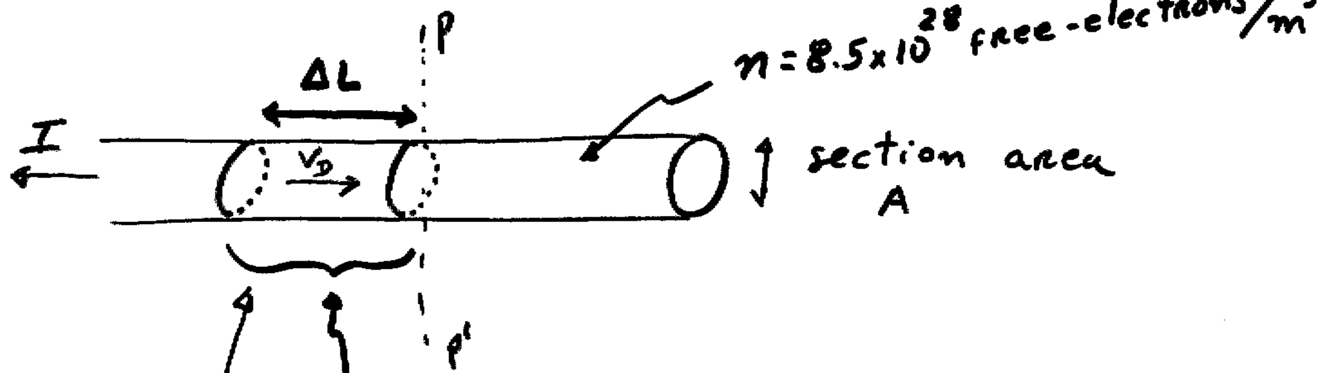
MAGNETIC FORCE ON A CURRENT-CARRYING WIRE.-

First, let's make a connection between the current I and the drift speed.



amount of charge in this volume $\Delta q = ne \times \text{volume}$ ①

$= ne(\Delta L)A$

All this charge will cross the plane PP' in a time Δt $\qquad \Delta t = \dfrac{\Delta L}{V_D}$ ②

From ① and ② we can find I

$$\underbrace{I}_{\text{current}} = \frac{\Delta q}{\Delta t} = n \underbrace{e}_{\text{electron charge}} \underbrace{v_D}_{\text{drift speed}} \underbrace{A}_{\text{section area of the wire}}$$

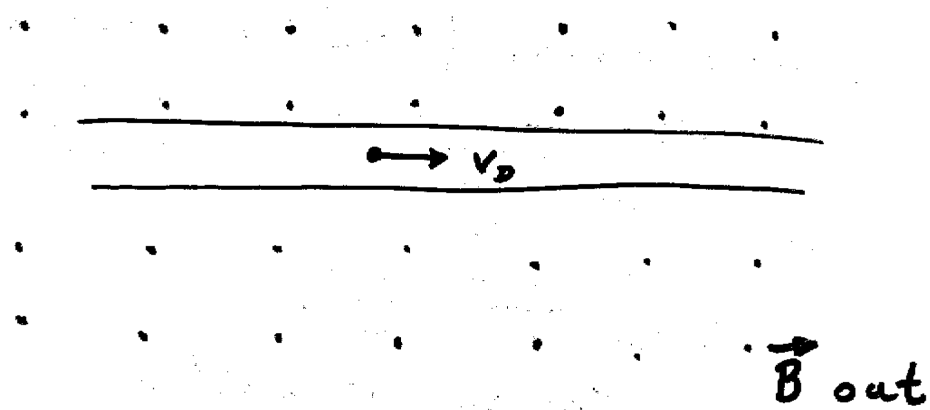
OR

$$\boxed{j = \frac{I}{A} = n e v_D}$$

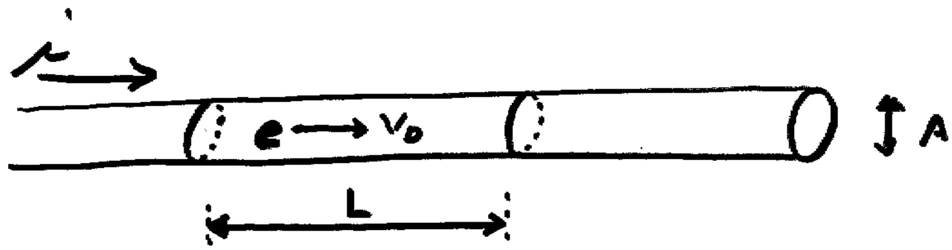
$e = 1.6 \times 10^{-19}$ C

n: # of free electrons per m^3

Let's place the wire in a region where it exists a magnetic field



What happens ?



Let's consider just a segment of length "L" of the wire

the force on each charge e is

$$e v_D B$$

the number of charge in the wire segment is

$$n L A$$

So, the total force is

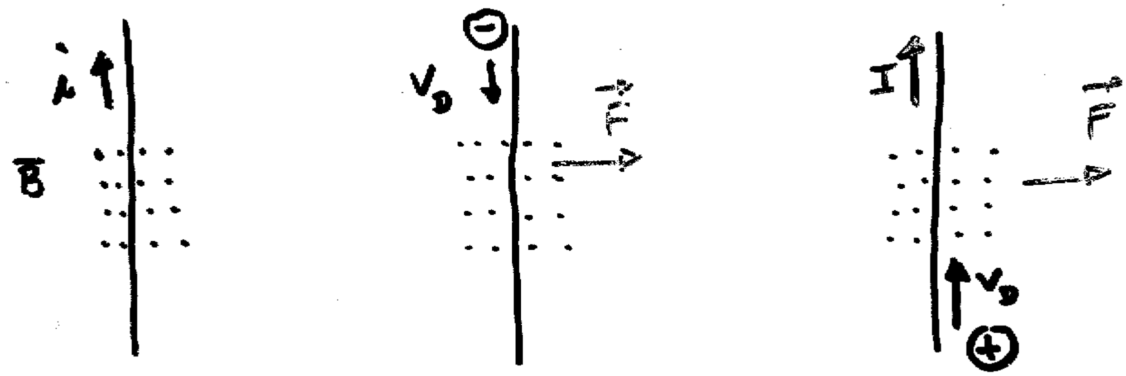
$$F = (e v_D B) n L A$$

Re-arranging the terms, we obtain

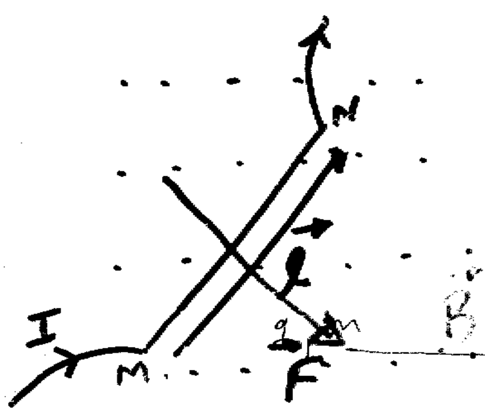
$$F = \underbrace{n e v_D A}_{I} \cdot B L$$

$$F = I B L$$

$$\vec{F} = q\,\vec{v} \times \vec{B}$$



Force on the segment MN

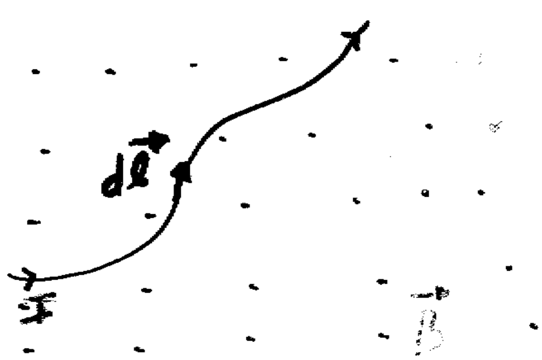


$$\vec{F} = I\,\vec{l} \times \vec{B}$$

current

magnetic field (vector)

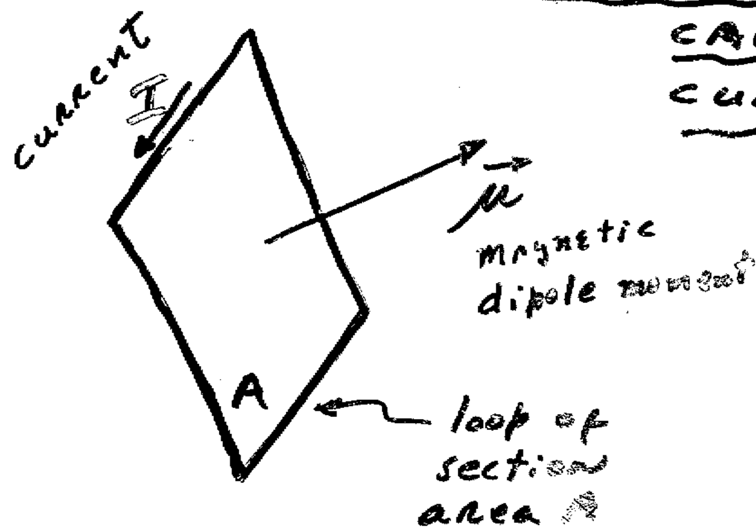
Force on the small segment $d\vec{l}$



$$\Delta\vec{F} = I\,d\vec{l} \times \vec{B}$$

practice: checkpoint #5, p.675
sample problem 29-6 p.675

TORQUE produced by an external MAGNETIC FIELD on a loop that carries a current I



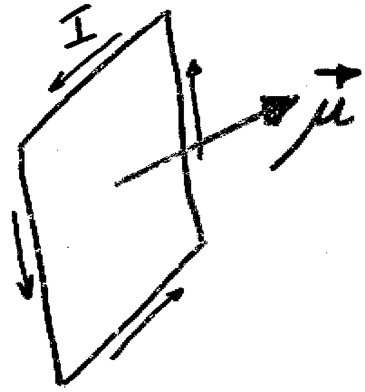
A current-carrying loop is characterized by its magnetic-dipole moment $\vec{\mu}$, defined as:

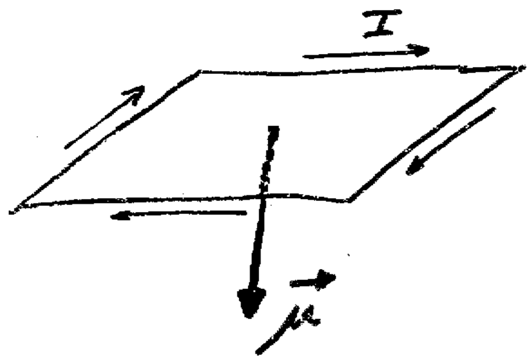
$$\vec{\mu} = N I A \hat{n}$$

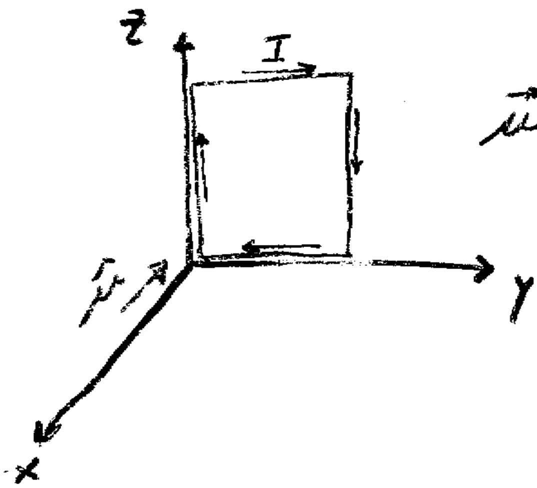
- $\hat{n}$: unit vector, $\perp$ to the loop
- A: cross section area of the loop
- I: current flowing through each coil in the loop.
- N: Number of turns in the loop

examples:

RULE





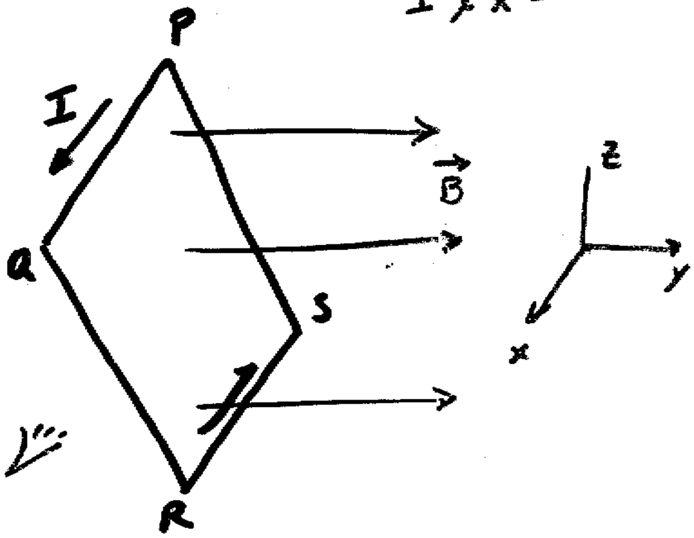


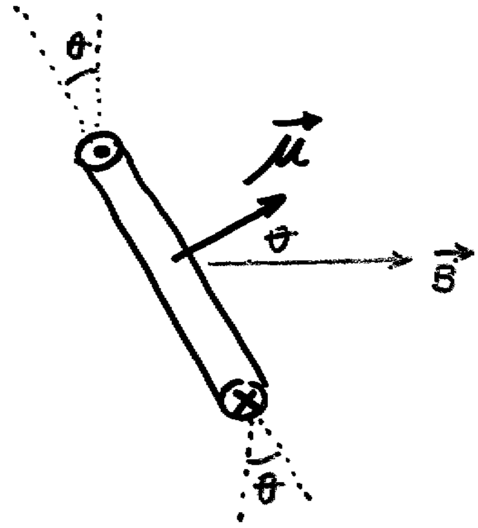
$\vec{\mu} = \mu \hat{x}$?

which direction?

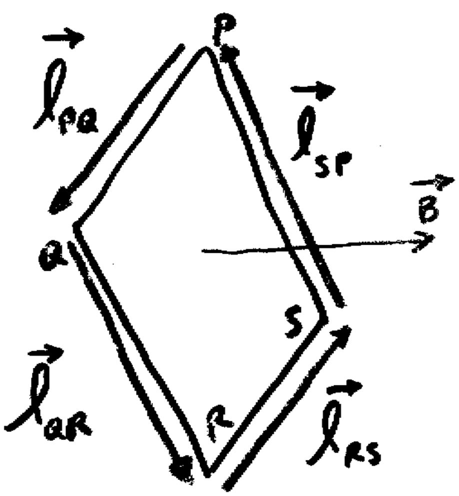
Draw the forces acting on a rectangular coil immersed in a uniform magnetic field

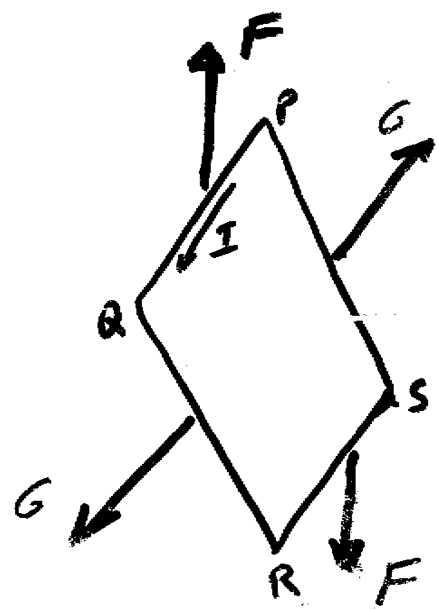
$I \vec{l} \times \vec{B}$

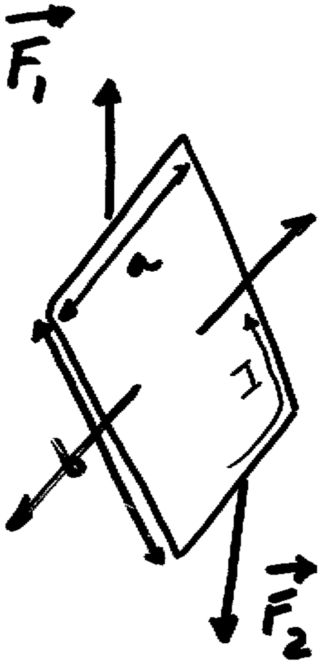


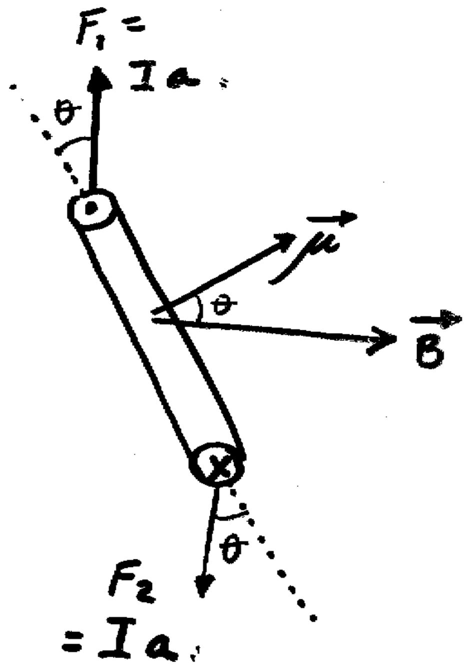


On the segment PQ: $\vec{F} = I \vec{l}_{PQ} \times \vec{B}$



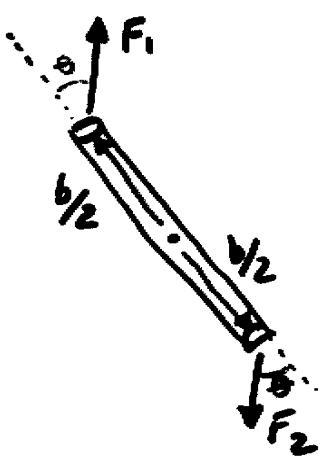






NOTICE $F_1 = F_2$

$\vec{F_1}$ and $\vec{F_2}$ produce a <u>torque</u> on the loop



$$\tau_1 = \left(\frac{b}{2}\right) F_1 \sin\theta$$

Likewise

$$\tau_2 = \left(\frac{b}{2}\right) F_2 \sin\theta$$

total torque $\tau = \tau_1 + \tau_2 \Rightarrow \tau = b\, I a B \sin\theta$

But

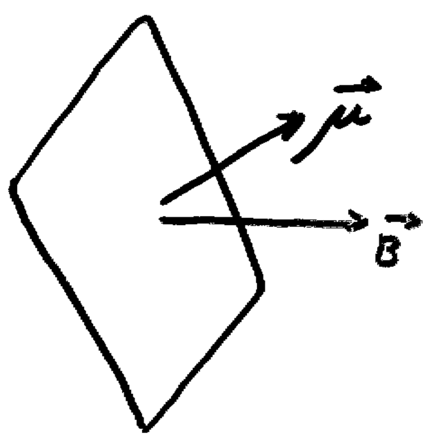
ab = area of the loop. = A

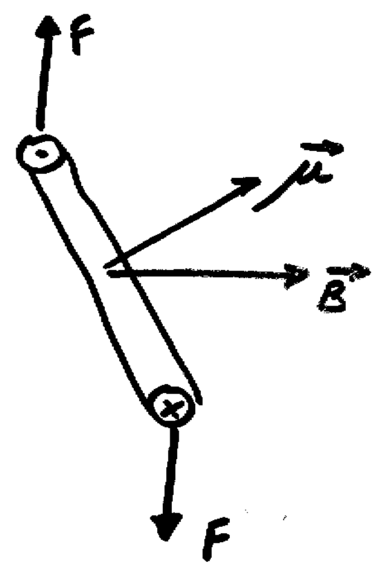
so,

$$\tau = \underbrace{I A}_{\vec{\mu}} B \sin\theta$$

$\vec{\mu}$ magnetic dipole

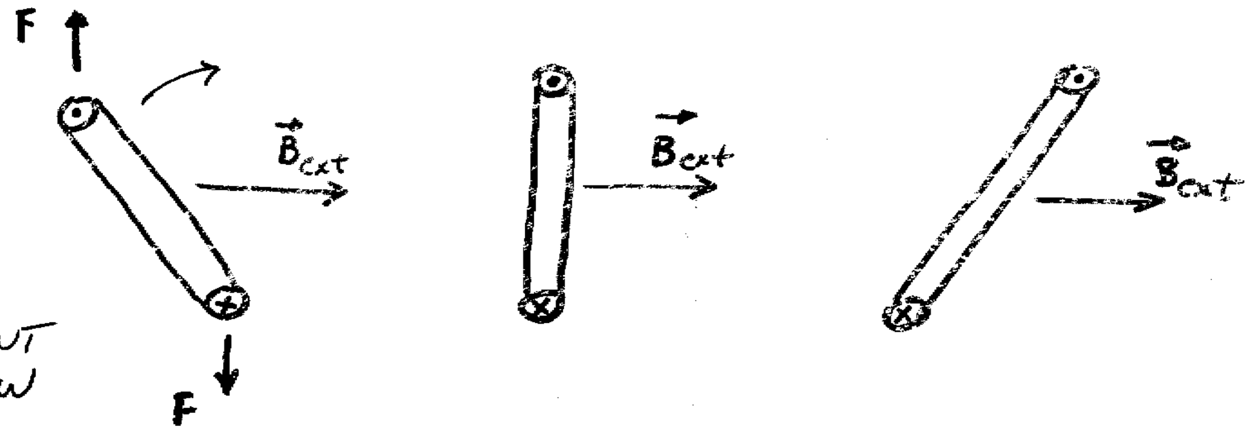
$$\boxed{\vec{\tau} = \vec{\mu} \times \vec{B}}$$



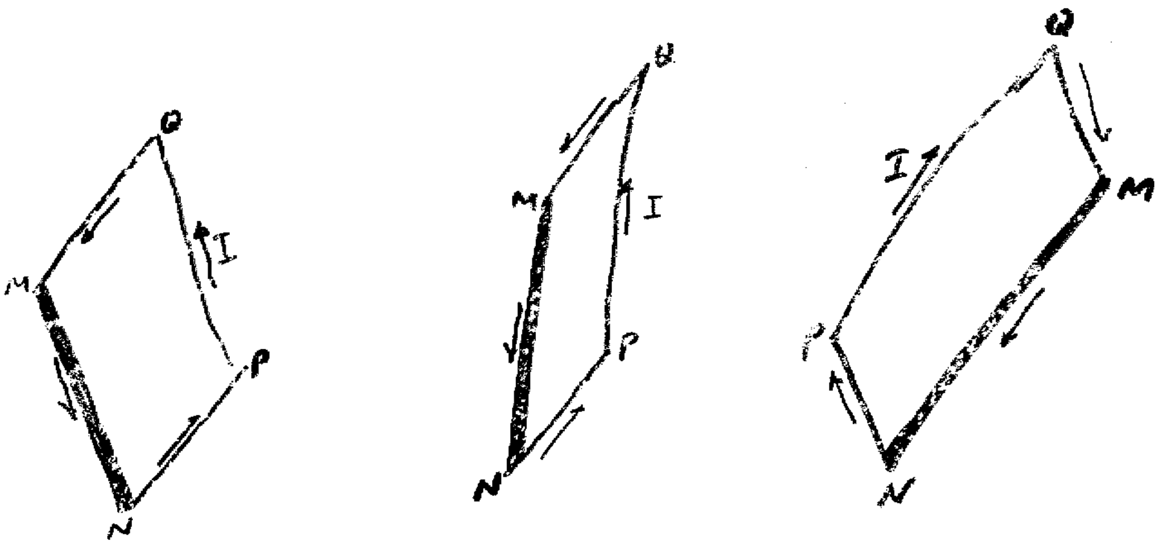


Conclusion: the loop experiences a torque that tends to make the magnetic dipole $\vec{\mu}$ aligned along the external magnetic field $\vec{B}$

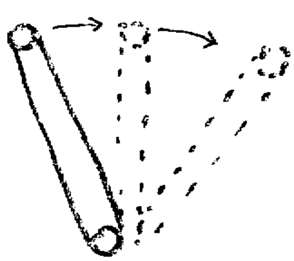
Mechanical motion of a coil that is immersed in a region where exists an external magnetic field.



FRONT VIEW



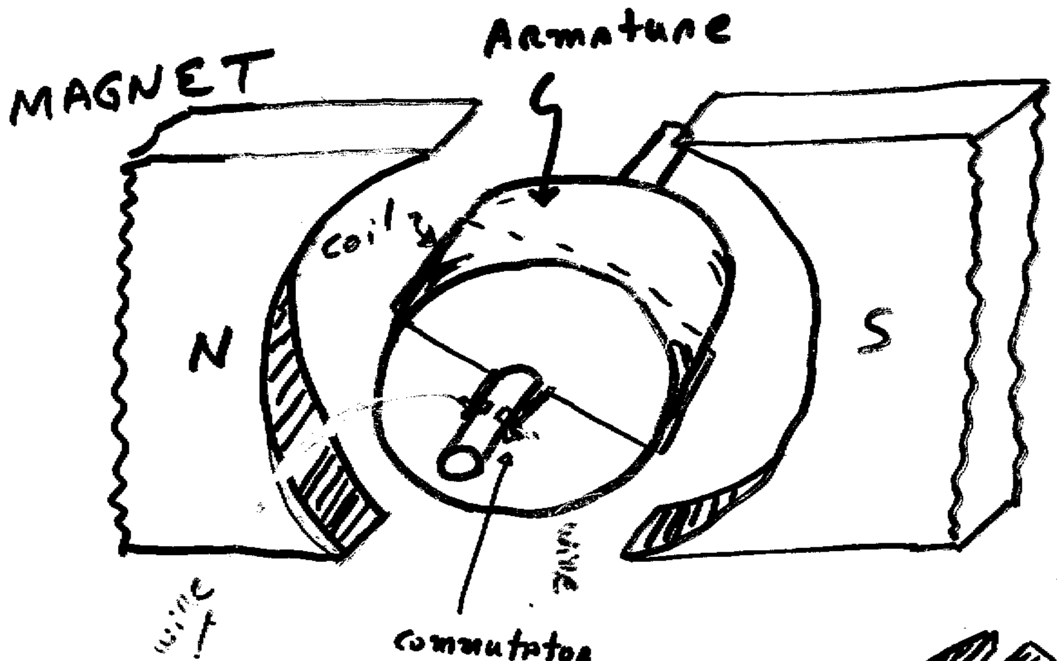
3D view

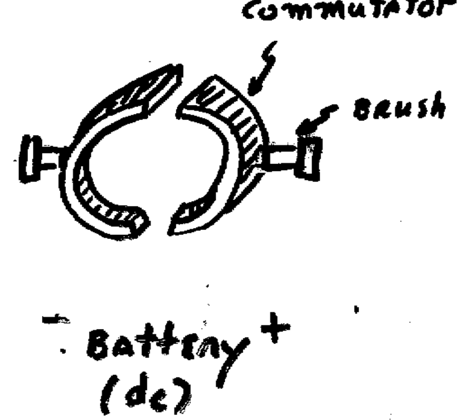


The Electric Motor

An electric motor changes electric energy into (rotational) mechanical enegy

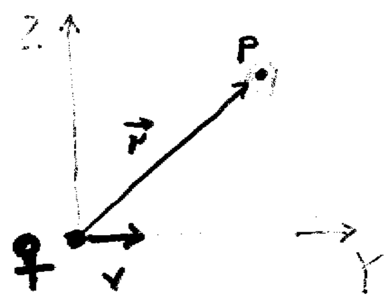
$\vec{F} = I\,\vec{l} \times \vec{B}$





- Coil is attached to the armature. they rotate together.
- Commutater and wires remain fixed.

MAGNETIC FIELD PRODUCED BY A MOVING POINT CHARGE



$$\vec{B} = \frac{\mu_0}{4\pi} \frac{q\,\vec{v} \times \vec{r}}{r^3}$$

⊗ means a vector is pointing INTO the plane

⊙ means a vector is pointing OUT of the plane

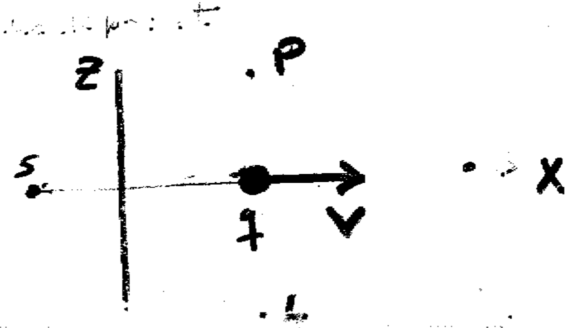
q charge (coulomb)

$\vec{v}$ velocity (m/s)

$\vec{r}$ vector position (m)

$\vec{B}$ Magnetic field unit is TESLA [T]

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{T\,m}{Amp.}$$

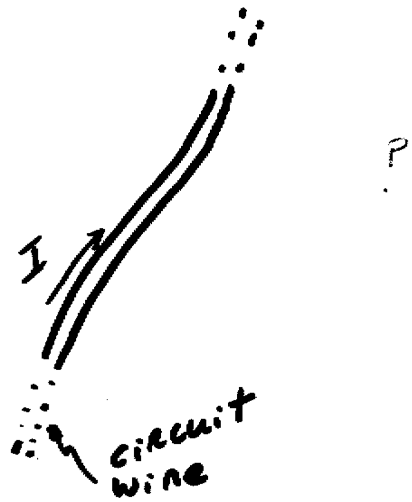


B_P =

B_L =

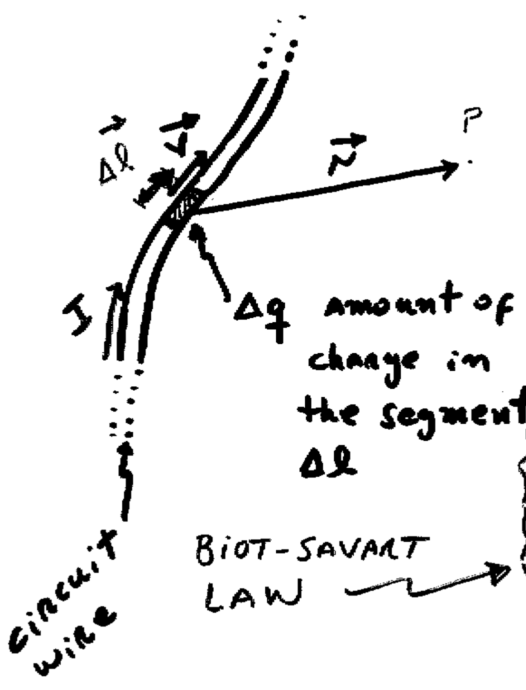
B_S =

MAGNETIC FIELD PRODUCED BY A CURRENT I



What is the magnetic field at point P ?

Strategy: To divide the wire in small sections of length $\Delta \ell$



$$\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{\Delta q \, \vec{v} \times \vec{r}}{r^3}$$

but notice:

$$\Delta q \, \vec{v} = \Delta q \frac{\Delta \vec{\ell}}{\Delta t} = I \Delta \vec{\ell}$$

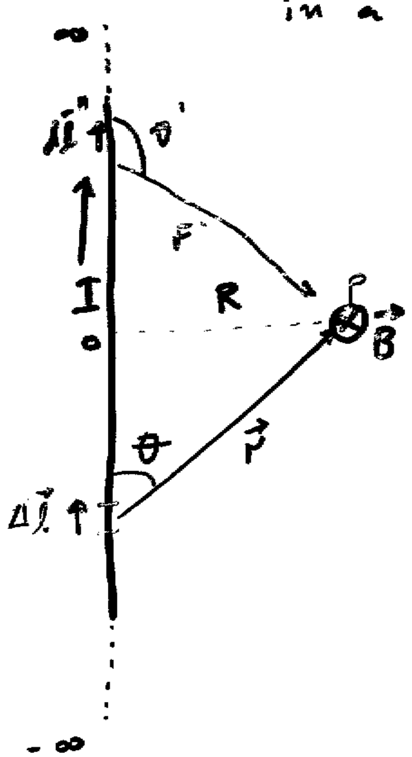
$$\boxed{\Delta \vec{B} = \frac{\mu_0}{4\pi} \frac{I \, \Delta \vec{\ell} \times \vec{r}}{r^3}}$$

BIOT-SAVART LAW

contribution to the magnetic field due to the segment $\Delta\vec{\ell}$ of the circuit

3

Example: Magnetic field due to a current I in a long straight wire



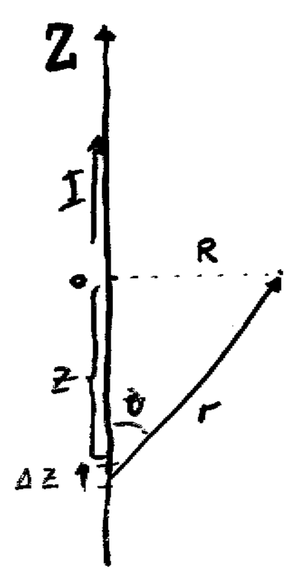
$$\Delta\vec{B} = \frac{\mu_0}{4\pi}\,\frac{I\,\Delta\vec{l}\times\vec{r}}{r^3}$$

We want to evaluate the magnetic field at a point "p" a distance "R" far away from the wire.

$$\Delta B = \frac{\mu_0}{4\pi}\,\frac{I(\Delta l)\,r\,\sin\theta}{r^3}$$

$$\left(\sin\theta = \frac{R}{r} \quad \text{or} \quad r\sin\theta = R\right.$$

$$\Delta B = \frac{\mu_0}{4\pi}\,\frac{I(\Delta l)\,R}{r^3} \qquad r^3 = (r^2)^{3/2}$$



$$\Delta B = \frac{\mu_0}{4\pi}\,\frac{I R\,\Delta z}{(z^2+R^2)^{3/2}}$$

contribution from just one segment Δz

For a infinitely long wire:

$$B = \frac{\mu_0 I}{4\pi} R \int_{-\infty}^{\infty} \frac{dz}{(z^2+R^2)^{3/2}}$$

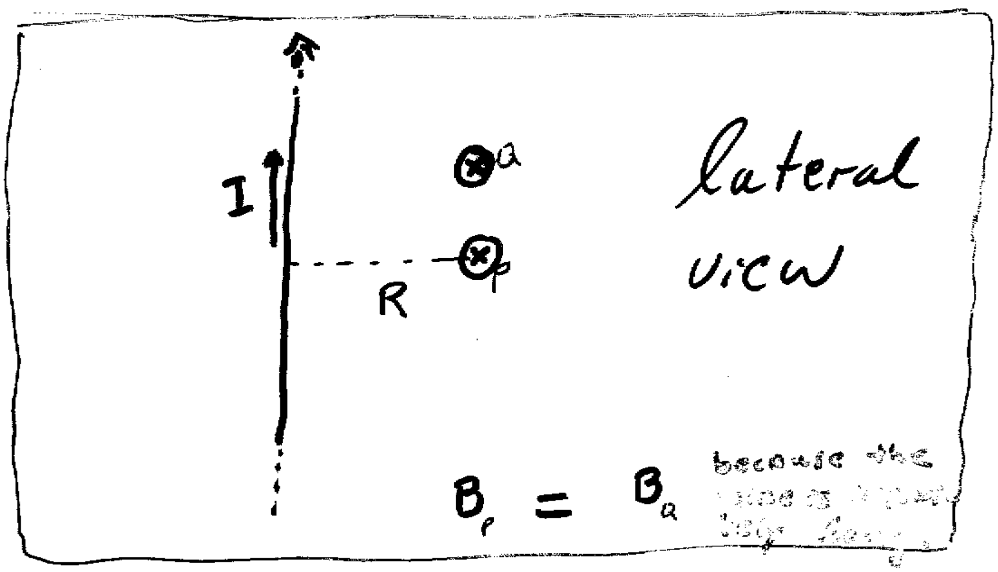
$$B = \frac{\mu_0 I R}{4\pi R^2} \underbrace{\left[\frac{z}{\sqrt{z^2 + R^2}} \right]_{-\infty}^{\infty}}$$

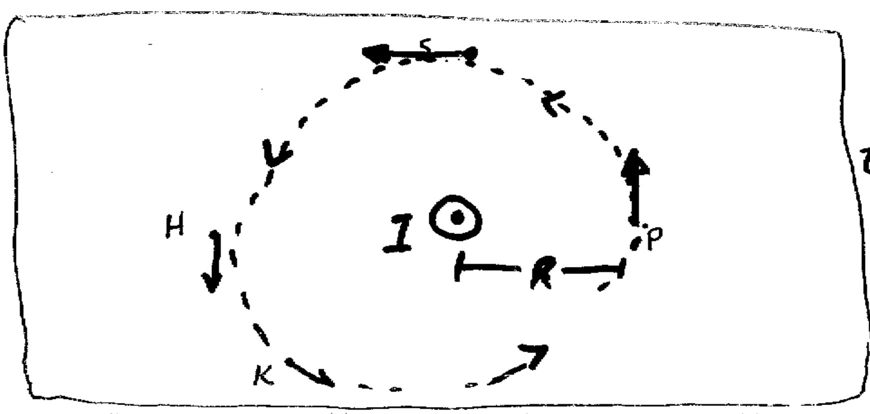
see also page 689

Notice $\lim_{z \to \infty} \frac{z}{\sqrt{z^2+R^2}} = +1$

$\lim_{z \to -\infty} \frac{z}{\sqrt{z^2+R^2}} = -1$

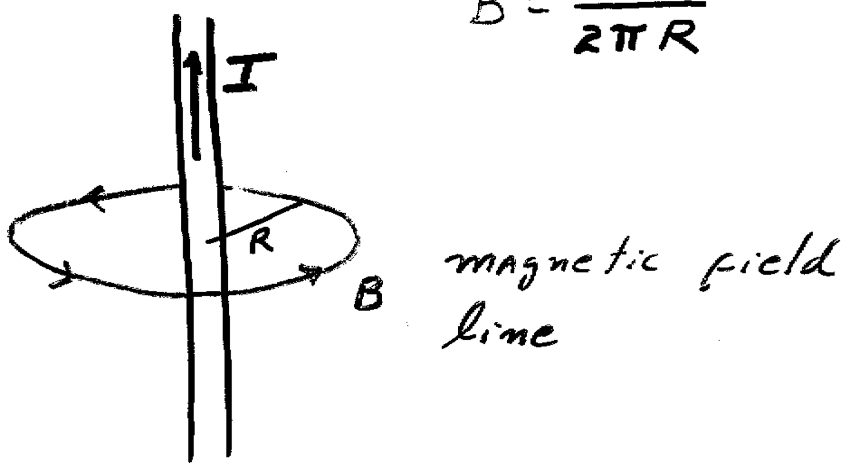
$$B = \frac{\mu_0 I}{2\pi R}$$

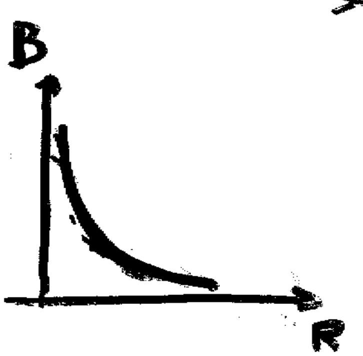




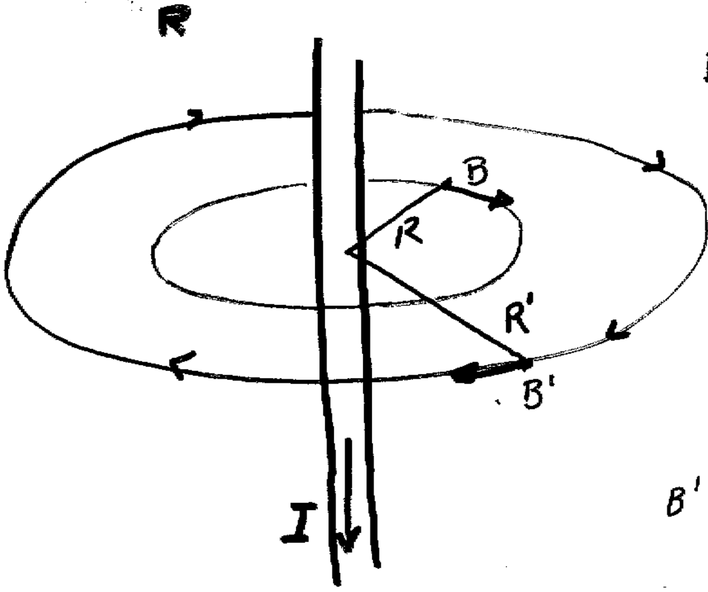
top view

$$B = \frac{\mu_0 I}{2\pi R}$$





$$B = \frac{\mu_0 I}{2\pi R}$$



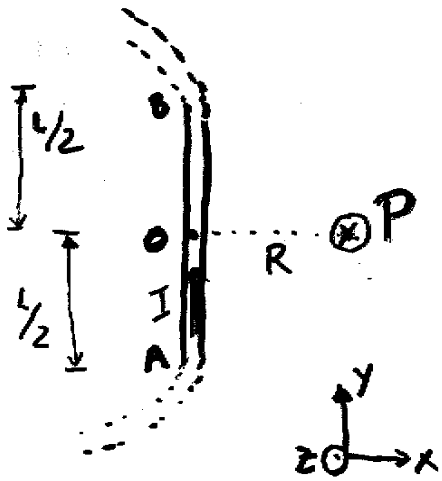
$$B' = \frac{\mu_0 I}{2\pi R'}$$

QUESTION

$B < B'$ $B = B'$ $B > B'$

QUESTION

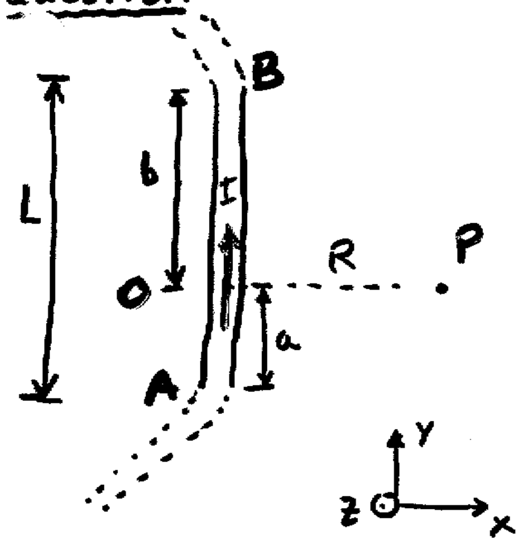
What is the contribution to the magnetic field by a finite wire segment AB of length L.



$$B = \frac{\mu_0 I R}{4\pi} \int_{-L/2}^{L/2} \frac{dz}{(z^2 + R^2)^{3/2}}$$

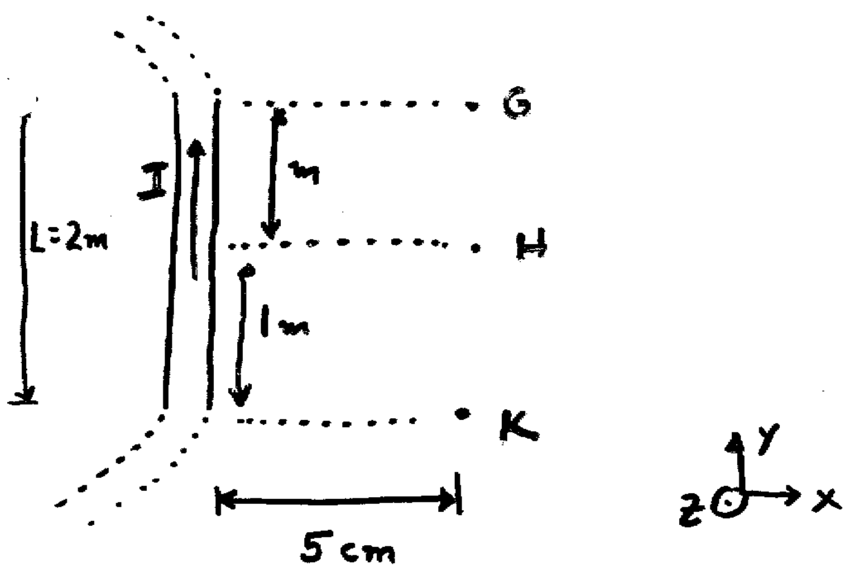
QUESTION

What is the magnetic field, due to the segment AB, at the point "P" ?



$$B_P = \text{——} \int_{-a}^{b}$$

* Practice problem: Find the magnetic field at "G", "H" and "K".

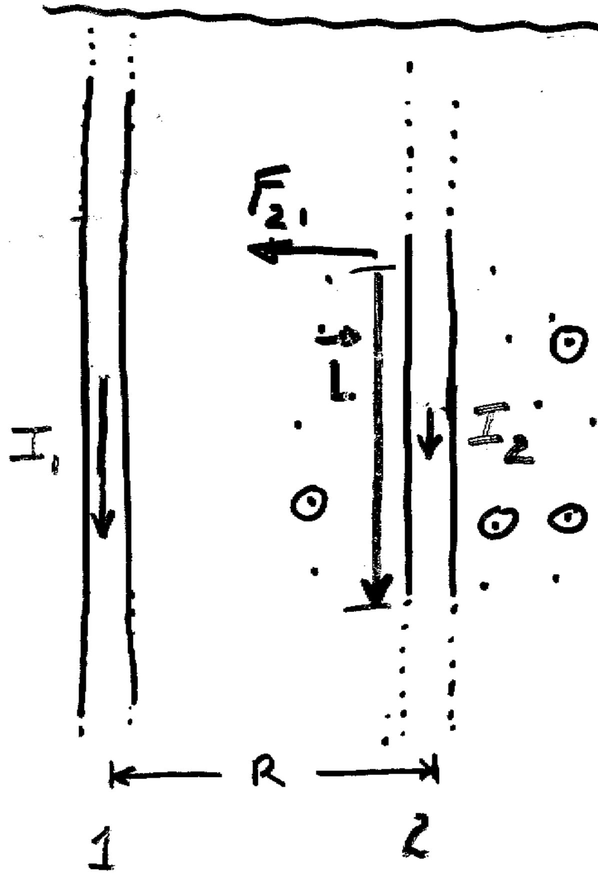


I = 0.5 Amp.

8

FORCE BETWEEN TWO parallel CURRENTS. -

see also page 693

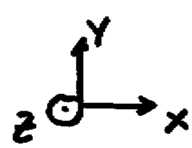


- The magnetic field B_1, produced by the current I_1, at the site 2 is

$$B_1 = \frac{\mu_0 I_1}{2\pi R}$$

magnetic field affecting current I_2.

- A segment $\vec{L}$ of wire 2 is immersed in a magnetic field B_1. So, it will experience a force



we are assuming the wires are infinitely long.

$$\vec{F}_{21} = I \; \vec{L} \times \vec{B}_1$$

?

$$B_1 = \frac{\mu_0 I_1}{2\pi R}$$

$$\vec{F}_{21} = -F_{21}\ \hat{\imath} \qquad \text{(attractive force)}$$

where $F_{21} = \underbrace{I_2}_{\substack{\text{current flowing}\\ \text{through wire 2}}} L\ \underbrace{\frac{\mu_0}{2\pi}\frac{I_1}{R}}_{\substack{\text{magnetic field produced by an infinitely long}\\ \text{wire carring a current } I_1}}$

the longer the segment L the stronger the force.

$$\boxed{\text{Force per unit length} \quad \frac{F_{21}}{L} = \frac{\mu_0}{2\pi R} I_1 I_2}$$

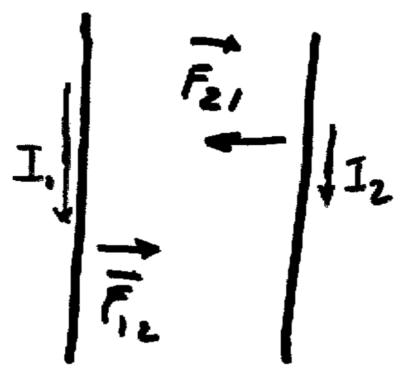
$\vec{F}_{21} = -F_{21}\ \hat{\imath}$

Notice the symmetry of the result with respect to the currents I_1 and I_2.

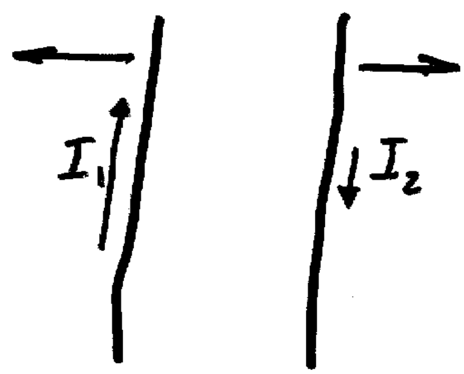
The symmetry of the previous result indicates that

$$\frac{F_{12}}{L} = \frac{\mu_0}{2\pi R} I_2 I_1$$

and $\vec{F}_{12} = F_{12}\,\hat{r}$



Parallel currents attract each other



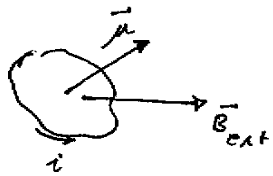
Antiparallel currents repel each other

* Solve checkpoint 2, page 694

①

The magnetic dipole moment

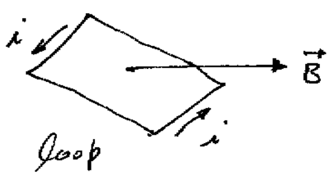
$\vec{\tau} = \vec{\mu} \times \vec{B}$

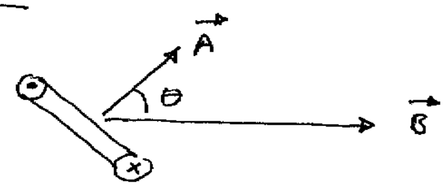


$\mu = i \times AREA$

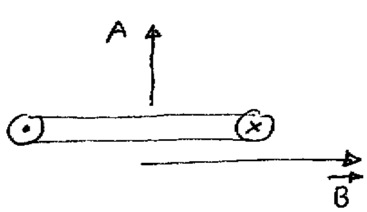
$\vec{\tau}$ tends to make $\vec{\mu}$ be aligned along the external magnetic field $\vec{B}$

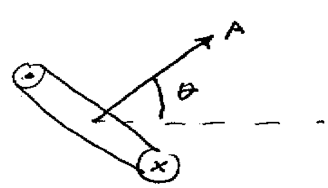
Magnetic Potential Energy of a dipole immersed into a external magnetic field





As usual, potential energy is always given with respect to a configuration REFERENCE.



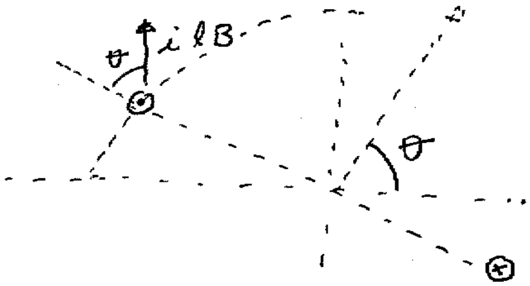


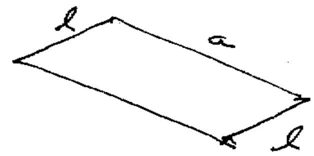
$\theta_{ref} = \pi/2$

①

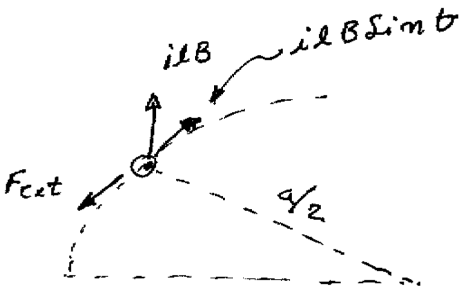
②

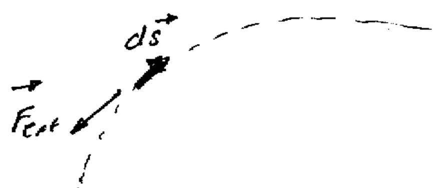
How much external work is need to take the loop from configuration ① to configuration ② ?





Area = la





$F_{ext} = i l B \sin\theta$

$$W_{ext} = -\underbrace{\int_{\pi/2}^{\theta} |F_{ext}|\,|ds|} = -\int_{\theta}^{\pi/2} |i l B \sin\theta|\,|ds| = -\int_{0}^{\pi/2} i l B \sin\theta \, \frac{a}{2}\, d\theta$$

$$= -i l B \frac{a}{2} \underbrace{\int_{\theta}^{\pi/2} \sin\theta \, d\theta}_{-\cos\theta \Big|_{\theta}^{\pi/2} = (-0) - (-\cos\theta) = \cos\theta} = -i l B \frac{a}{2} \cos\theta$$

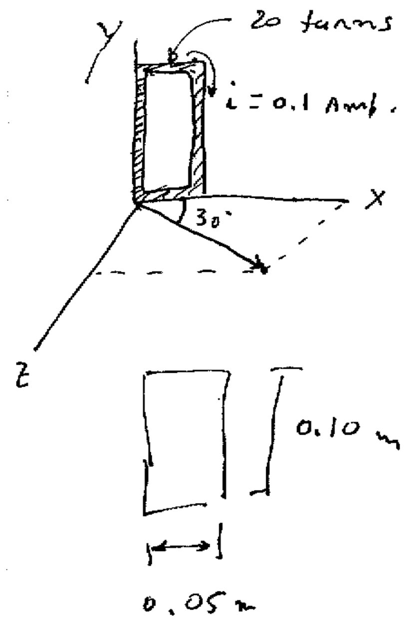
the total external work will be twice this value

$$W_{ext\ \text{Total}} = -i l B a \cos\theta = -B \underbrace{iA}\cos\theta = -B\mu\cos\theta$$

$$\boxed{W_{ext} = -\vec{\mu}\cdot\vec{B}}$$

(4)

Problem 39E CH - 29



B = 0.5T

$\vec{\mu} = N i A$

$= \cancel{20} \times 0.1 \text{ Amp} \times 10^{-1} \times \cancel{5} \times \cancel{10}^{-2}$

$\vec{\mu} = 10^{-2} \text{ Amp m}^2 \ \hat{k}$

$\vec{B} = 0.5 \times \cos 30° \hat{i} + 0.5 \sin 30° \hat{k}$

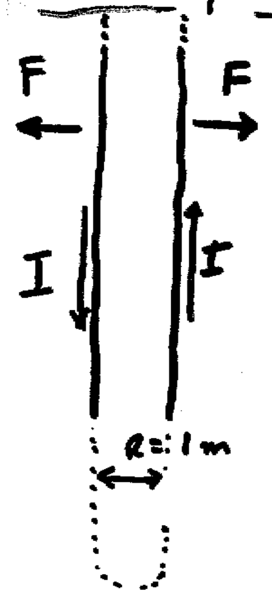
$\tau = \vec{\mu} \times \vec{B}$

$= (-\hat{j})\, 10^{-2} \times 0.5 \times \frac{\sqrt{3}}{2}$

$= -0.42 \times 10^{-2} \ \hat{j}$

$= -4.2 \times 10^{-3}$

Definition of the UNIT of CURRENT: The ampere



$$\frac{F}{L} = \frac{\mu_0}{4\pi}\frac{2}{R} I I$$

The ampere is that constant current wich, if maintained in two straight parallel conductors, placed one meter apart, the conductor will exert a force on each other of 2×10^{-7} newtons per meter of length.

If $R = 1\,m$ and $\frac{F}{L} = 2 \times 10^{-7}$ newtons then $I = 1$ ampere.

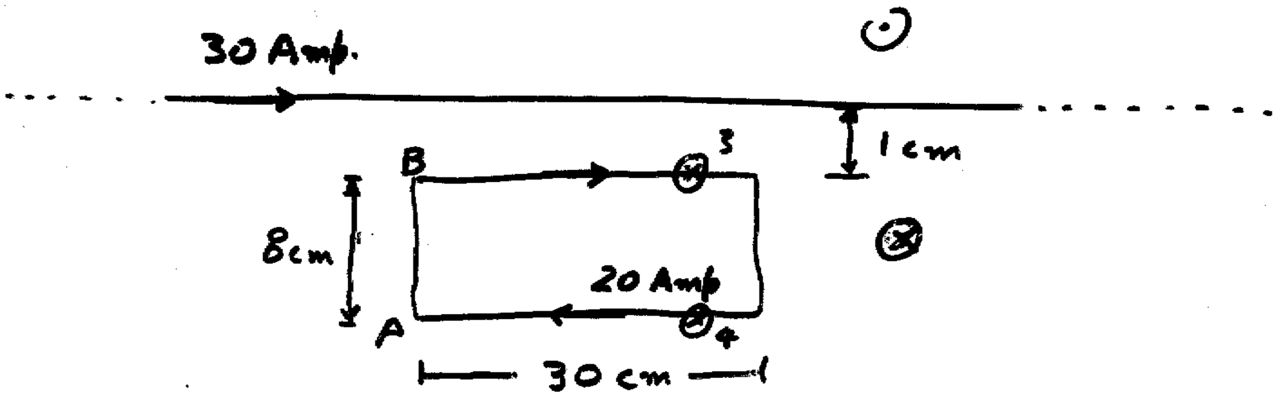
Having defined the ampere (unit of current) and knowing that $i = \frac{q}{t}$, <u>the unit of charge</u> is defined as follows:

The COULOMB (C) is the amount of charge flowing throw a wire in $t = 1$ sec when the current in wire is 1 ampere

coulomb = 1 ampere . second

Practice: Problems 39E, 40E, 47P, 12
50E, 35E, 28P
Chapter 29

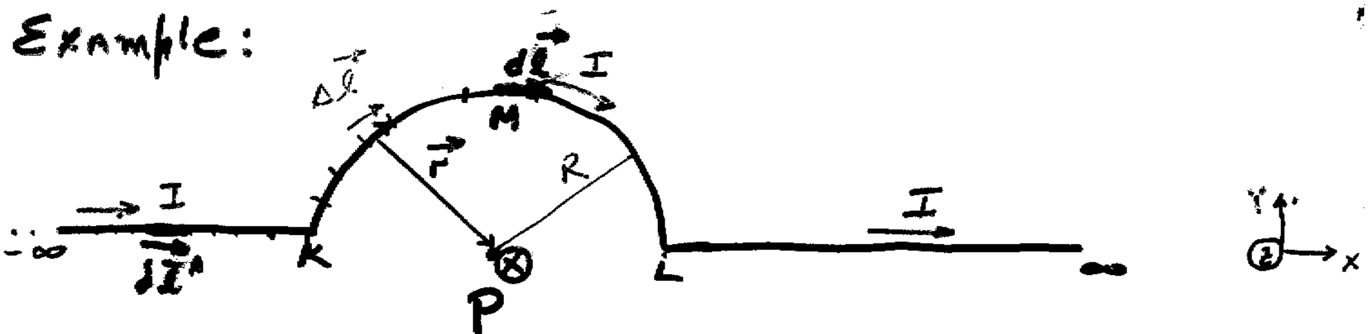
* checkpoint   Problem 29P, page 707



calculate the force acting on the loop

$$\vec{F} = I\,\vec{l} \times \vec{B}$$

Example:



$$\Delta\vec{B} = \frac{\mu_0 I}{4\pi}\frac{\Delta\vec{l}\times\vec{r}}{r^3}$$

what is the magnetic field at point "P" ?

KL is a semi-circle of radius "R"

<u>solution</u>

- contribution to the magnetic field from the wire-section $-\infty$ to K $\Delta\vec{B} =$
- contribution to the magnetic field from the wire-section L to ∞ $\Delta\vec{B} =$
- Contribution to the magnetic field from the semicircle KL

  Along the circular path what is the angle between $\Delta\vec{l}$ and $\vec{r}$?

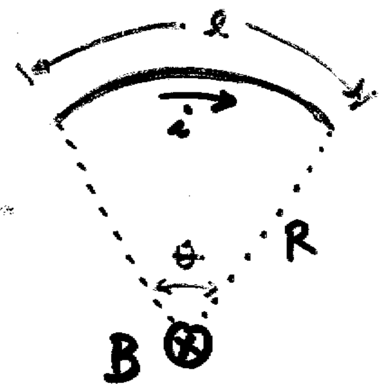
  what will be the direction of the total magnetic field ?

$$\Delta B = \frac{\mu_0 I}{4\pi}\frac{|\Delta\vec{l}\times\vec{r}|}{r^3} = \frac{\mu_0 I}{4\pi}\frac{\Delta l}{R^3}R = \frac{\mu_0 I}{4\pi}\frac{\Delta l}{R^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int\frac{\Delta l}{R^2} = \frac{\mu_0 I}{4\pi}\frac{1}{R^2}\underbrace{\int \Delta l}_{\pi R} \longrightarrow B = \frac{\mu_0 I}{4R}$$

TOTAL magnetic field at point "P"

Example circular arc



Following a similar procedure from the previous example we obtain.

$$B = \frac{\mu_0 i}{4\pi}\frac{l}{R^2}$$

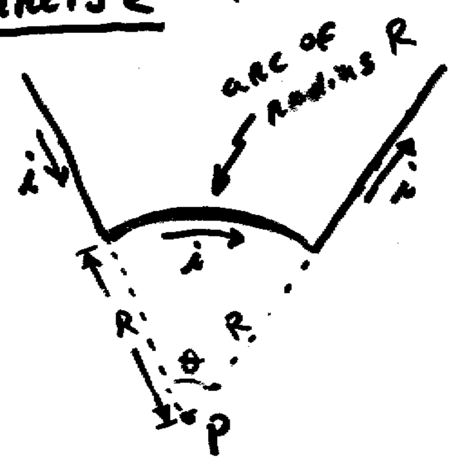
magnetic field at the center of the arc

or, equivalent

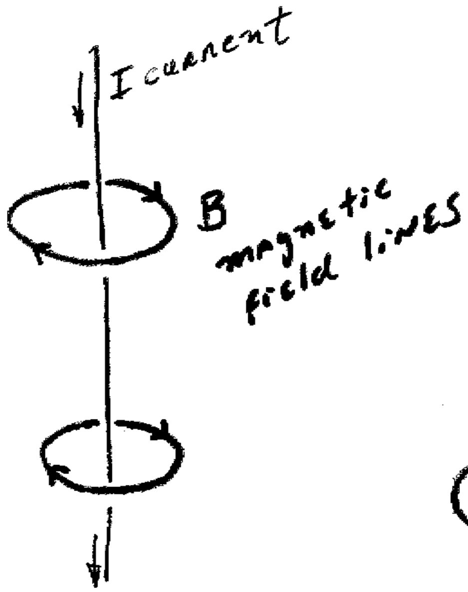
$$B = \frac{\mu_0 i}{4\pi}\frac{\theta}{R}$$

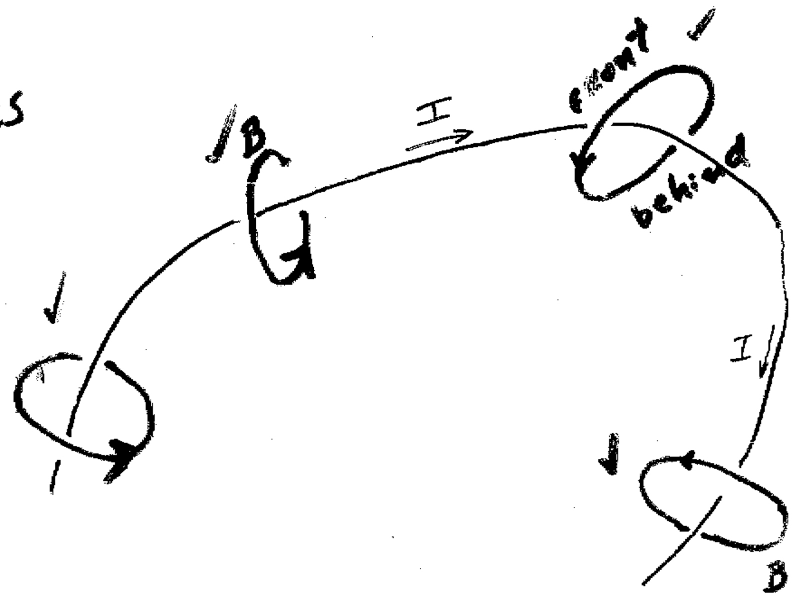
θ in radians

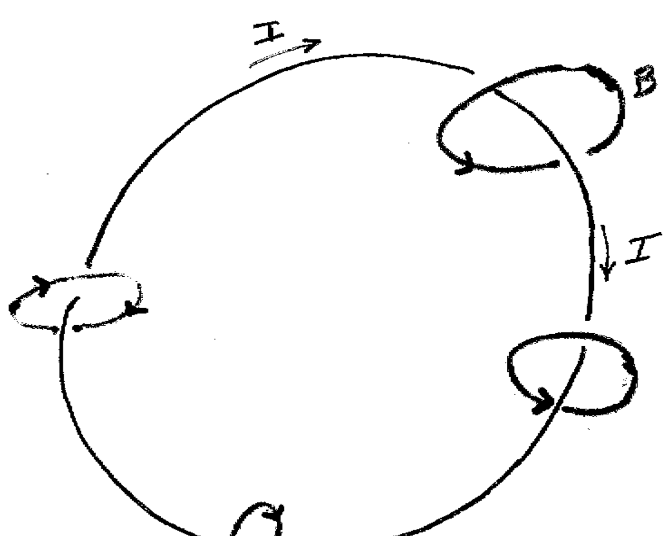
EXERCISE Find the magnetic field at point "P" (the center of the arc of radius R)



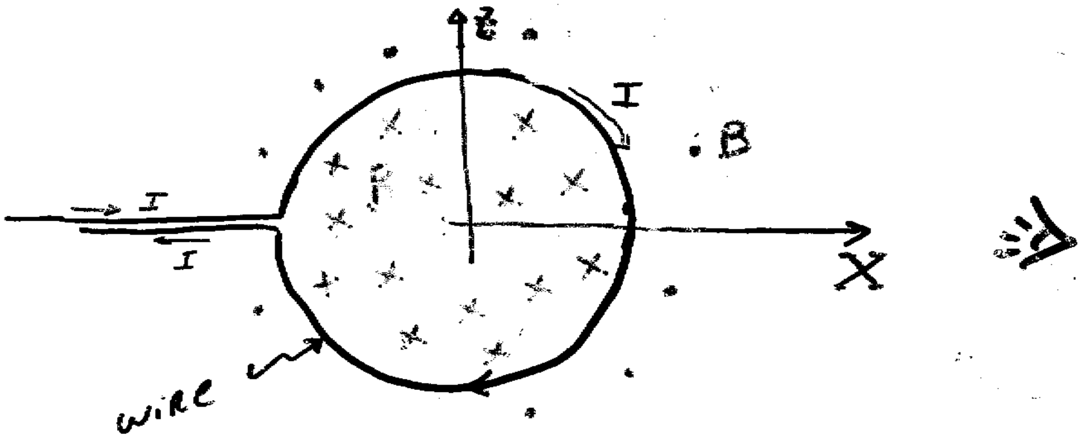
Practice: - sample problem 30-1
- checkpoint 1 on page 691
- Problem 56P (page 709)

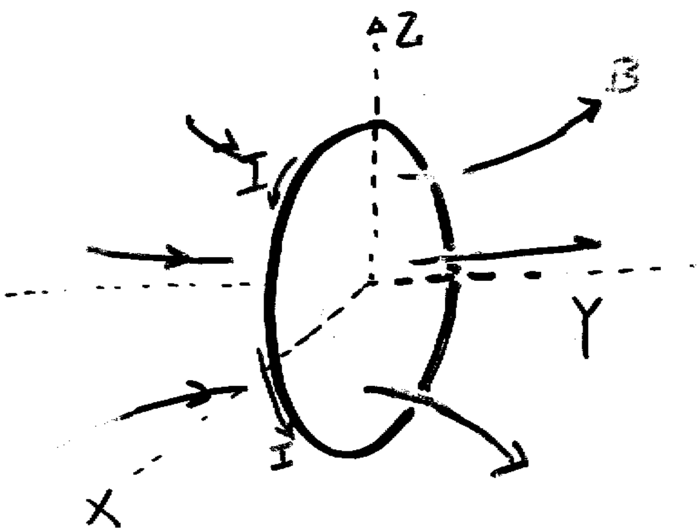


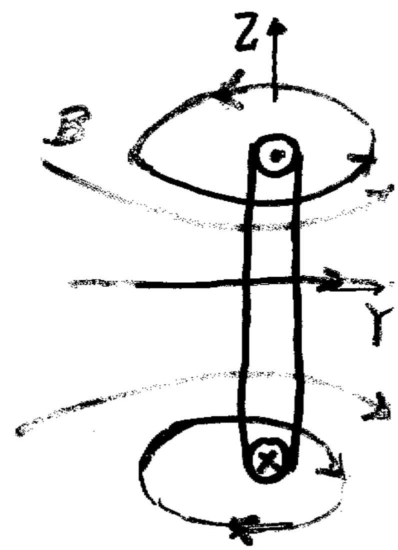




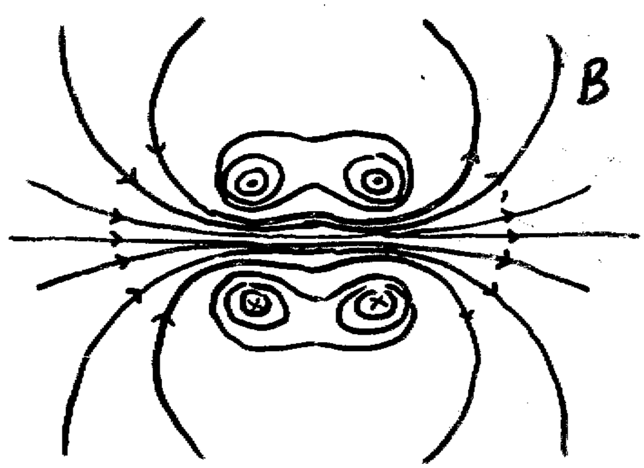
QUESTION Sketch the magnetic field of a circular coil of radius "R"

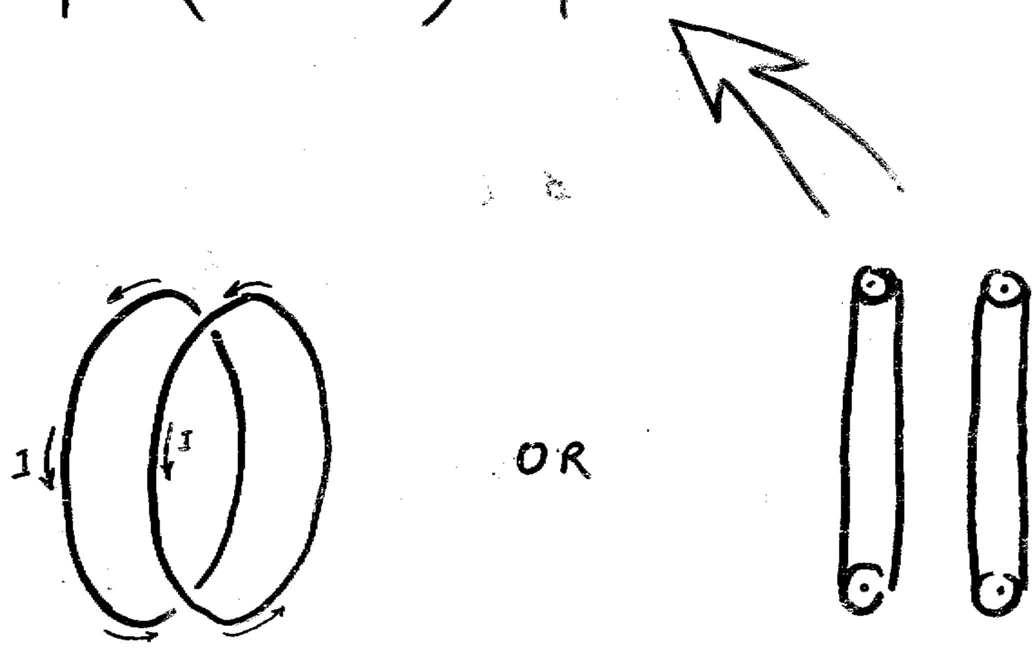


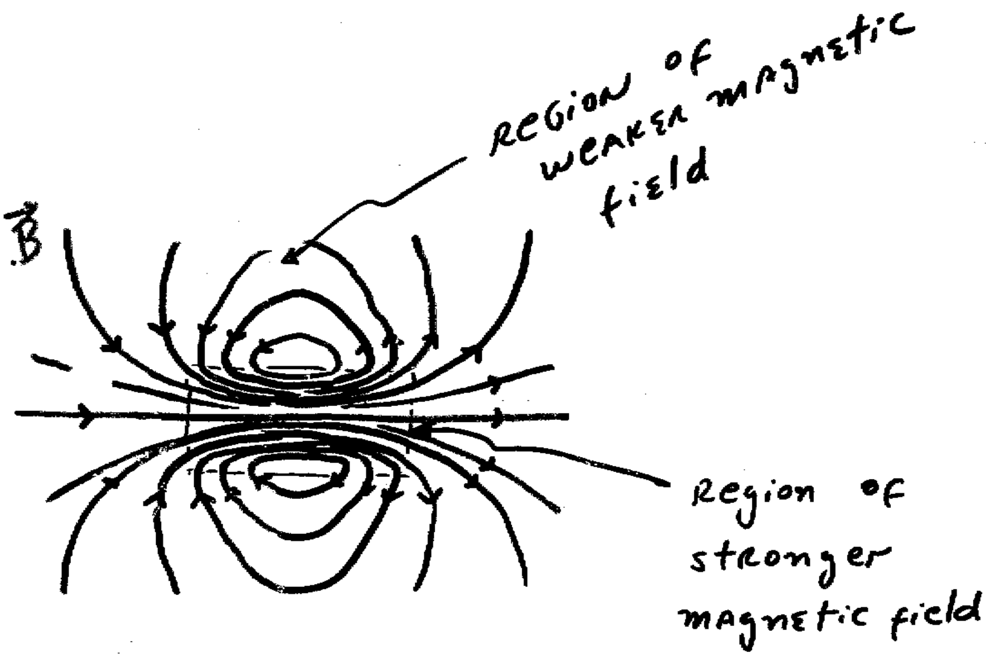


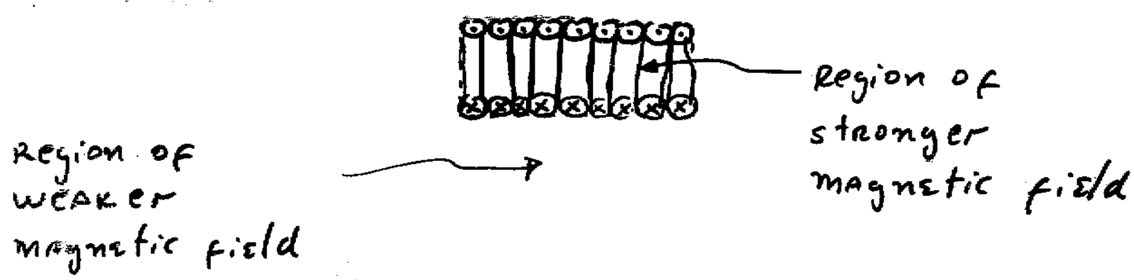


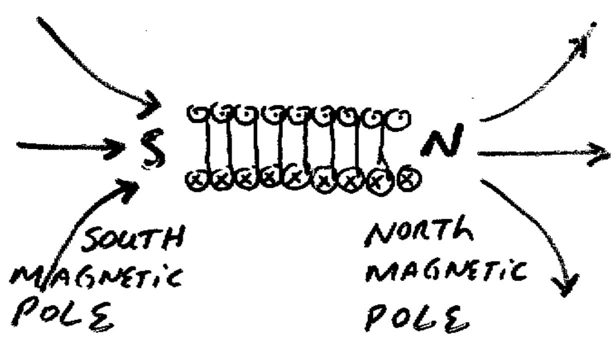
Magnetic field lines established by two loops

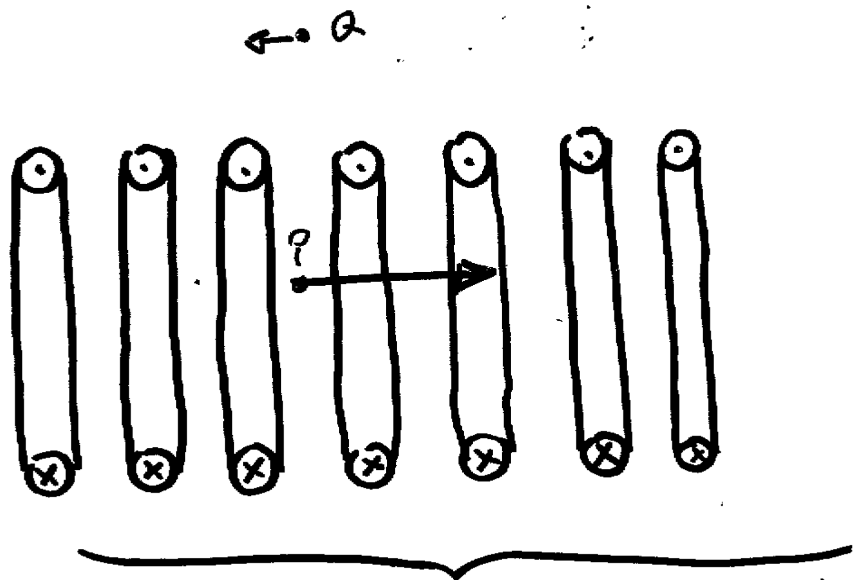




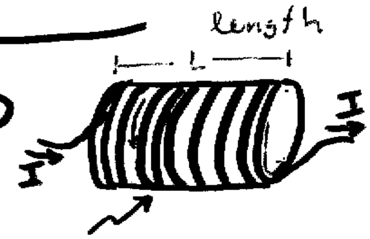

REGION OF
WEAKER MAGNETIC
field
$\vec{B}$
Region of
stronger
magnetic field


Region of
stronger
magnetic field
Region of
weaker
magnetic field


S
N
SOUTH
MAGNETIC
POLE
NORTH
MAGNETIC
POLE



SOLENOID

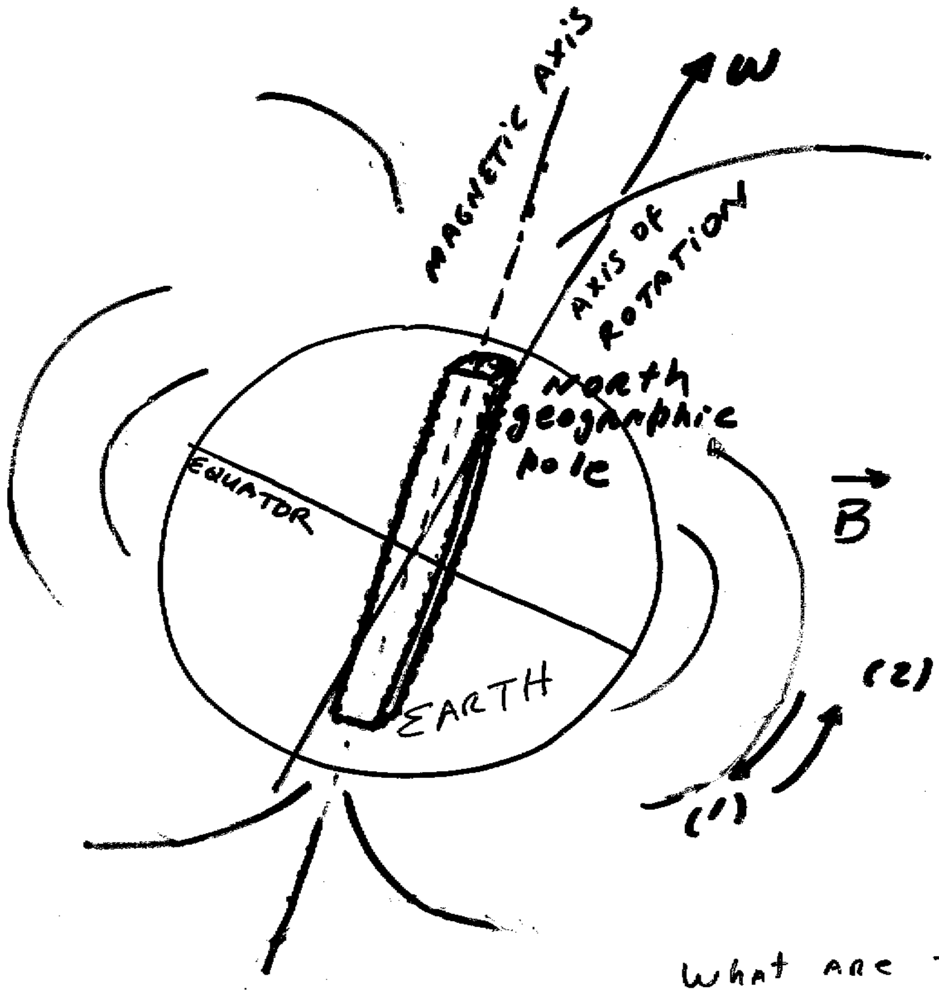


N turns

Notation: $\boxed{n = \frac{N}{L} = \text{\# of turns per unit length}}$

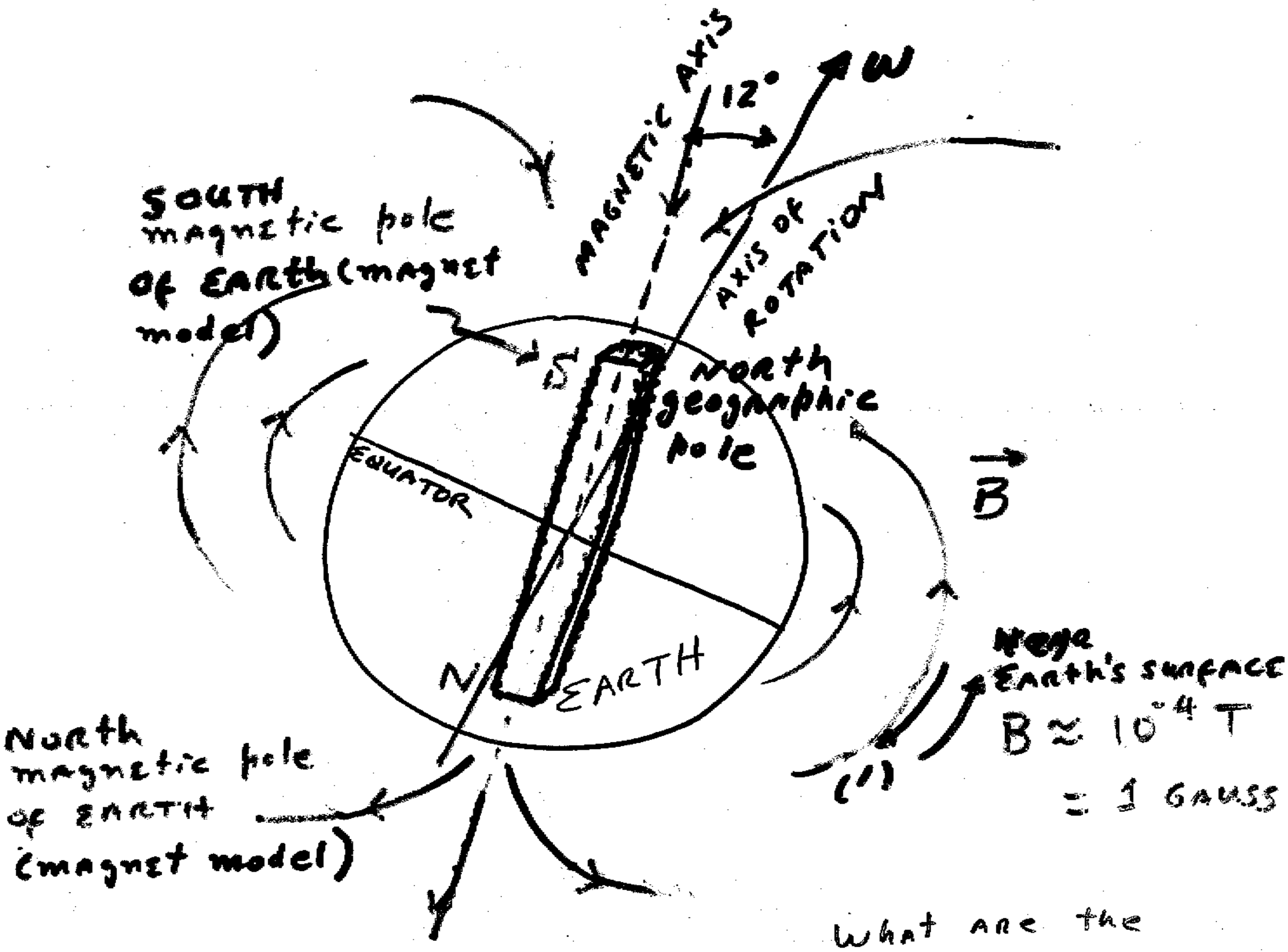
(see also page 69B)

QUESTION: In the figure above, where does the magnitude of the magnetic field higher: at point "P" or at point "Q"?



What are the orientation of the magnetic field lines ?

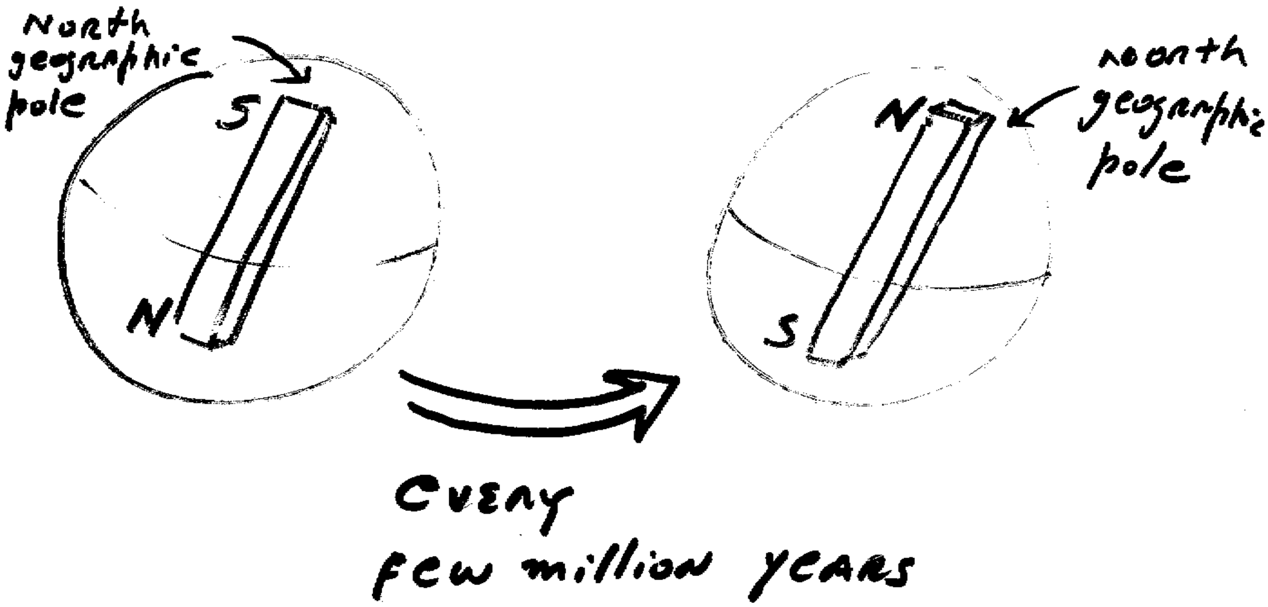
The EARTH AS A BIG MAGNETIC DIPOLE



What are the orientation of the magnetic field lines?

The EARTH AS A BIG MAGNETIC DIPOLE

The magnetic field of the Earth has undergone reversals of magnetic polarity



Age of the Universe = 1.6×10^4 million years
$= (5 \times 10^{17}$ seconds$)$