,

$$\frac{q}{c} - Ri = 0$$

$$\Rightarrow \boxed{\frac{q}{c} + R\frac{dq}{dt} = 0}$$ capacitor discharging

$$\frac{dq}{dt} + \frac{1}{RC}q = 0 \Rightarrow q(t) = Be^{-at}$$

$$\frac{dq}{dt} = -aBe^{-at}$$

$$-aBe^{-at} + \frac{1}{RC}Be^{-at} = 0$$

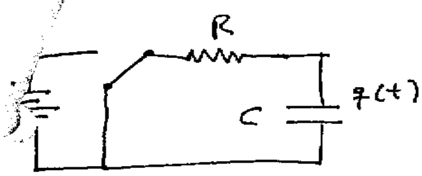
$$\left(-a + \frac{1}{RC}\right)Be^{-at} = 0 \Rightarrow \boxed{a = \frac{1}{RC}}$$

$$q(t) = Be^{-t/RC}$$

At $t=0$ $q(0) = q_{max} = \mathcal{E}C$

$$\mathcal{E}C = q(0) = B \Rightarrow \boxed{B = \mathcal{E}C}$$

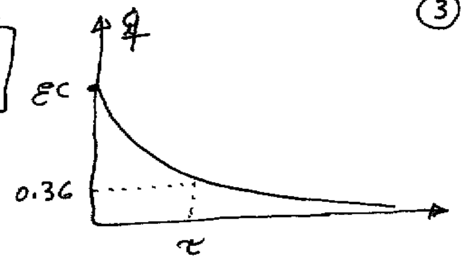
$$\boxed{q(t) = \mathcal{E}Ce^{-t/RC}}$$



$$\boxed{q(t) = \mathcal{E} C e^{-t/RC}}$$

$$\tau = RC$$

$$q(t) = \mathcal{E} C e^{-t/\tau}$$



when $t = \tau$

$$q(\tau) = \mathcal{E} C \times e^{-1} = 0.$$
$$= 0.36\, \mathcal{E} C$$

$$i(t) = -\frac{dq}{dt}$$
$$= -\frac{d}{dt}\left(\mathcal{E} C e^{-t/RC}\right)$$
$$= \frac{\mathcal{E} C}{RC} e^{-t/RC} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$\boxed{i(t) = \frac{\mathcal{E}}{R} e^{-t/RC}}$$