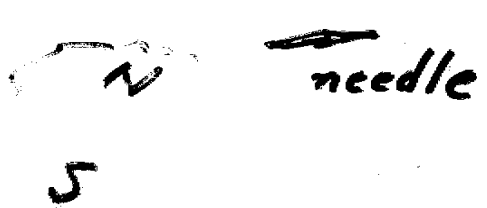


MAGNETISM

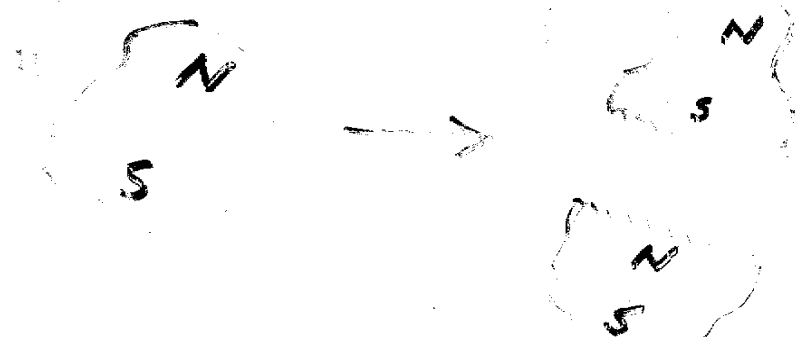
2000 years ago: Greeks were aware of natural magnets "MAGNETITE" stones attract pieces of iron

1269, Pierre de Maricourt:



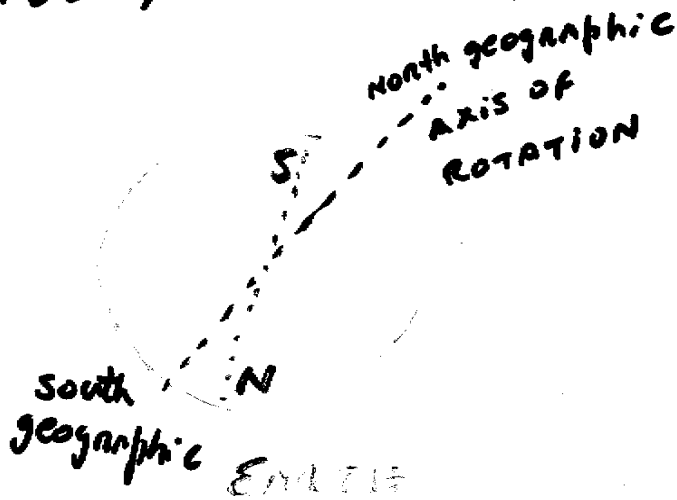
Every magnet has 2 poles

magnetic poles always occur in pairs



So far ^{there is} NO conclusive evidence of the existence of isolated magnetic poles (monopole)

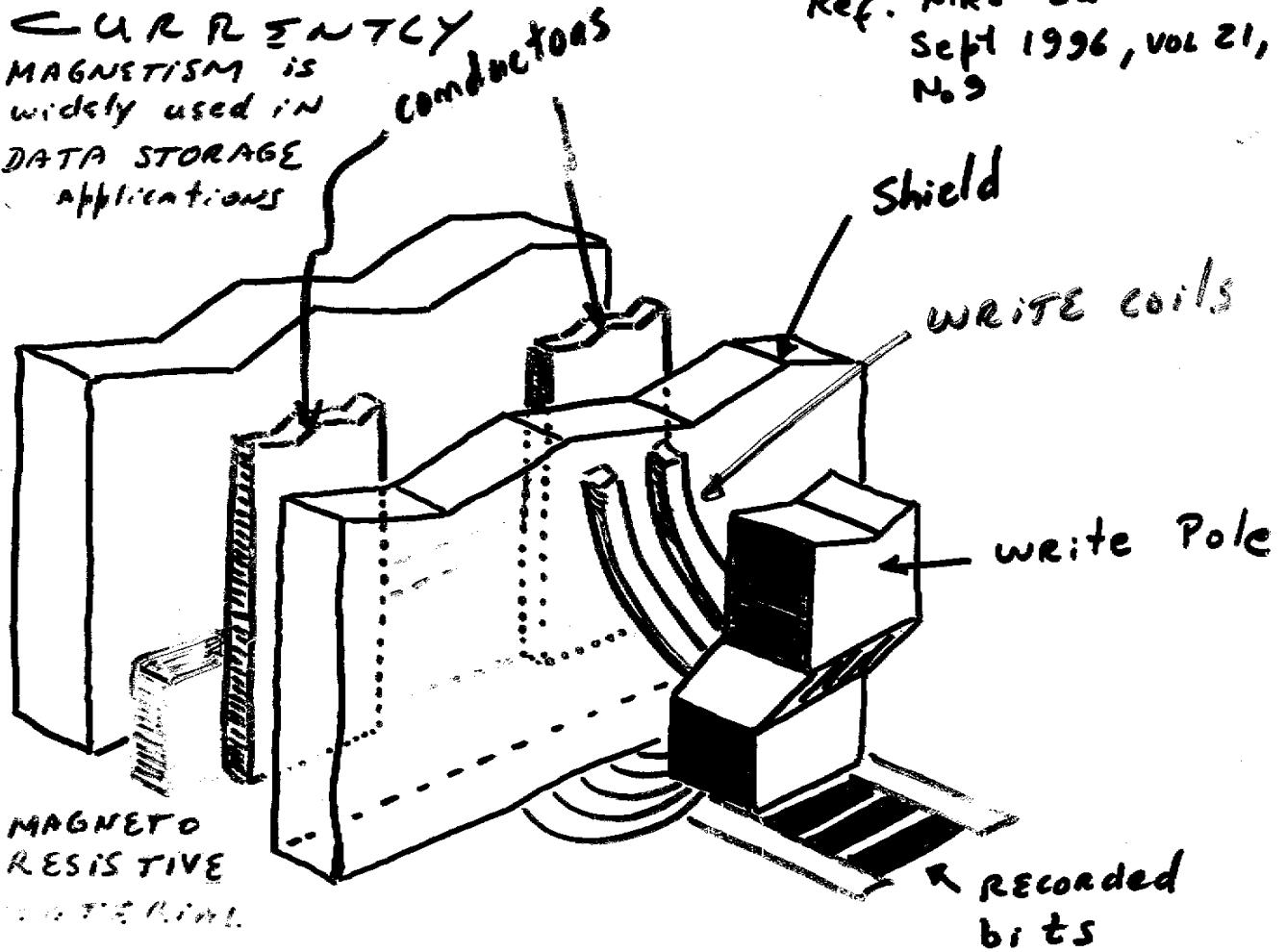
1600, William Gilbert



EARTH itself is a natural magnet

CURRENTLY
MAGNETISM is
widely used in
DATA STORAGE
Applications

Ref: MRS Bulletin
Sept 1996, vol 21,
No 9

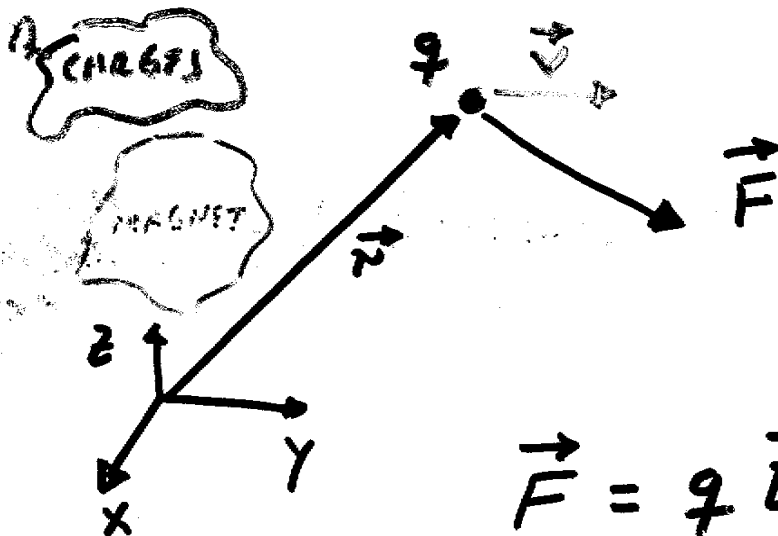


The Magnetic Field \vec{B}

The force on an electric charge q depends

↳ Not only on WHERE it is: (\vec{r})

↳ but also on how is it moving (\vec{v})



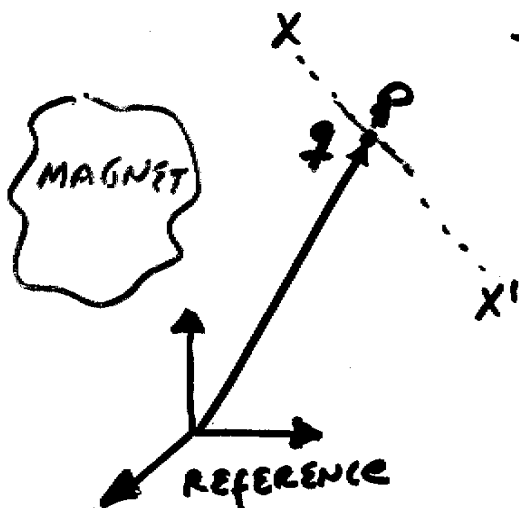
$$\vec{F} = q \vec{E}(\vec{r}, t)$$

FORCE
independent
of the
motion of
the charge

$$q \vec{v} \times \vec{B}(\vec{r}, t)$$

FORCE that
depend on
the velocity
 \vec{v} of the
charge.

DEFINITION of the MAGNETIC FIELD



First, some facts:

- At a particular point "P" there exist a direction XX' such that:

- a) No matter how the charge q moves, it experiences a force \vec{F} perpendicular to XX' (except if \vec{v} is parallel to XX')

When the charge q at "P" is moving with velocity \vec{v} it experiences a force \vec{F} perpendicular to \vec{v} (except if \vec{v} is parallel to XX')

- If q is stationary at "P"

it experiences no force.

From these facts we conclude:

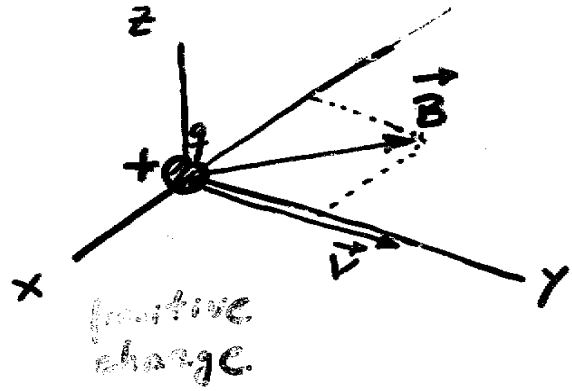
CONCLUSION

XX' defines the direction of the magnetic field at the point P.

And

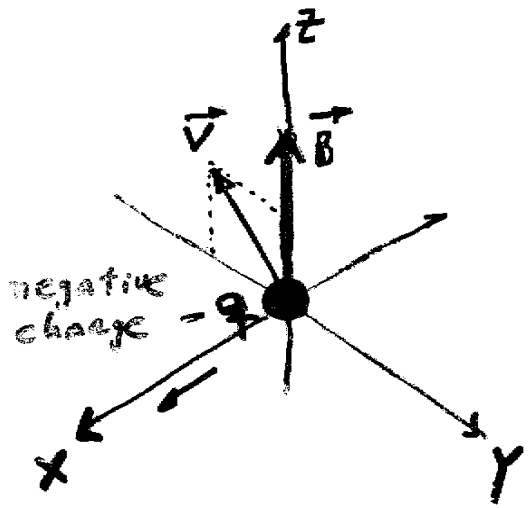
$$\vec{F}_B = q \vec{v} \times \vec{B}$$

Definition of the MAGNETIC FIELD. \vec{B}



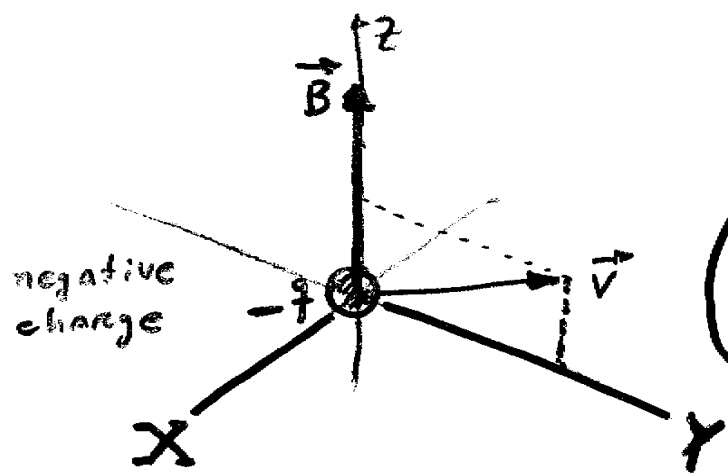
$$\vec{F} = F\hat{r}$$

(\vec{B} and \vec{v} are in the XY plane)



$$\vec{F} = F\hat{z}$$

(\vec{B} and \vec{v} are in the YZ plane)

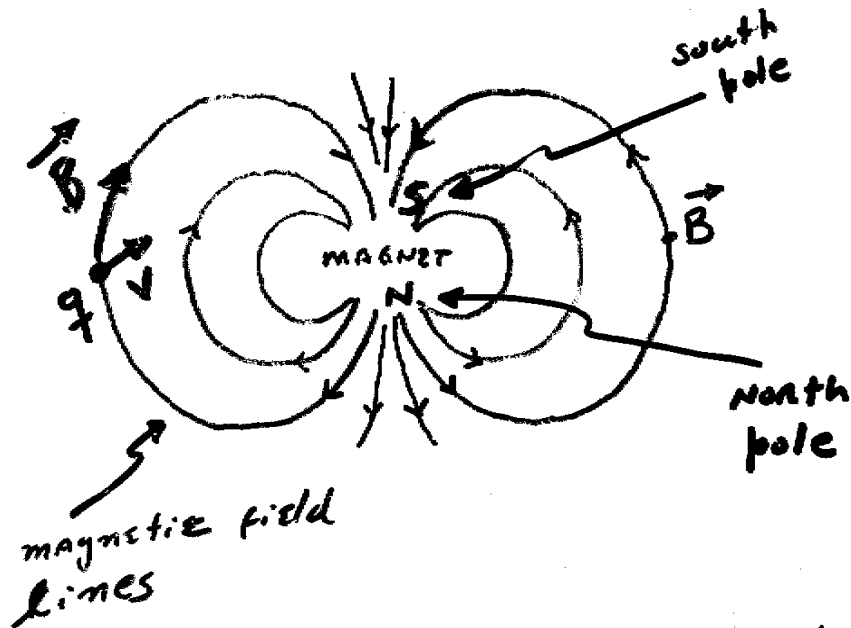


$$\vec{F} = -F\hat{z}$$

(\vec{B} and \vec{v} are in the YZ plane)

$$\vec{F} = q \vec{v} \times \vec{B}$$

Magnetic Field Lines



MAGNETIC
DIPOLE

A magnet produces a magnetic field \vec{B} in its surrounding region.

A charge q , moving with velocity \vec{v} passing by a region where it exists a magnetic field \vec{B} , experiences a force $\vec{F} = q \vec{v} \times \vec{B}$

indicates
vector product

F (Newtom)

v (m/s)

q (coulomb)

B (TESLA)

UNITS

Practice: check-point 1, page 661

7

For the moment, let's analyze those situations in which the magnetic field \vec{B} is given (without worrying about how it is generated).

Later on, we will learn how to calculate the magnetic field produced by currents flowing along a line or a ring. But, for the moment we will consider that $\vec{B} = \vec{B}(\vec{r})$ is given (without caring how it got there).

The simplest case to analyze is the motion of a punctual charge q moving across a uniform magnetic field.

MOTION OF A POINT CHARGE q
IN A UNIFORM MAGNETIC FIELD

$$\vec{F} = q \vec{v} \times \vec{B}$$

\vec{F} perpendicular to \vec{v}

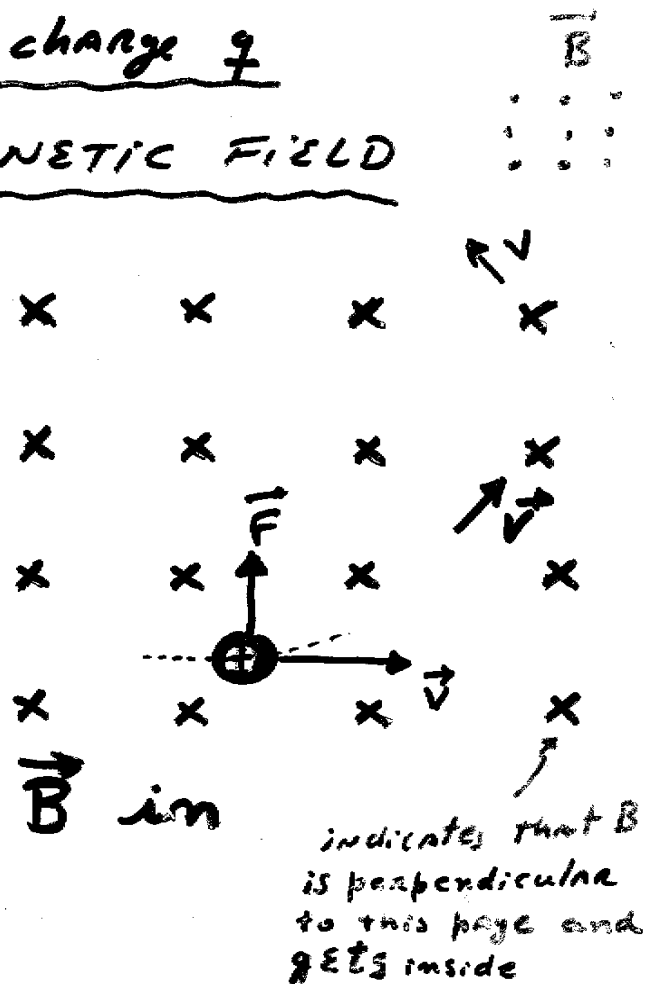
so:

\vec{F} changes the direction of \vec{v} but not its magnitude

this is, the kinetic energy does not change

thus,

MAGNETIC FIELDS do not do WORK on the charge particles



Let's consider those cases in which

$$v < c$$

↑
speed of light

$$F = q v B \quad (1)$$

q is a proton
 $q = 1.6 \times 10^{-19} \text{ C}$

$R = 0.21 \text{ m}$ (radius)

$B = 0.4 \text{ Tesla}$

$v = ?$

If q is undergoing circular motion, the magnetic force F must be responsible of the centripetal acceleration.

$$F = m \frac{v^2}{R} \quad (2)$$

From (1) and (2) we obtain

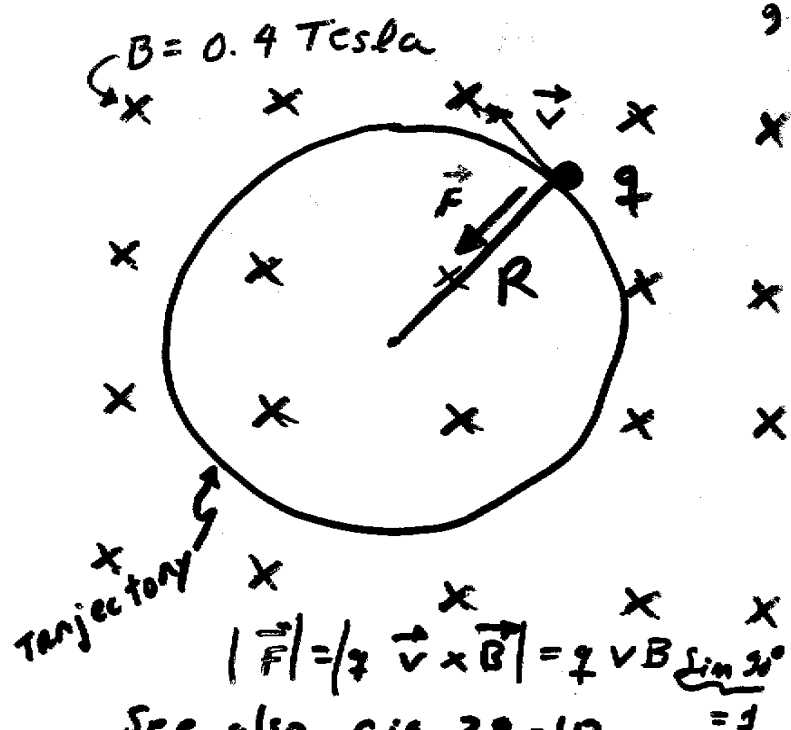
$$q v B = m \frac{v^2}{R} \Rightarrow$$

$$v = \frac{q B R}{m}$$

$$v = 8 \times 10^6 \text{ m/s}$$

$$\text{use } m = 1.67 \times 10^{-27} \text{ kg}$$

* Follow up section 29.5, page 667

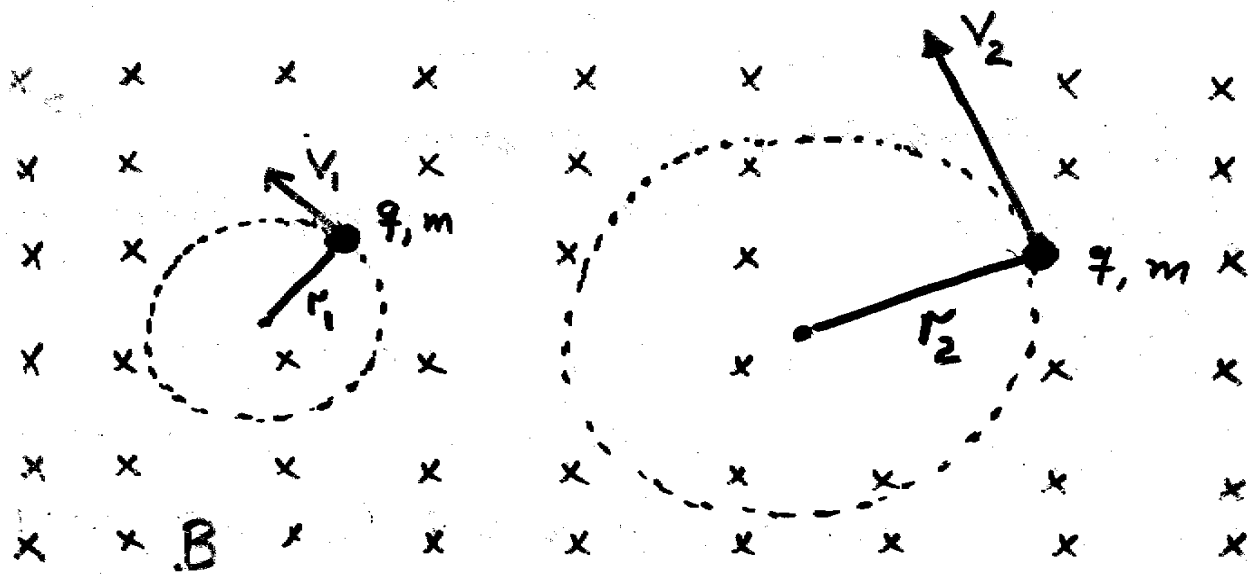


See also fig 29-10 page 668 (be careful that there, in the text-book, the charge is negative)

The result

$$r = \frac{1}{(q/m) B} v$$

indicates that, for a given charged particle and a fixed value of the magnetic field, particles moving at higher speed describe circular paths of bigger radii



$$r_1 < r_2$$

$$v_1 < v_2$$

How long does the proton take to complete "ONE revolution" (from the previous example)

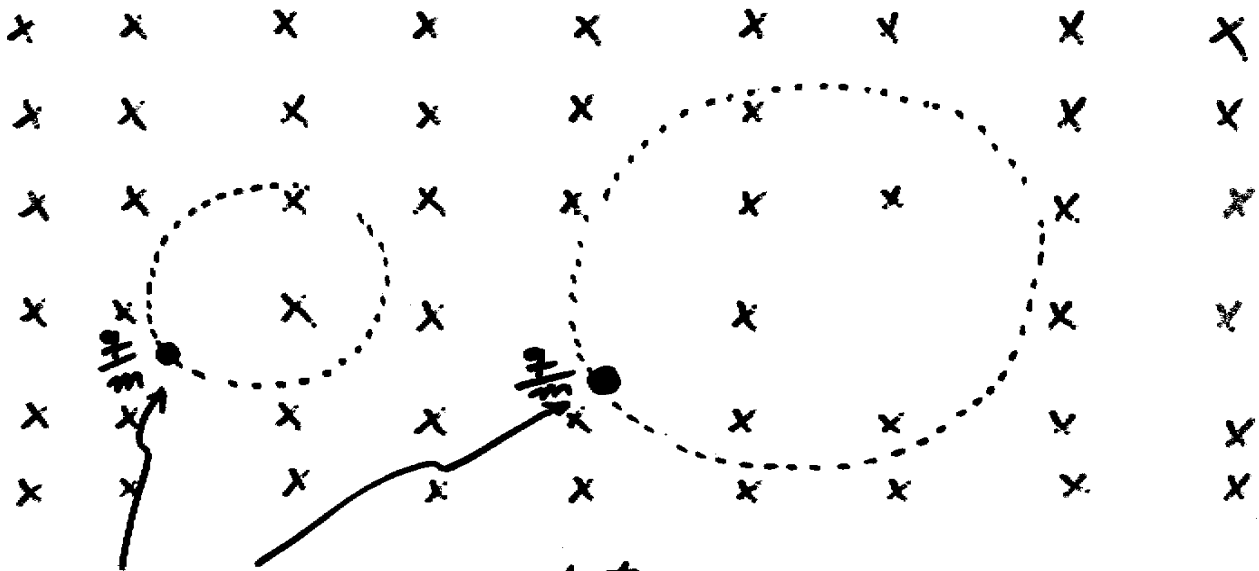
$$T = \frac{\text{length}}{\text{velocity}} = \frac{2\pi R}{v}$$

period

$$= \frac{2\pi R}{\frac{q}{m} B R} = 2\pi \frac{1}{\frac{q}{m} B} = T$$

PERIOD
of the
circular
orbit

Notice: T is independent
of the radius.



Both complete ONE orbit
in the same amount
of time T

How many turns does the proton ^{make} in 1 sec? ¹²
 In other words what is the frequency f ?

$$f = \frac{1}{T}$$

$$f = \frac{1}{2\pi} \frac{q}{m} B \quad \text{frequency}$$

People typically use angular frequency ω :

$$\omega = \frac{\text{radians}}{\text{second}} = \frac{\# \text{ revolutions} \times 2\pi \text{ rad}}{\text{sec}}$$

$$\omega = \frac{q}{m} B$$

Notice: For a given q/m

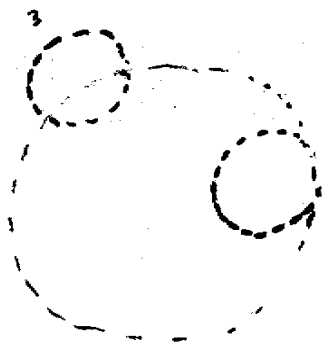
$$\frac{v}{R} = \frac{q}{m} B$$

For a constant magnetic field:
the higher v speed → the higher the radius R

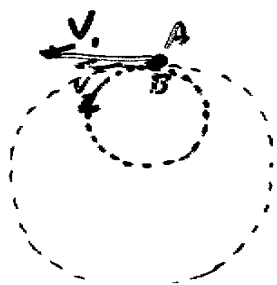
$$\frac{1}{T} = f = \frac{1}{2\pi} \frac{q}{m} B$$

for a constant magnetic field:

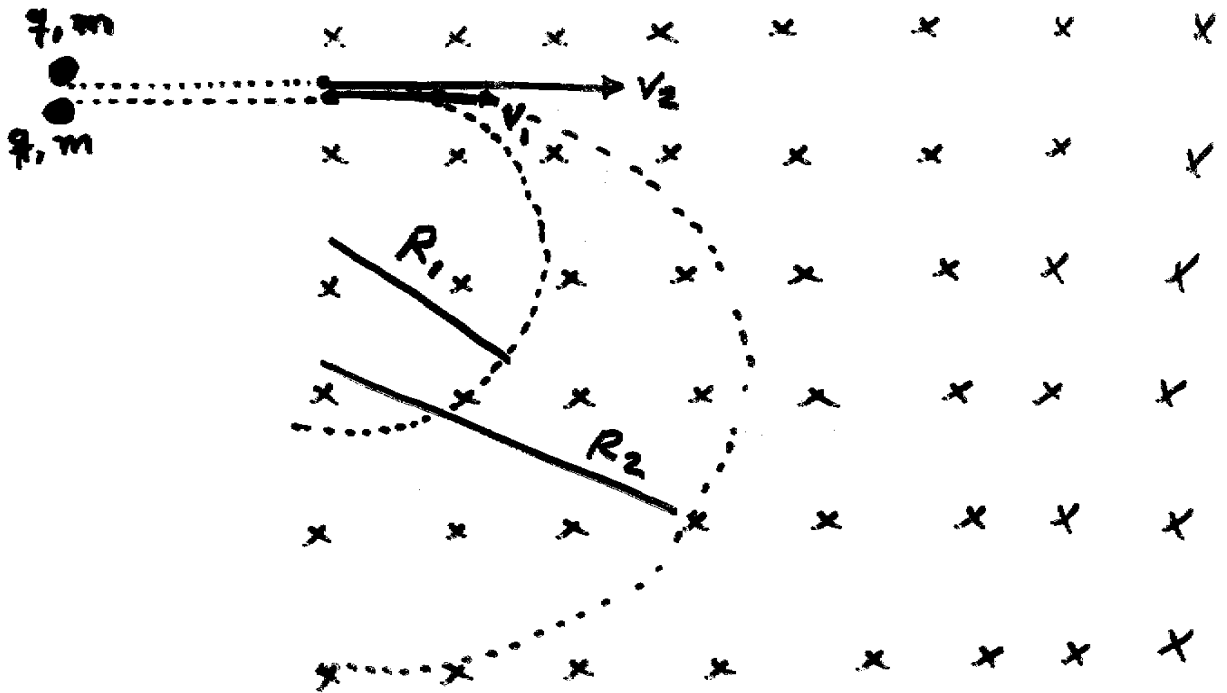
T and f are independent of the velocity



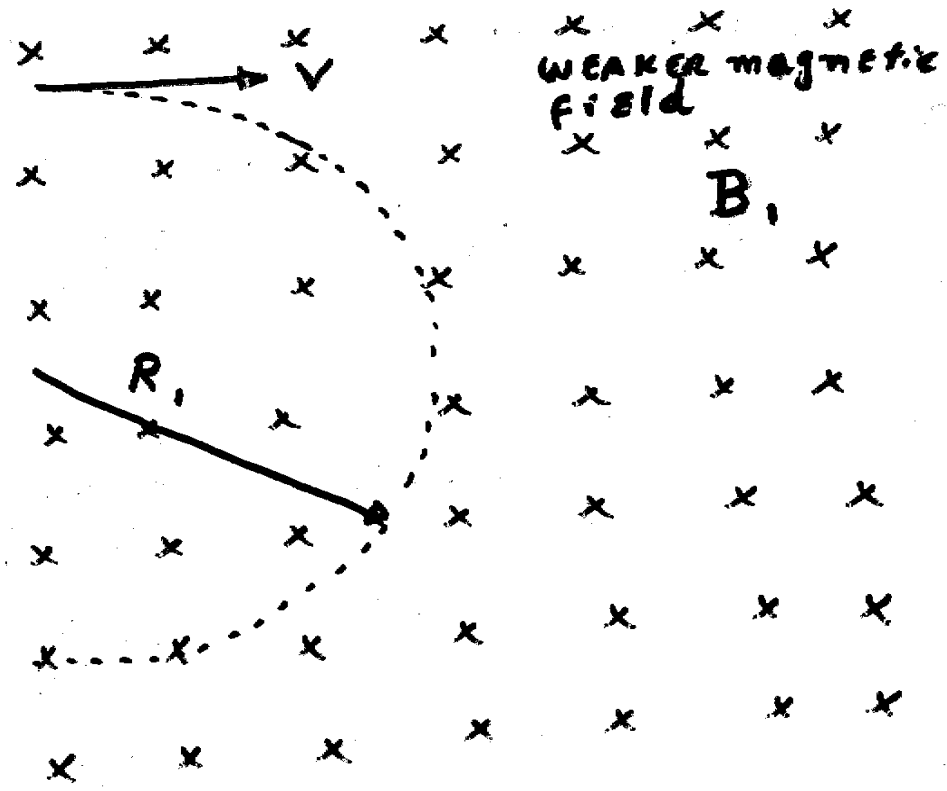
Particles with higher speed move in large circles, slower particles move in smaller circles



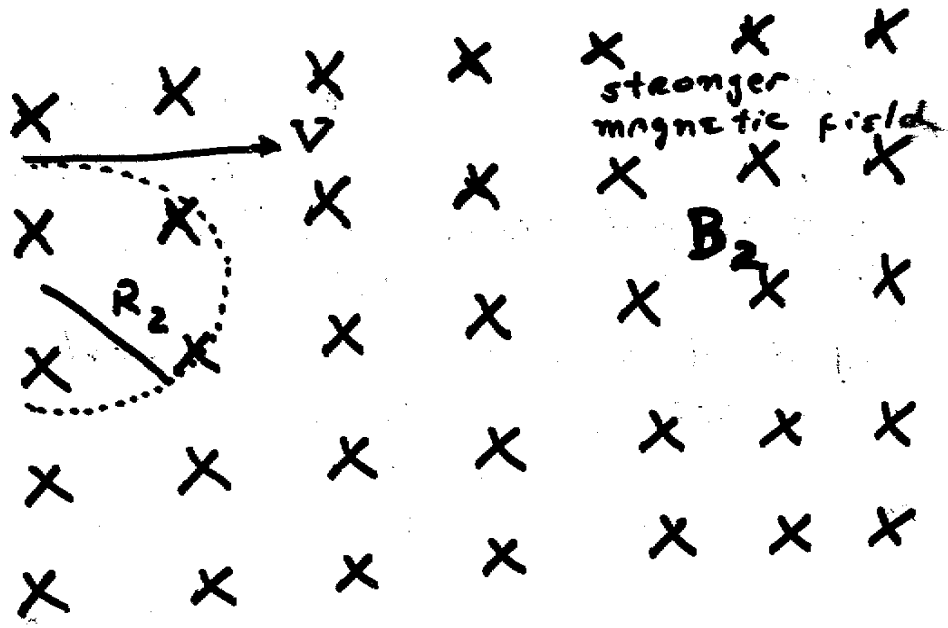
All particles with the same q/m take the same time to complete one revolution



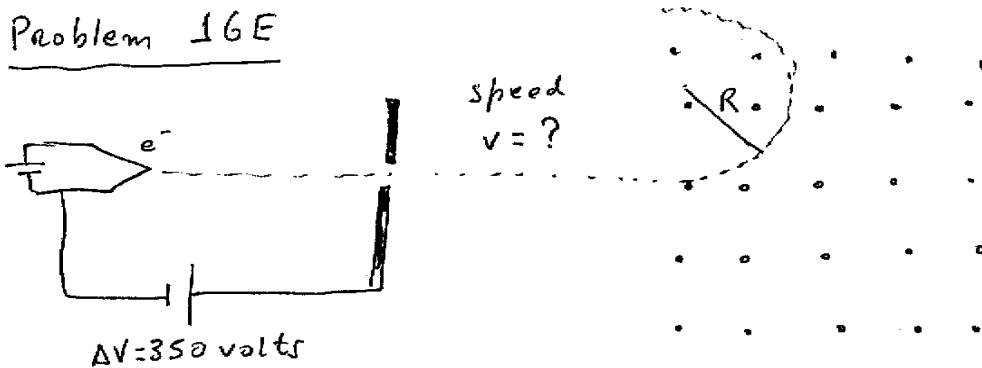
- B does not control \vec{v}
- changing B does not change $|\vec{v}|$
(the speed $v = |\vec{v}|$ remains the same)



$B_1 < B_2$
 implies
 $R_1 > R_2$



Problem 16E



change in electric potential = change in kinetic energy

$$e\Delta V = \frac{1}{2}mv^2$$

$$v^2 = \frac{2e\Delta V}{m} = \frac{2 \times 1.6 \times 10^{-19} \text{ C} \times 350 \text{ volts}}{9.1 \times 10^{-31} \text{ kg}} =$$

$$= \frac{2 \times 1.6 \times 3.5 \times 10^{14}}{9.1} = 1.23 \times 10^{19} \frac{\text{m}}{\text{s}}$$

$$v = 1.11 \times 10^7 \text{ m/s}$$

$$\frac{mv^2}{R} = e v B$$

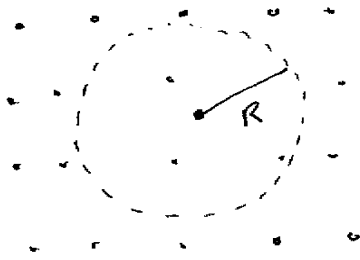
$$\frac{mv}{eR} = B \quad \text{or} \quad \boxed{\frac{mv}{eB} = R}$$

$$R = \frac{9.1 \times 10^{-31} \text{ kg} \times 1.11 \times 10^7 \text{ m/s}}{1.6 \times 10^{-19} \text{ C} \times 0.2}$$

$$= \frac{9.1 \times 1.1 \times 10^{-5}}{1.6 \times 0.2} \text{ m}$$

$$= 31.55 \times 10^{-5} \text{ m}$$

$$R = 315.5 \times 10^{-6} \text{ m}$$

Problem 17 E

$$R = 25 \text{ cm} \\ = 0.25 \text{ m}$$

$$K = 1.2 \text{ keV} \\ = 1.2 \times 10^3 \text{ eV} \\ = 1.2 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ Joules}$$

$$K = 1.92 \times 10^{-16} \text{ Joules}$$

$$a) \frac{1}{2} m v^2 = K$$

$$v^2 = \frac{2K}{m} = \frac{2 \times 1.92 \times 10^{-16} \text{ Joules}}{9.1 \times 10^{-31} \text{ kg}} = 0.42 \times 10^{15} = 4.2 \times 10^{14}$$

$$v = 2.05 \times 10^7 \text{ m/s}$$

$$b) B = \frac{mv}{eR} = \frac{9.1 \times 10^{-31} \text{ kg} \times 2.05 \times 10^7 \text{ m/s}}{1.6 \times 10^{-19} \text{ C} \times 0.25} = \frac{9.1 \times 2.05}{1.6 \times 0.25} \times 10^{-5} \text{ T}$$

$$B = 4.6 \times 10^{-4} \text{ Tesla} \\ = 4.6 \text{ Gauss}$$

Equivalence

$$1 \text{ Tesla} = 10^4 \text{ Gauss}$$

c) frequency of circling

$$f = \frac{1}{2\pi} \frac{q}{m} B = \frac{1}{2\pi} \frac{1.6 \times 10^{-19} \text{ C}}{9.1 \times 10^{-31} \text{ kg}} \times 4.6 \times 10^{-4} \text{ T} = 0.129 \times 10^8$$

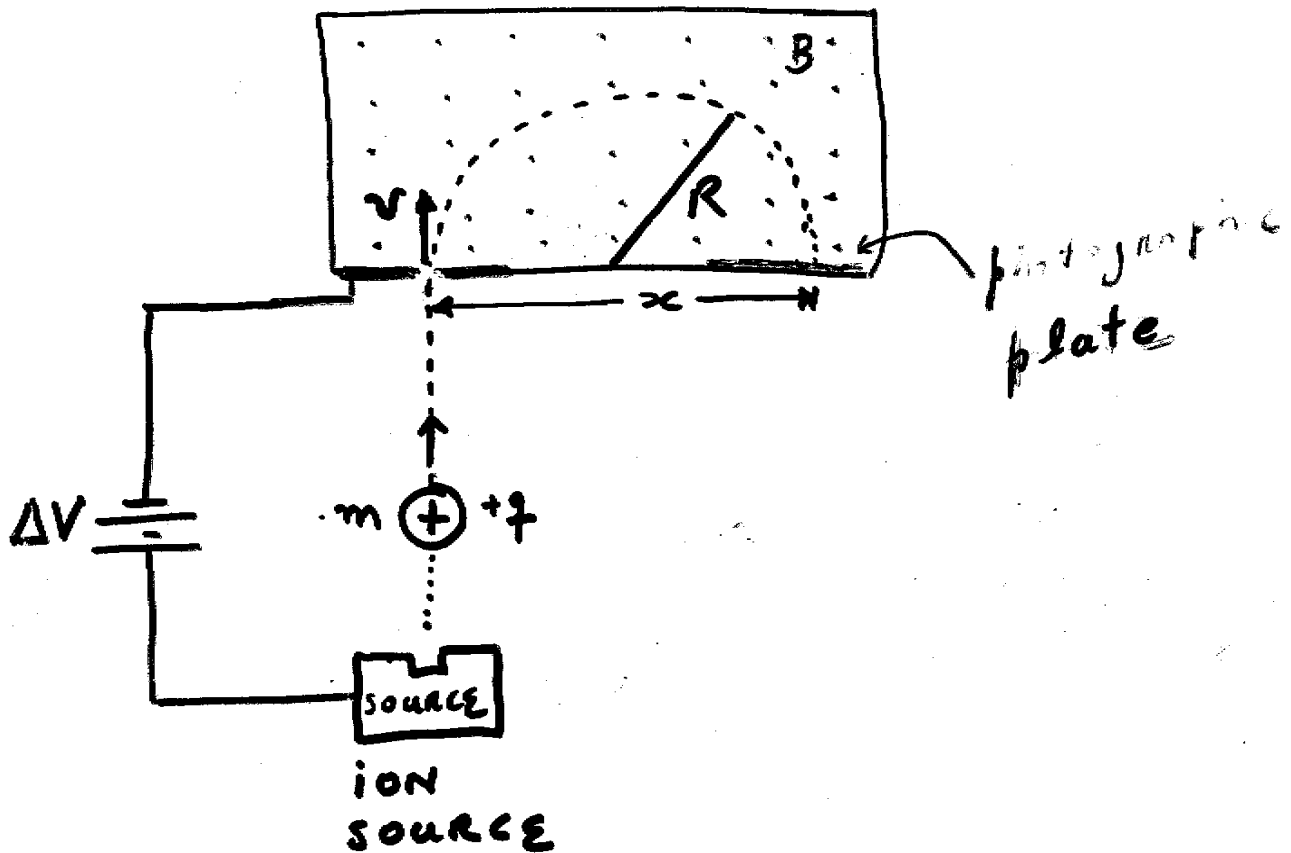
$$f = 12.9 \times 10^6 \text{ Hz}$$

$$\text{or } f = 13 \text{ MHz}$$

$$d) T = \frac{1}{f} = \frac{1}{12.9 \times 10^6 \text{ Hz}} = 77 \times 10^{-9} \text{ sec}$$

$$= 77 \text{ ns} = T \text{ period}$$

The Mass Spectrometer



$$q \Delta V = \frac{1}{2} m v^2$$

$$v = \sqrt{\frac{2 q \Delta V}{m}}$$

$$\frac{m v^2}{R} = q v B$$

$$R = \frac{m v}{q B} = \frac{x}{2}$$

$$x = 2 \frac{m}{qB} v$$

$$= 2 \frac{m}{qB} \sqrt{\frac{2q\Delta V}{m}} = \sqrt{\frac{8m\Delta V}{qB^2}}$$

$$\boxed{\frac{m}{q} = \frac{B^2}{8\Delta V} x^2}$$

Isotopes of different mass m (same q) will strike the photographic plate at different values of x .

Example A ^{58}Ni ion of charge $+e$ and mass $m = 9.62 \times 10^{-26} \text{ kg}$ is accelerated through a potential difference of 3 kV and deflected in a magnetic field of 0.12 T

a) Find the radius of curvature of the orbit of the ion.

$$\frac{9.62 \times 10^{-26} \text{ kg}}{1.6 \times 10^{-19} \text{ C}} = \frac{(0.12 \text{ T})^2}{8 \times 3 \times 10^3 \text{ V}} x^2 \Rightarrow \begin{aligned} x &= 1.0020 \text{ m} \\ R &= 0.5010 \text{ m} \end{aligned}$$

b) Find the difference in the radii of curvature of ^{58}Ni and ^{60}Ni

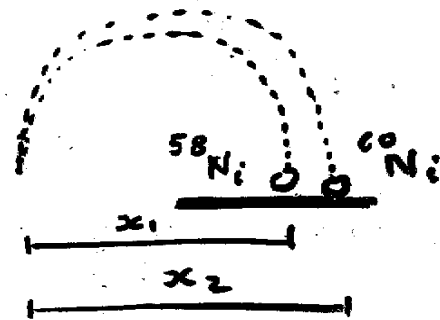
$$\frac{m_1}{m_2} = \frac{x_1^2}{x_2^2}$$

We take $\frac{m_1}{m_2} = \frac{58}{60}$

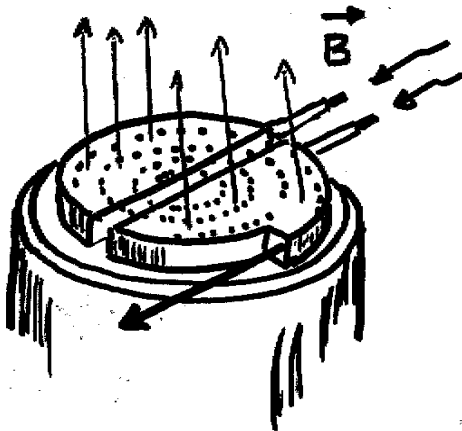
$$\Rightarrow \frac{x_1}{x_2} = 0.983 = \frac{R_1}{R_2}$$

Since $R_1 = 0.5010 \text{ m}$

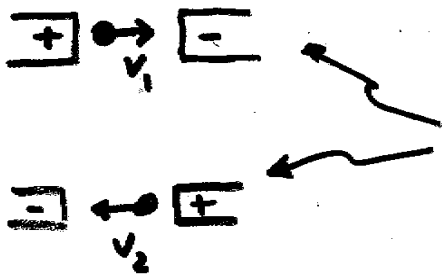
then $R_2 = 0.5095 \text{ m} \Rightarrow R_2 - R_1 = 8.56 \text{ mm}$



The Cyclotron



high frequency
alternating voltage
 f_{osc}



the polarity of the
electrodes is changed
in a synchronized way
with the motion of the
particle

We already know that, given a particle of mass m and charge q , it will circle inside a uniform magnetic field with

$$\text{a frequency } f = \frac{1}{2\pi} \frac{q}{m} B$$

regardless of the particle's speed

So, in a cyclotron the alternating voltage (see previous figure) is tuned

until
$$f_{osc} = \frac{1}{2\pi} \frac{q}{m} B$$

Example A cyclotron uses a magnetic field $B = 1.5 \text{ T}$ and it is going to be used to accelerate protons.

a) What should be the frequency of the alternating voltage?

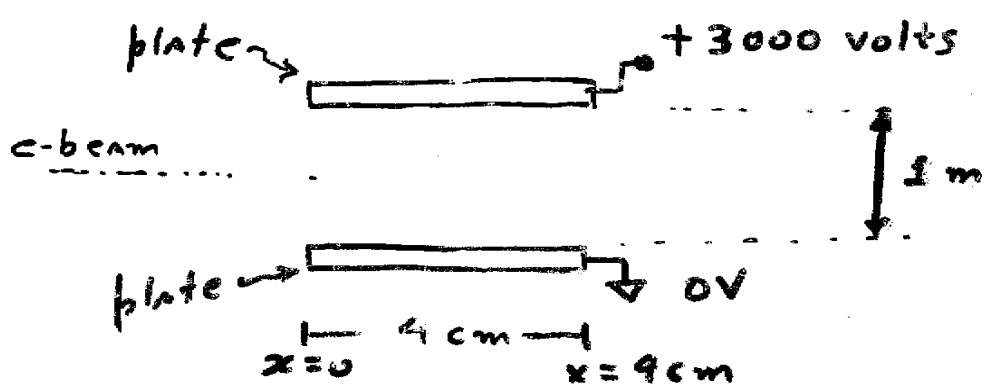
$$f_{osc} = \frac{1}{2\pi} \frac{1.6 \times 10^{-19} \text{ C} \times 1.5 \text{ T}}{1.67 \times 10^{-27} \text{ kg}} = 23 \text{ MHz}$$

b) If the radius of the D-shape electrodes is 0.5 m , what is the kinetic energy of the protons when they emerge?

PRACTICE:
Read sample problem 29-5

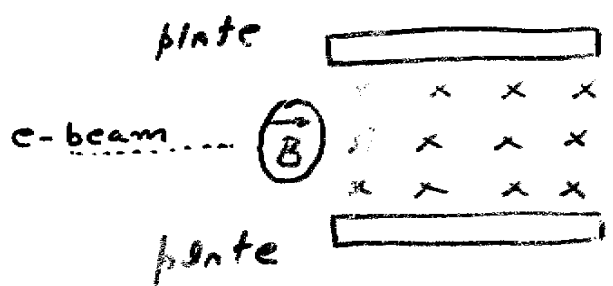
Practice problems: 16E, 19E, 20E, 21E
 (page 683) 22P, 28P

EXERCISE: DRAW schematically the trajectory followed by the indicated electron beam

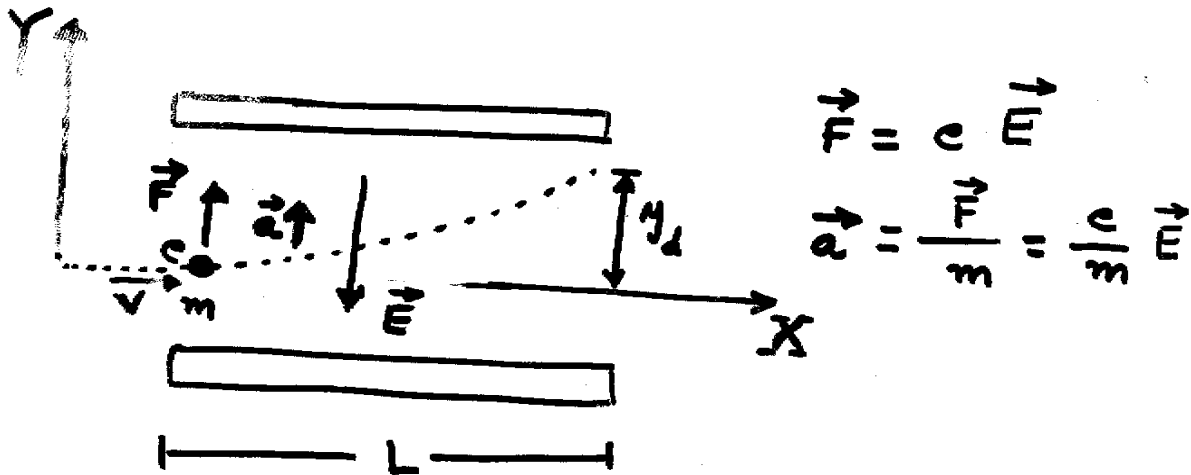


Find the position of the beam when it exists the plates (at $x = 4 \text{ cm}$)

EXERCISE: DRAW schematically the trajectory followed by the electron beam



(No electric field are applied in this case)



$$y = \frac{1}{2} a t^2$$

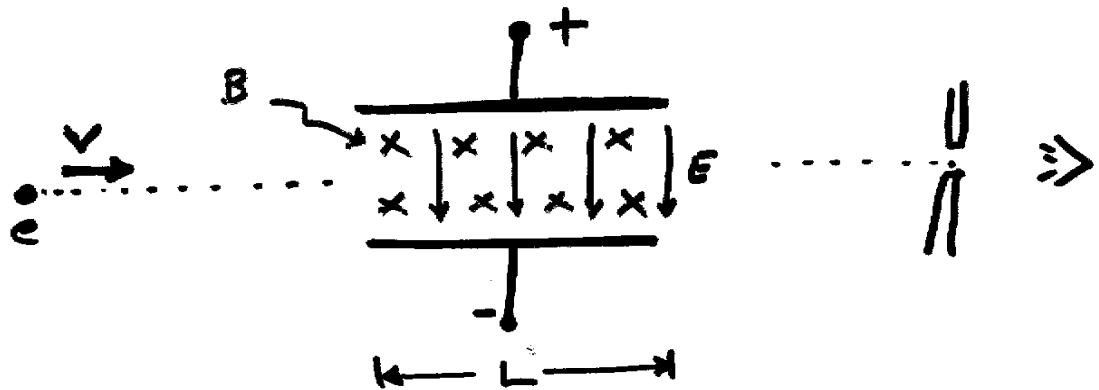
How much does the electron deflect after passing the plates? $y_d = ?$

Answer: We obtain y_d when $t = \frac{L}{v}$

$$y_d = \frac{1}{2} a \left(\frac{L}{v} \right)^2$$

$$y_d = \frac{1}{2} \frac{e}{m} E \frac{L^2}{v^2}$$

Discovery of the electron



- The strength of the magnetic field is increased and adjusted until the incident particle does not experience any vertical deflection.

No deflection implies:

$$eE = evB$$

$$v = \frac{E}{B} \quad (1)$$

- When the magnetic field is turned off the particle is deflected vertically by a distance y_d whose relationship with v is

$$v^2 = \left(\frac{e}{m}\right) \frac{E}{2} \frac{L^2}{y_d} \quad (2)$$

From experiments (1) and (2) we obtain²⁶

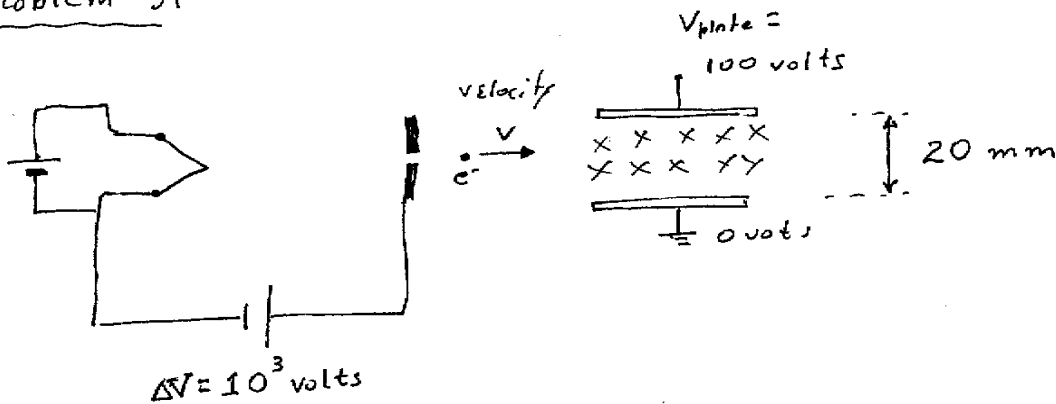
$$\frac{E^2}{B^2} = \left(\frac{c}{m}\right) \frac{E}{2} \frac{L^2}{\gamma_d}$$

$$\Rightarrow \boxed{\frac{c}{m} = \frac{B^2}{E} \frac{L^2}{2\gamma_d}}$$

J. J. Thomson
1897

Problem 9P

(20)



Let's find first the velocity:

change in electric potential = change in kinetic energy

$$e \times \Delta V = \frac{1}{2} m v^2$$

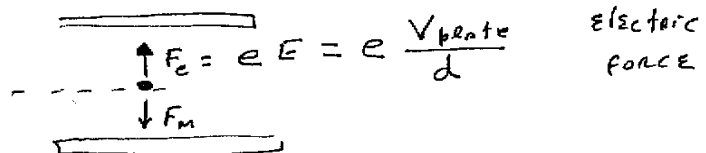
$$1.6 \times 10^{-19} \text{ C} \times 10^3 \text{ volts} = \dots$$

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

$$v = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \text{ C} \times 10^3 \text{ volts}}{9.1 \times 10^{-31} \text{ kg}}}$$

$$= \sqrt{\frac{32 \times 10^{-14}}{9.1}}$$

$$= 1.87 \times 10^7 \text{ m/s}$$



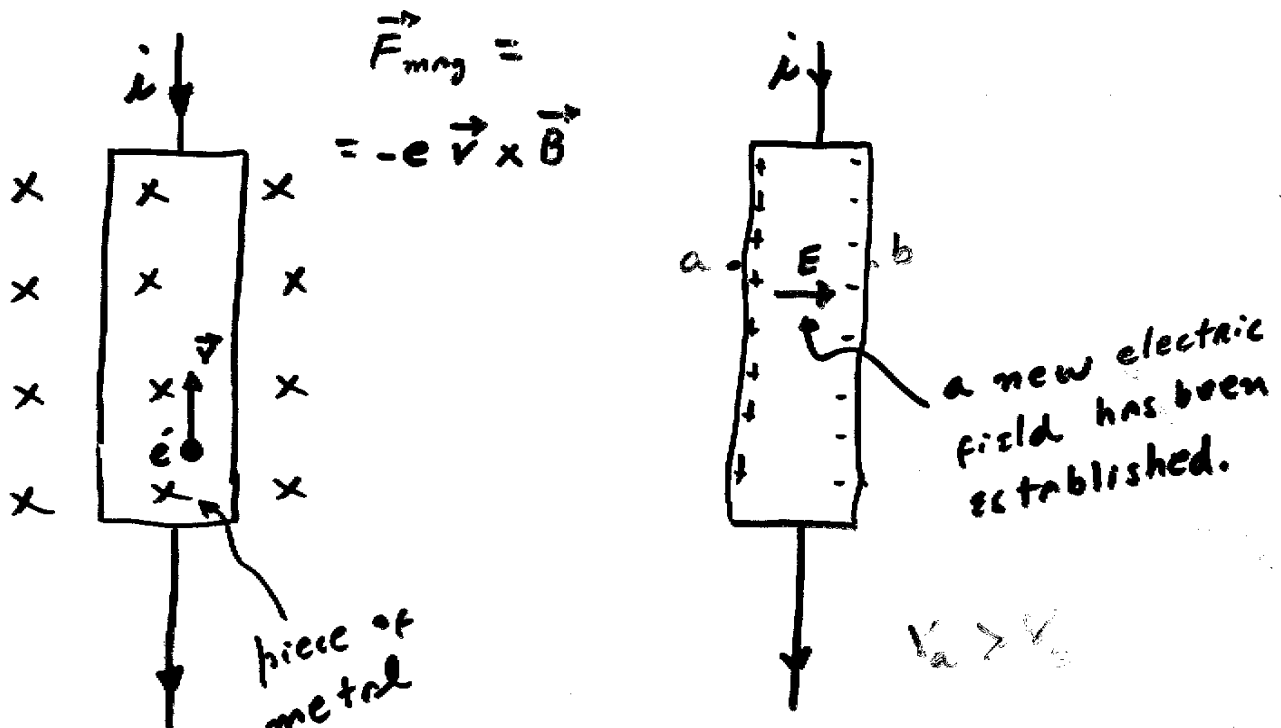
$$F_m = e v B$$

electron travels along a straight line $\Rightarrow F_e = F_m$

$$e \frac{V_{\text{plate}}}{d} = e v B$$

$$\Rightarrow \boxed{B = \frac{V_{\text{plate}}}{d v}} = \frac{100}{20 \times 10^{-3} \times 1.87 \times 10^7} = 2.67 \times 10^{-4} \text{ T}$$

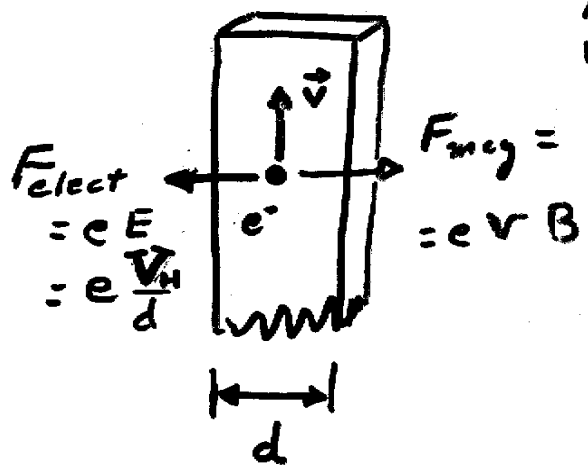
The Hall Effect



$$\vec{F}_{mag} = -e \vec{v} \times \vec{B}$$

$$V_a > V_b$$

Let's call $V_a - V_b = V_H$ ← Hall voltage



$$F_{\text{elect}} = F_{\text{mag}}$$

$$e \frac{V}{d} = e v B$$

$$v = \frac{V_H}{d B}$$

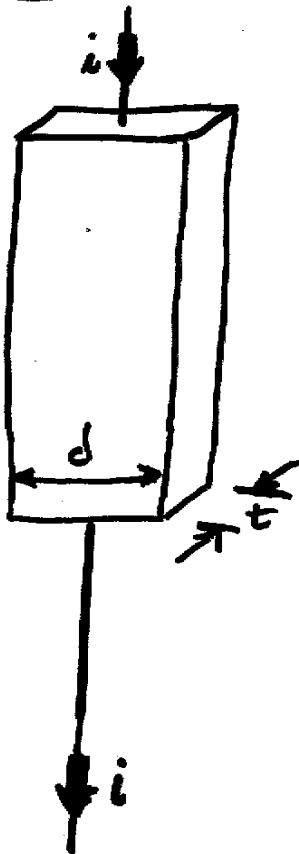
drift velocity

of charge carriers per unit volume

$$\frac{i}{t d} = j = n e v$$

$$\frac{i}{t d} = n e \frac{V_H}{d B}$$

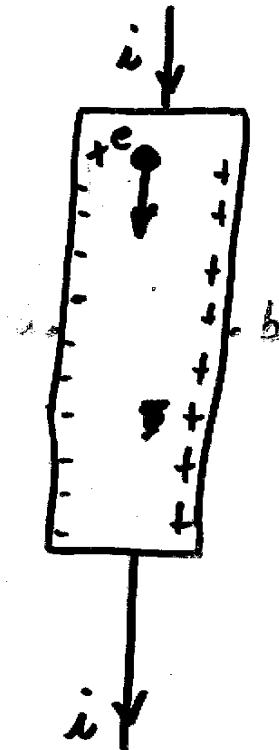
$$V_H = \frac{i}{n e t} B$$



The Hall voltage V_H provides a method to measure magnetic fields.

Had we assumed the current i was established by positive charges in motion, we would have obtain the following situation:

Since we can measure
 $V = V_b - V_a$
 we should have been able to determine which voltage, V_a or V_b is higher.



$V_a < V_b$
 measurable

• The situation

$$V_a < V_b$$

has not been observed in METALS, which confirm that the charge carriers in metals are negative charges.

• However, both situations $V_a < V_b$ and $V_a > V_b$ are observed in SEMICONDUCTORS.