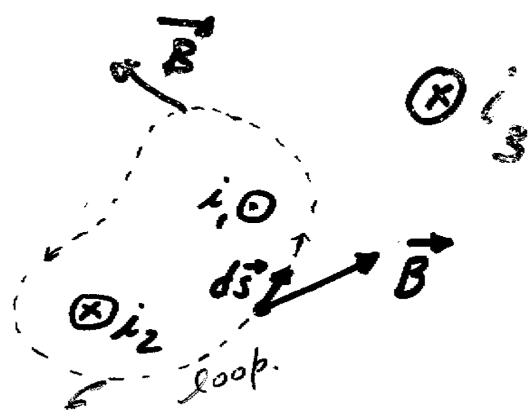
AMPERE's LAW and CALCULATION OF MAGNETIC FIELD FOR CASES OF HIGH SYMMETRY

In general, we use Biot-Savart law to evaluate $\vec{B}$ produced by currents flowing through wires of arbitrary shape.

But, if a case presents symmetry, we may use AMPERE'S LAW

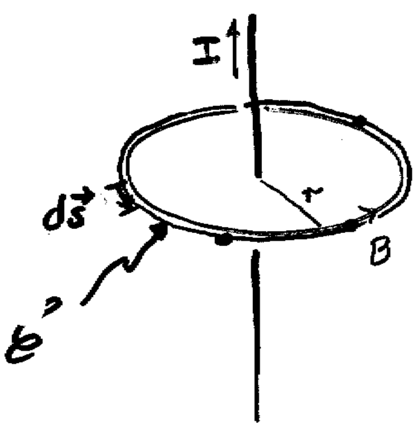
$$\oint_{\mathcal{C}} \vec{B}\cdot d\vec{s} = \mu_0 \underbrace{I_{enc}}_{\text{current encircled by the loop } \mathcal{C}}$$



$\mathcal{C}$ is a mathematical loop.

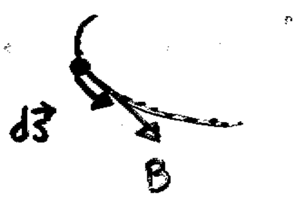
Example Use Ampere's law to find the magnetic field produced by a current I flowing through a infinitely long wire

- Some previous experience and/or insight indicates that the magnetic field lines go around in circle



this is to say that we know the "shape" of the magnetic field lines, but not its exact value.

So, we choose as our mathematical loop $\mathcal{C}$, a circumference of radius r



At each point along $\mathcal{C}$, $d\vec{s}$ and $\vec{B}$ point in the same direction

$$\int_{\mathcal{C}} d\vec{s} \cdot \vec{B} = \int_{\mathcal{C}} ds\, B = B \int_{\mathcal{C}} ds$$

$$= B\, 2\pi r$$

On the other hand, the current "encircled" by the loop $\mathcal{C}$ is I.

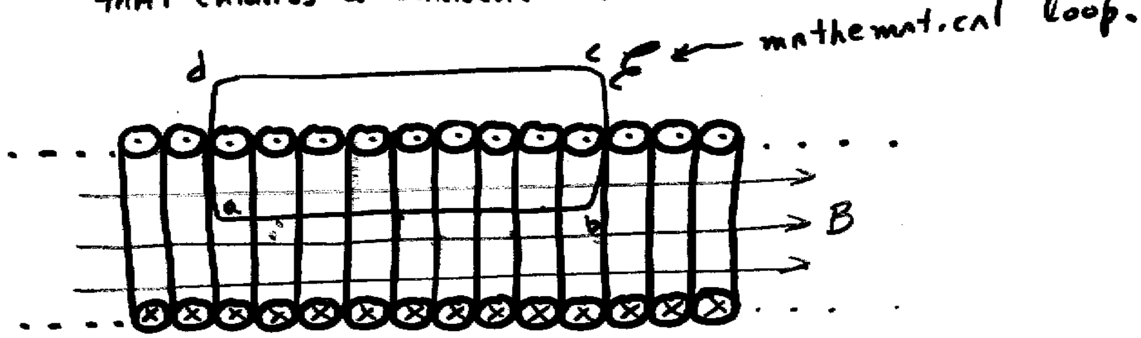
Therefore

$$\underbrace{\oint_{\mathcal{C}} \vec{B} \cdot d\vec{s}}_{2\pi r B} = \mu_0 \underbrace{I_{enc}}_{I}$$

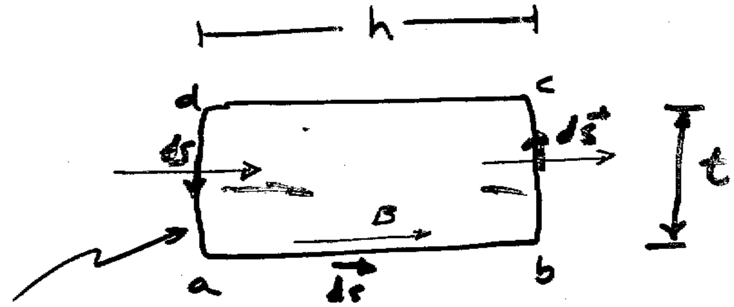
$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$

which is the same result we obtained using Biot-Savart law.

Example: Use Ampere's law to find the magnetic field inside an infinitely long SOLENOID that carries a current I.



Let n be the number of turns per unit length of the selenoid



mathematical loop $\mathcal{C}$

$$\oint_{\mathcal{C}} \vec{B}\cdot d\vec{s} = \int_a^b + \underbrace{\int_b^c}_{\vec{B}\perp d\vec{s}} + \underbrace{\int_c^d}_{\vec{B}=0} + \underbrace{\int_d^a}_{\vec{B}\perp d\vec{s}}$$

so

$$= \int_a^b \vec{B}\cdot d\vec{s} = B\,h \qquad (1)$$

On the other hand,

$$I_{encircled} = \underbrace{n h}_{\substack{\#\text{ of turns}\\ \text{in a segment}\\ \text{of length } h}} \; \underbrace{I}_{\substack{\text{current carried}\\ \text{by each turn}}} \qquad (2)$$

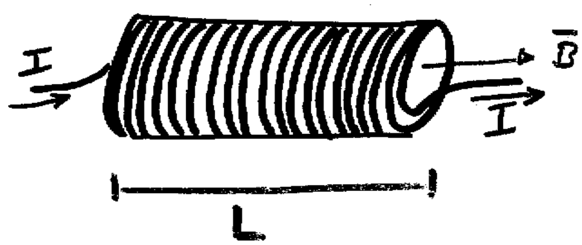
$$B\cancel{h} = \mu_0 n \cancel{h} I \qquad \boxed{B = \mu_0 n I}$$

From Ampere's law

$$\int_C \vec{B} \cdot d\vec{s} = \mu_0 \, I_{encircled}$$

$$B\,h \qquad\qquad n h \, I$$

$$\Rightarrow \quad \boxed{B = \mu_0 \, n \, I}$$



Magnetic field at the interior region of a infinitely long selenoid

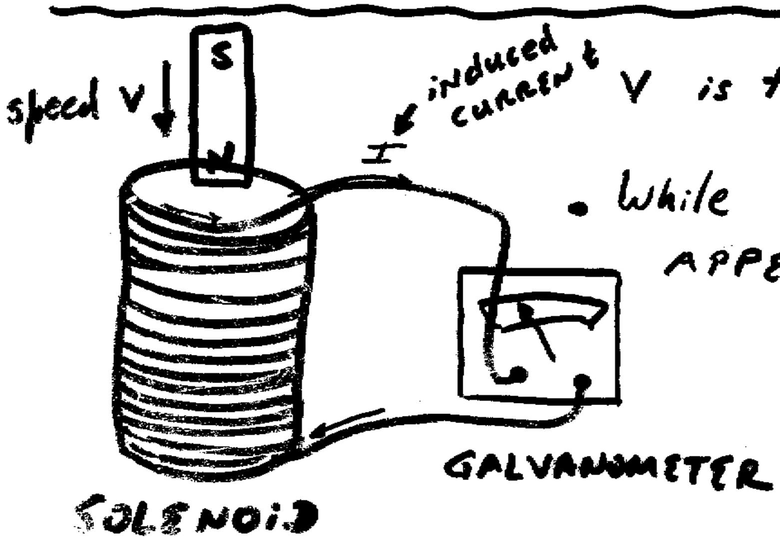
L very large compared to the cross section area

* Practice problems 40E, 41E on page 708.

* Sample problem 30-3 (page 697) Ampere's Law

MAGNETIC INDUCTION

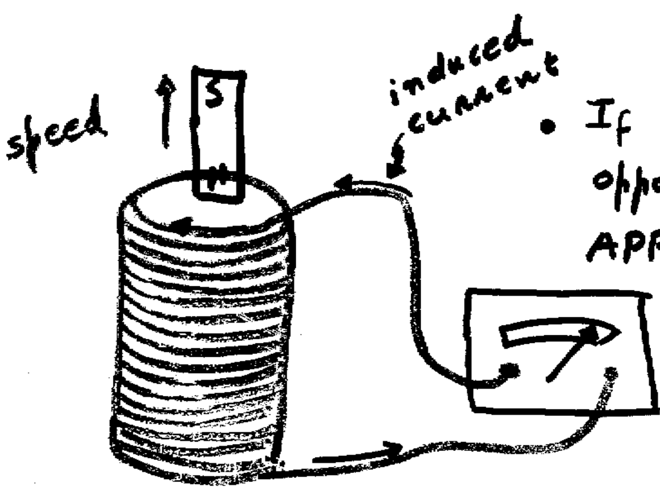
M. Faraday
J. Henry
1830



V is the speed of the magnet

- While $V \neq 0$ a current I APPEARS in the solenoid

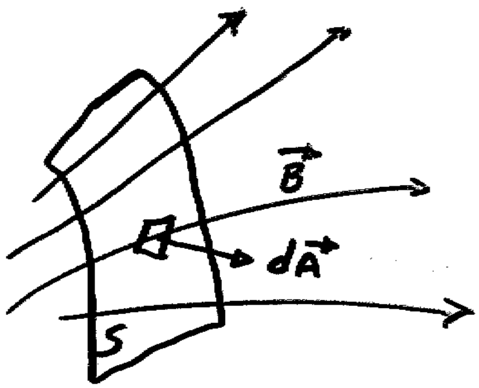
- If $V = 0$, no current is detected by the galvanometer



- If the magnet moves in the opposite direction, a current APPEARS in the solenoid,

  Notice the reversal of the current flow

To quantify this phenomena, it is convenient to define the magnetic flux Φ_m



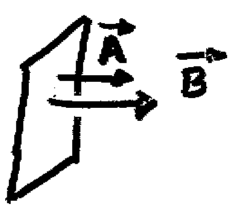
$$\Phi_m = \int_S \vec{B} \cdot d\vec{A}$$

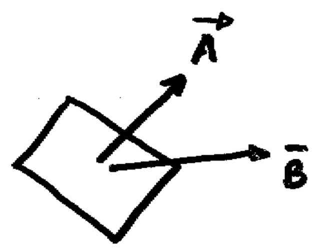
scalar product

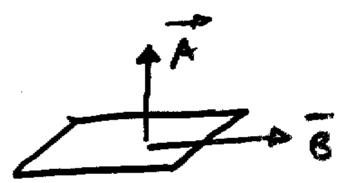
S : mathematical surface

$d\vec{A}$: an element of area on the surface S

Examples:



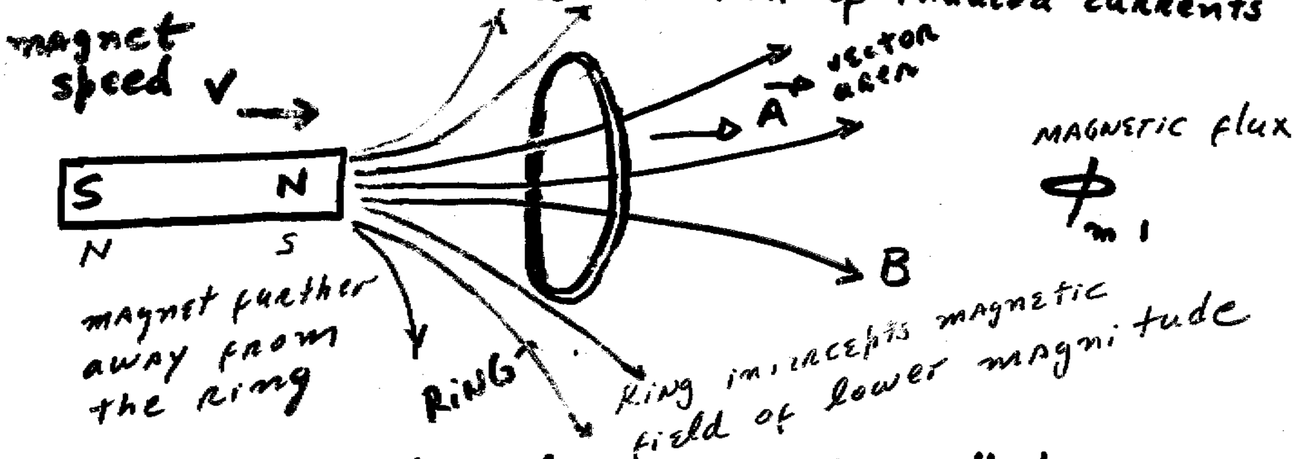




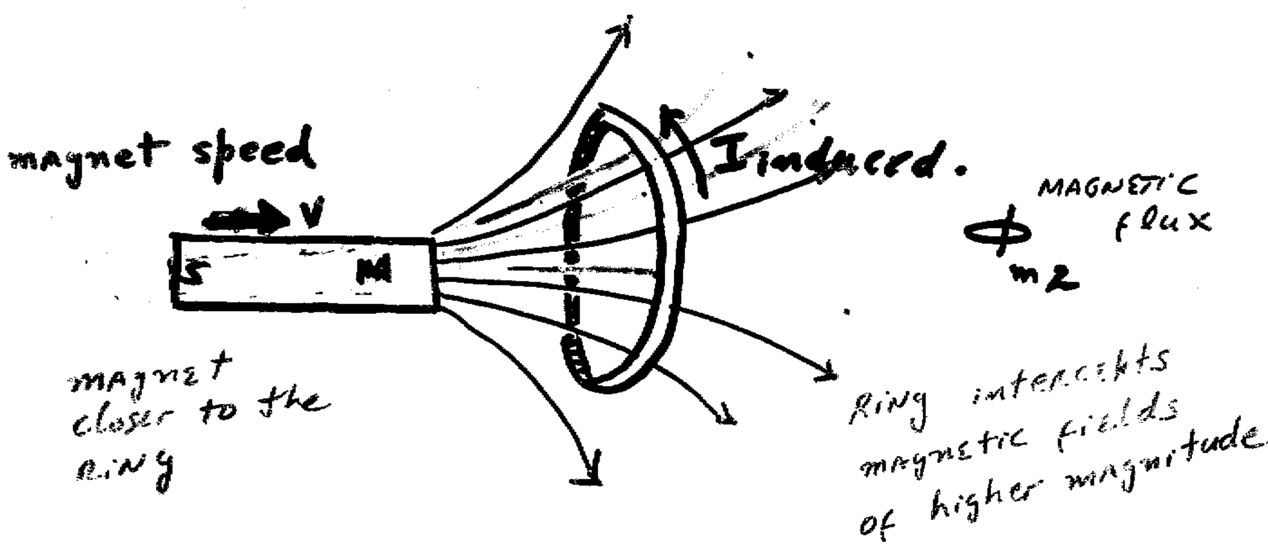
$\Phi_m > 0$	$\Phi_m > 0$	$\Phi_m > 0$
$= 0$	$= 0$	$= 0$
< 0	< 0	< 0

Lenz's Law

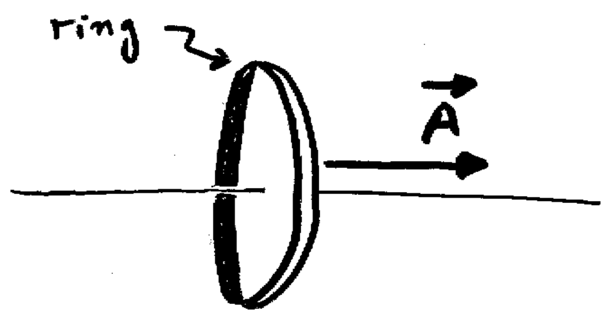
→ is a practical way to figure out the direction of induced currents



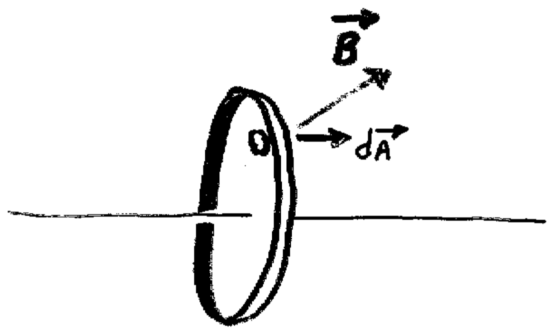
" the induced current will be directed so as to oppose the change in the magnetic flux that is taking place. "



Φ_{m2} $\quad$ Φ_{m0}

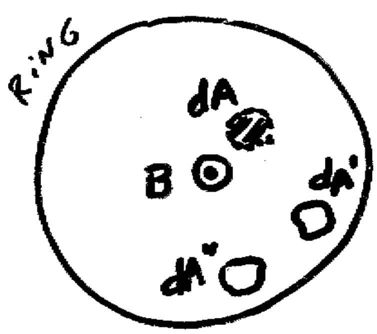


A: Area of the ring

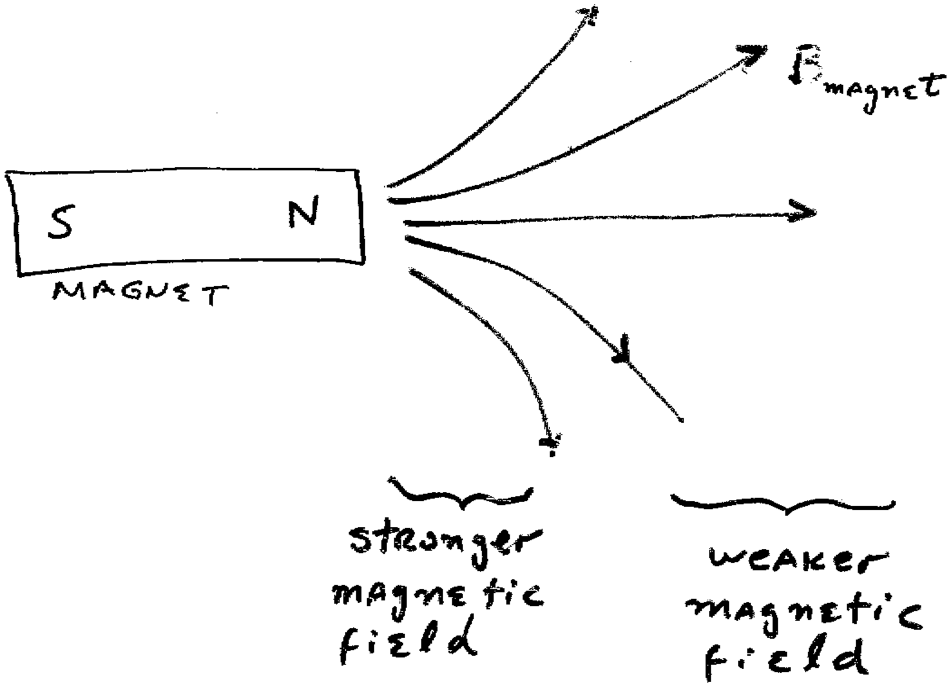


$d\vec{A} \cdot \vec{B} > 0$

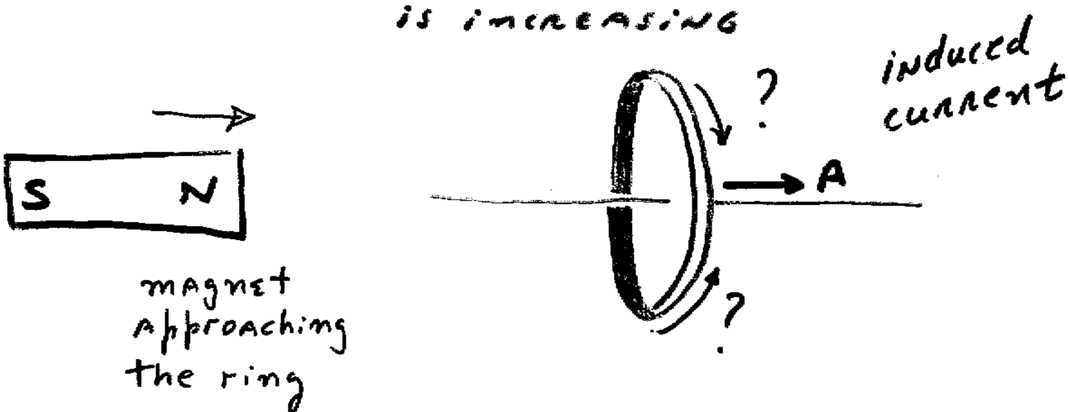
$\int \vec{B} \cdot d\vec{A}$



$\int \vec{B} \cdot d\vec{A} > 0$

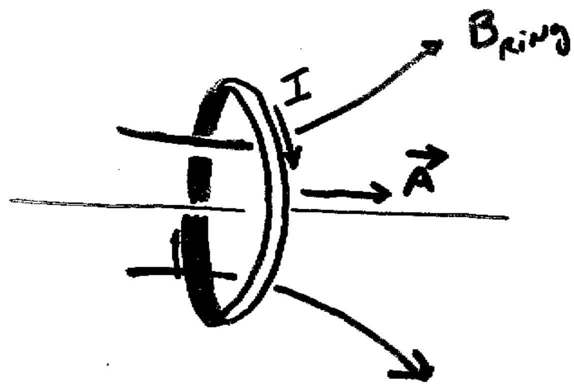


NOTICE: The magnetic flux through the ring Φ_m is increasing

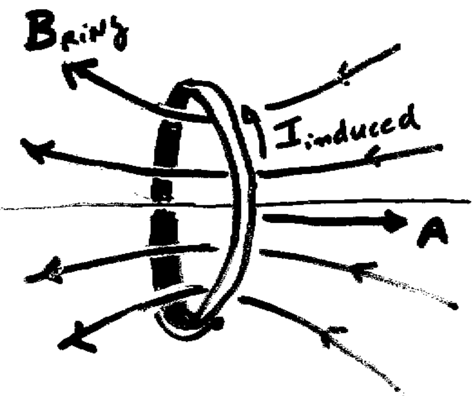


In which direction should the induced current I_{ind} should flow to decrease the magnetic flux

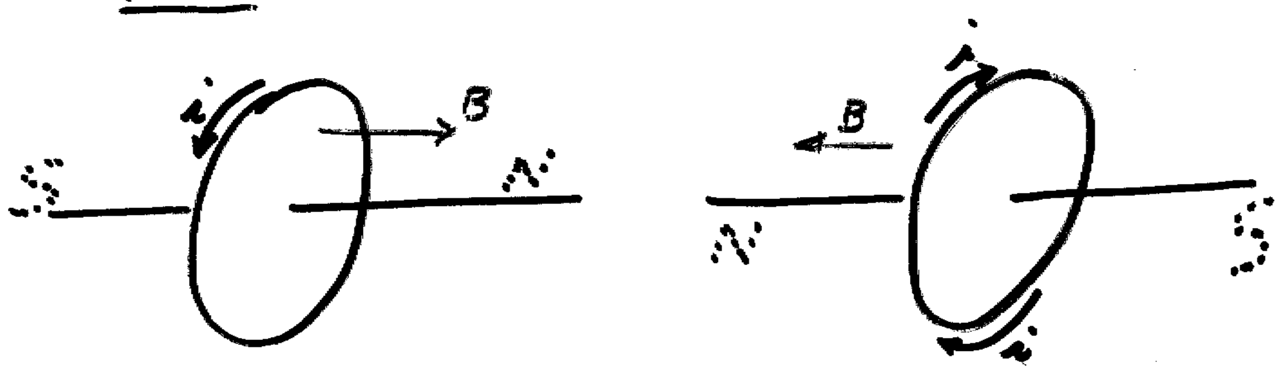
3''

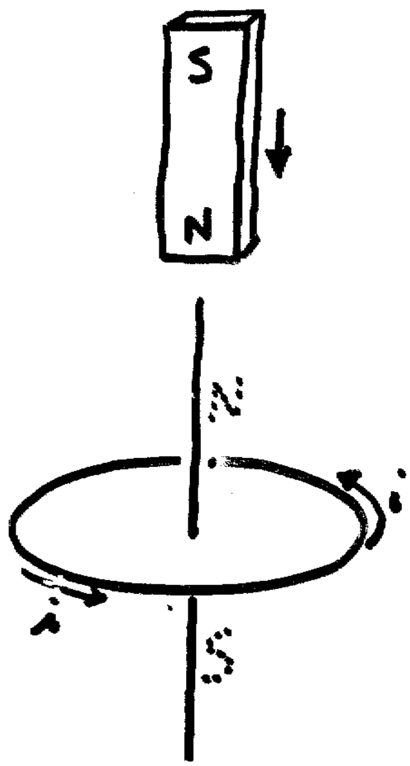


OR ?

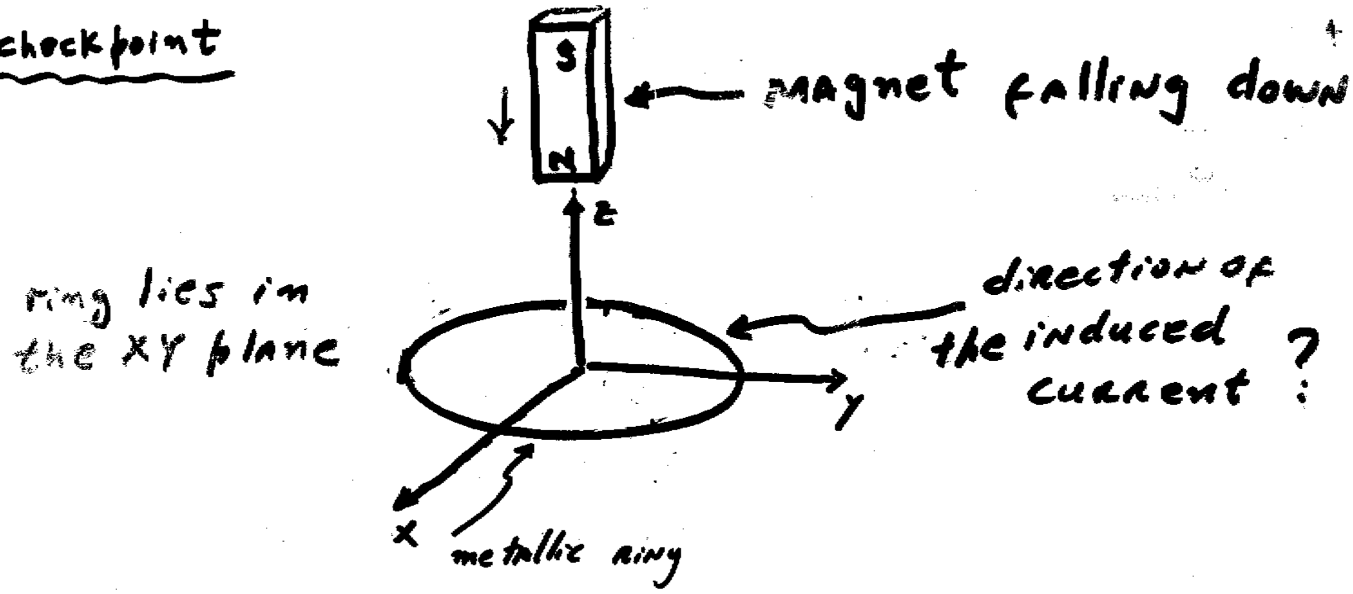


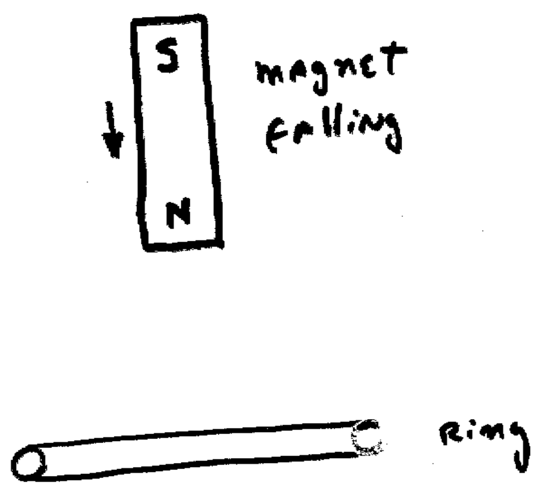
HINT



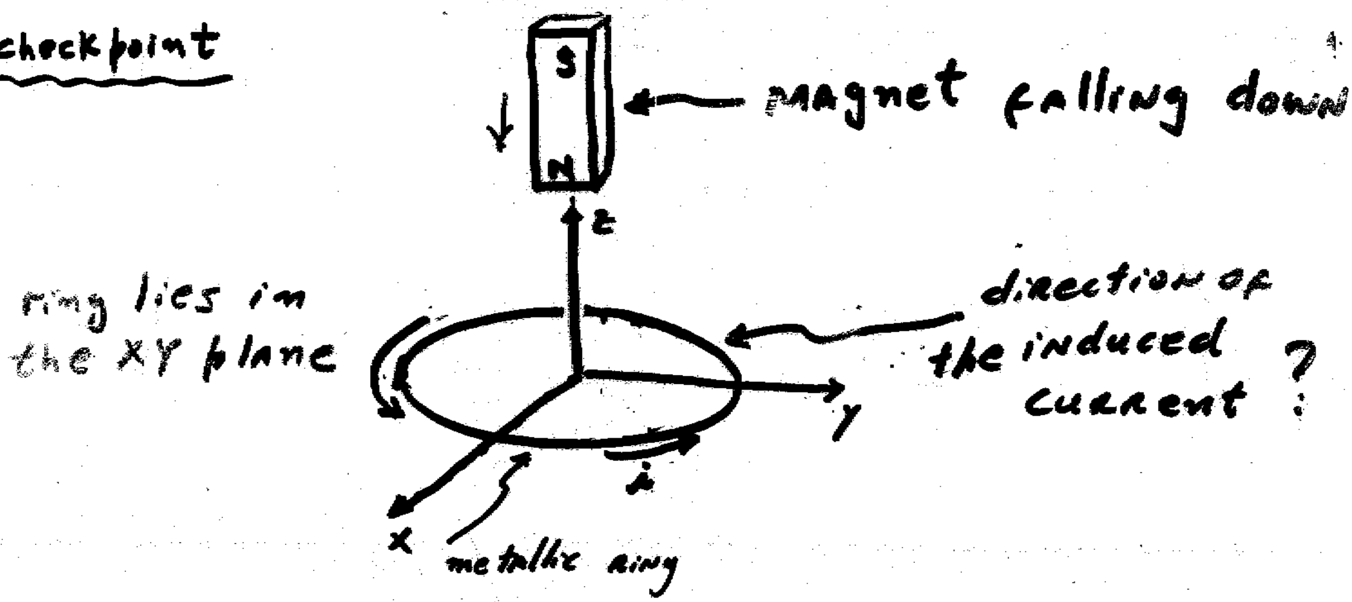


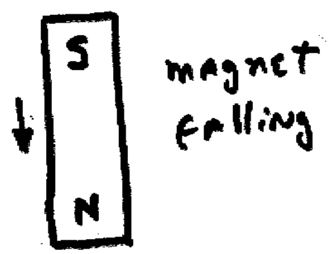
checkpoint

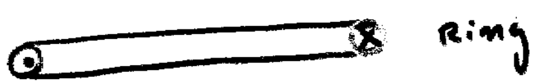

S
N
magnet falling down
z
ring lies in the XY plane
direction of the induced current ?
y
x
metallic ring


S
N
magnet falling
Ring

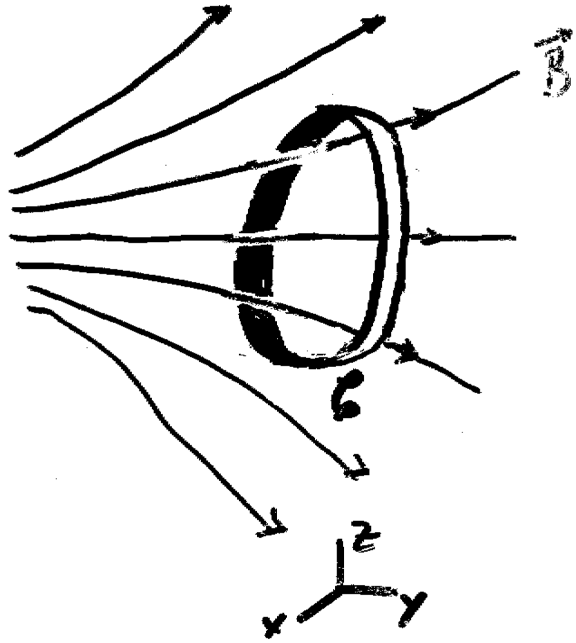
checkpoint


S
N
magnet falling down
z
ring lies in the XY plane
direction of the induced current ?
Y
x
i
metallic ring

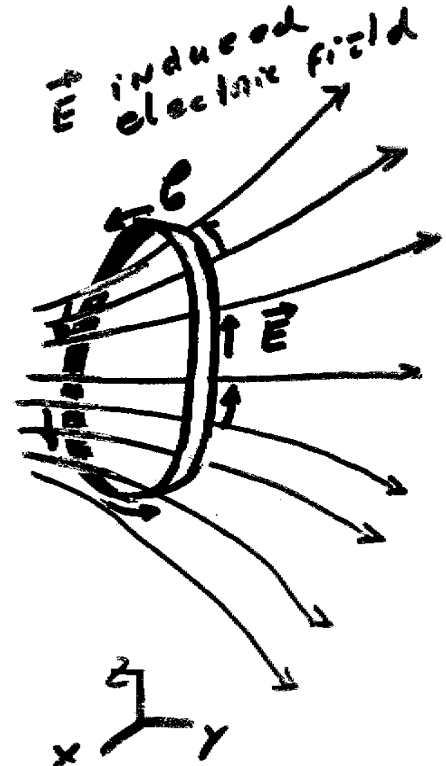

S
N
magnet falling


Ring

FARADAY'S LAW



time = t



time = t'

Magnetic field is changing with time

$$\vec{B} = \vec{B}(t)$$

which gives rise to $\longrightarrow$ $\vec{E}$

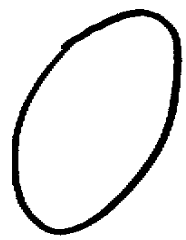
induced electric field.

An electric field $\vec{E}$ give rise to $\longrightarrow$ $\mathcal{E} = \oint_C \vec{E} \cdot d\vec{l}$

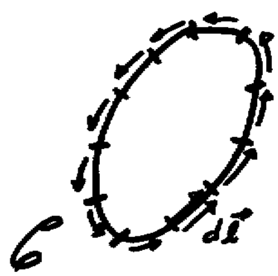
Induced "electromotive force"

What is "electromotive force" $\mathcal{E}$?

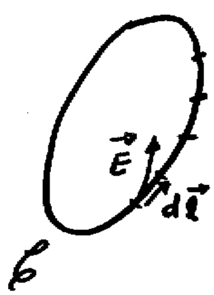
Given a path $\mathcal{C}$



We brake it down into small pieces of "small vectors" of length $\Delta\vec{l}$



In general, along the path $\mathcal{C}$ there may be a electric field $\vec{E}$



At each point along $\mathcal{C}$ we evaluate $\vec{E} \bullet d\vec{l}$

scalar product

The "electromotive force" $\mathcal{E}$ is defined as

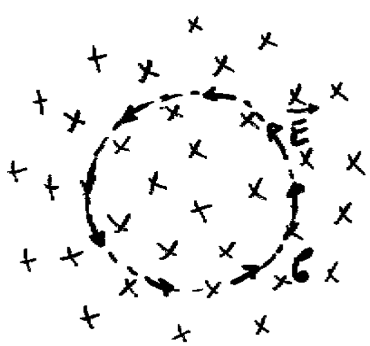
$$\mathcal{E} = \int_{\mathcal{C}} \vec{E} \bullet d\vec{l}$$

- $$\boxed{\mathcal{E} = -\frac{d}{dt}\Phi_m}$$ FARADAY'S LAW

Electromotive force — Magnetic flux

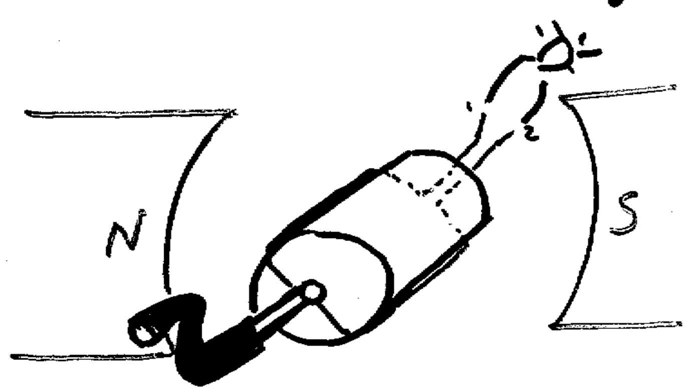
- Because $\Phi_m = \int_S \vec{B} \cdot d\vec{A}$

Φ_m can change because $\vec{B}$ is changing
or
the vector AREA of the circuit is changing



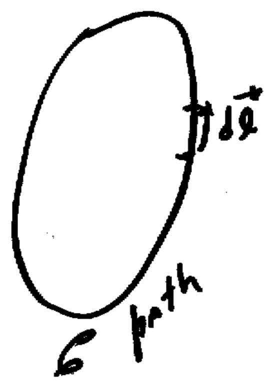
$B = B(t)$

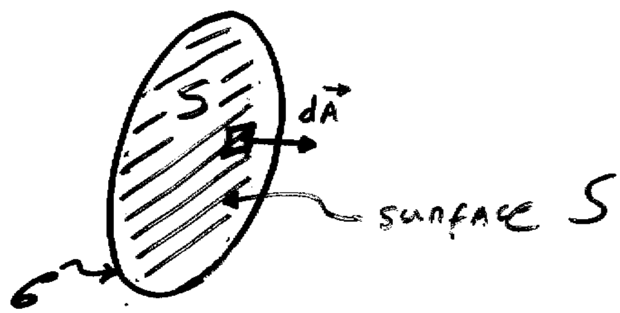
B is changing with time
(is B increasing or decreasing?)



$B = const$

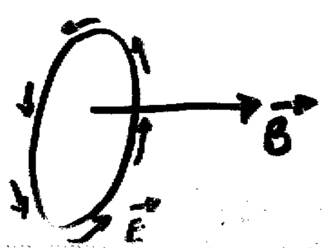
C is the boundary of the surface S





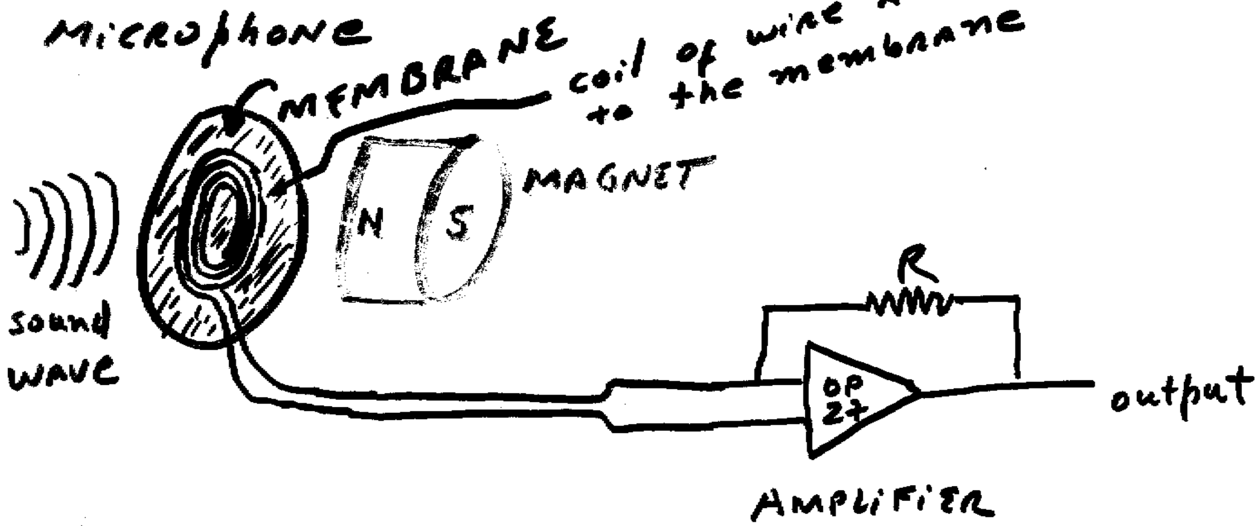
$$\mathcal{E} = -\frac{d}{dt}\Phi_M$$

$$\int_C \vec{E}\cdot d\vec{\ell} = -\frac{d}{dt}\int_S \vec{B}\cdot d\vec{A}$$



Applications

Microphone



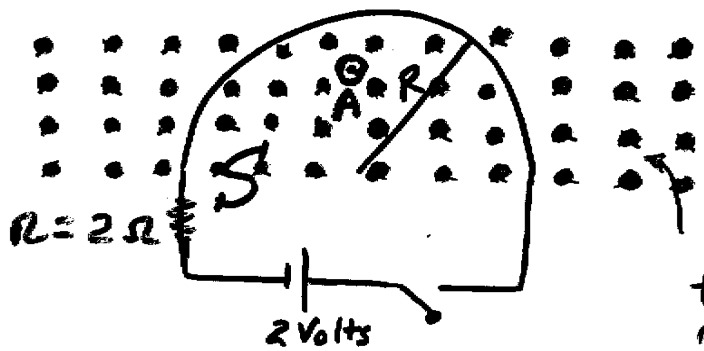
sound wave strikes the membrane

coil moves in a magnetic field
Magnetic flux changes, so there will be an $\mathcal{E}$

The relatively small current induced on the coil are further amplified.

Review SAMPLE PROBLEM 31-2, page 716

$$\Phi = \int_S \vec{B}\cdot d\vec{A} = \int_{\triangle} \vec{B}\cdot d\vec{A} = \int B\,dA = B\underbrace{\int dA}_{\frac{1}{2}\pi R^2} = BA$$



$$\mathcal{E} = -\frac{d}{dt}\Phi$$

$\overline{B}$ is uniform throughout this region.

$$A = \frac{1}{2}\pi R^2 \quad \text{(remains constant)}$$

$$B = 4t^2 + 2t + 3$$

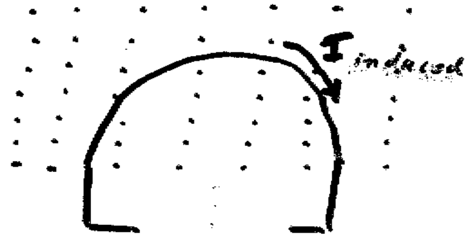
t in seconds
B in Tesla.

(B is changing with time)

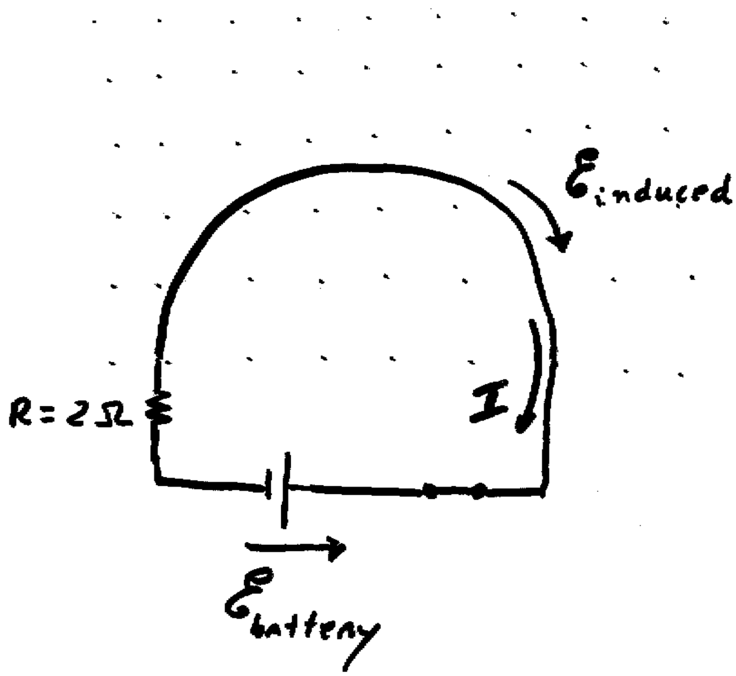
$$\mathcal{E}_{induced} = -\frac{d}{dt}AB = -A\underbrace{\frac{d}{dt}B}_{8t+2}$$

$$= -\frac{1}{2}\pi R^2 \times (8t + 2)$$

Now, use Lenz's law to figure out the "direction" of $\mathcal{E}_{induced}$



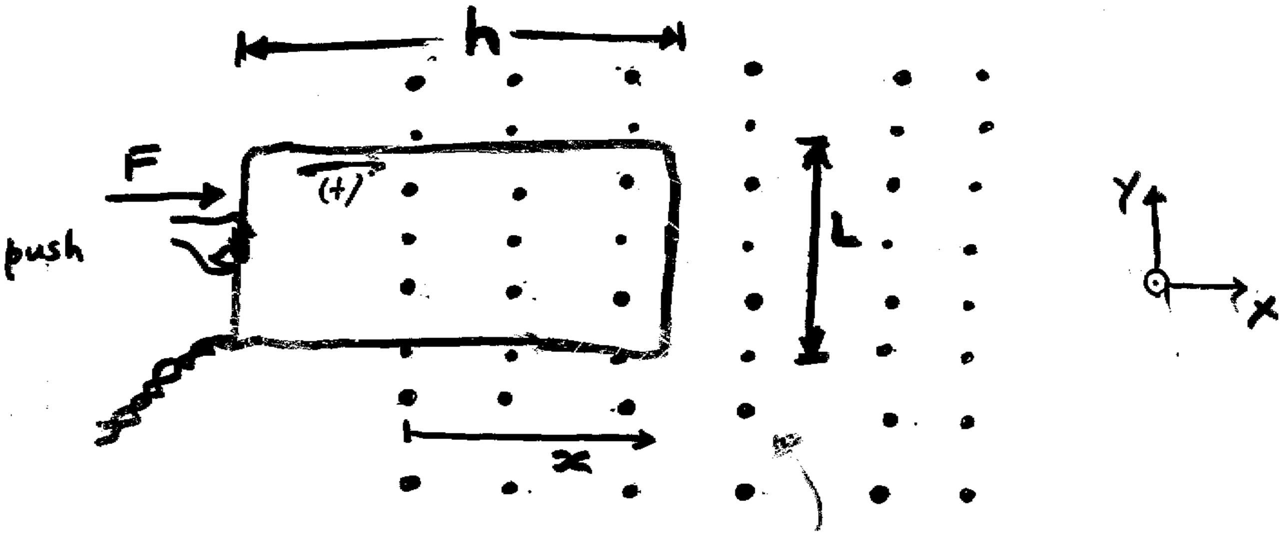
So, what we have is



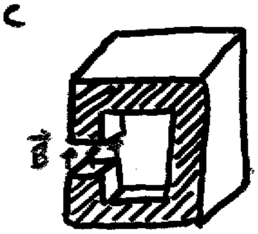
$$I = \frac{\mathcal{E}_{induced} - \mathcal{E}_{battery}}{R}$$

Notice, the current I will change with time.

Example: Wire loop mechanically pushed into a constant magnetic field



uniform magnetic field B and static (does not change with time)



$$\Phi_m = \int_{\square} \vec{B} \cdot d\vec{A}$$

$$= BhL \quad \text{or} \quad BxL$$

?

So, we notice that the magnetic flux changes as we introduce the loop into the region where there exists a magnetic field.

B doesn't change with time but $\int B \cdot d\vec{A}$ does, as far as the loop keeps moving

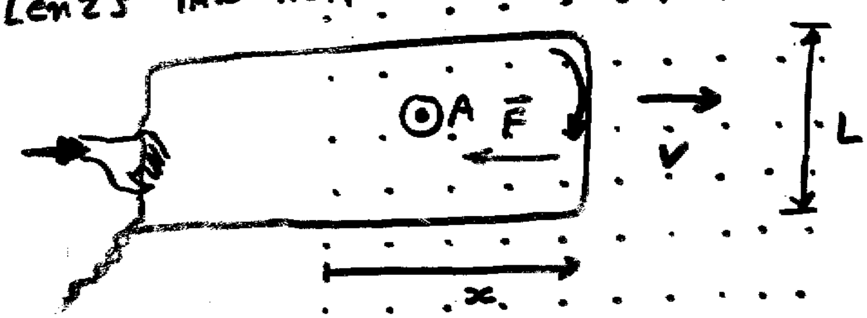
According to Faraday's law an electromotive force will appear along the loop

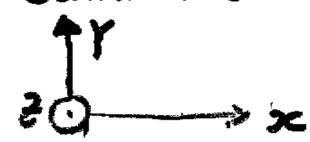
$$\mathcal{E}_{induced} = -\frac{d}{dt}\Phi_m = -\frac{d}{dt}BLx = -BL\underbrace{\frac{dx}{dt}}_{\text{velocity } V}$$

velocity V at which the wire-loop is being inserted

$$\mathcal{E} = -BLV$$

Lenz's law help us to figure out the direction of the induced current





Area $\vec{A} = A\,\hat{k}$

where $A = Lx$

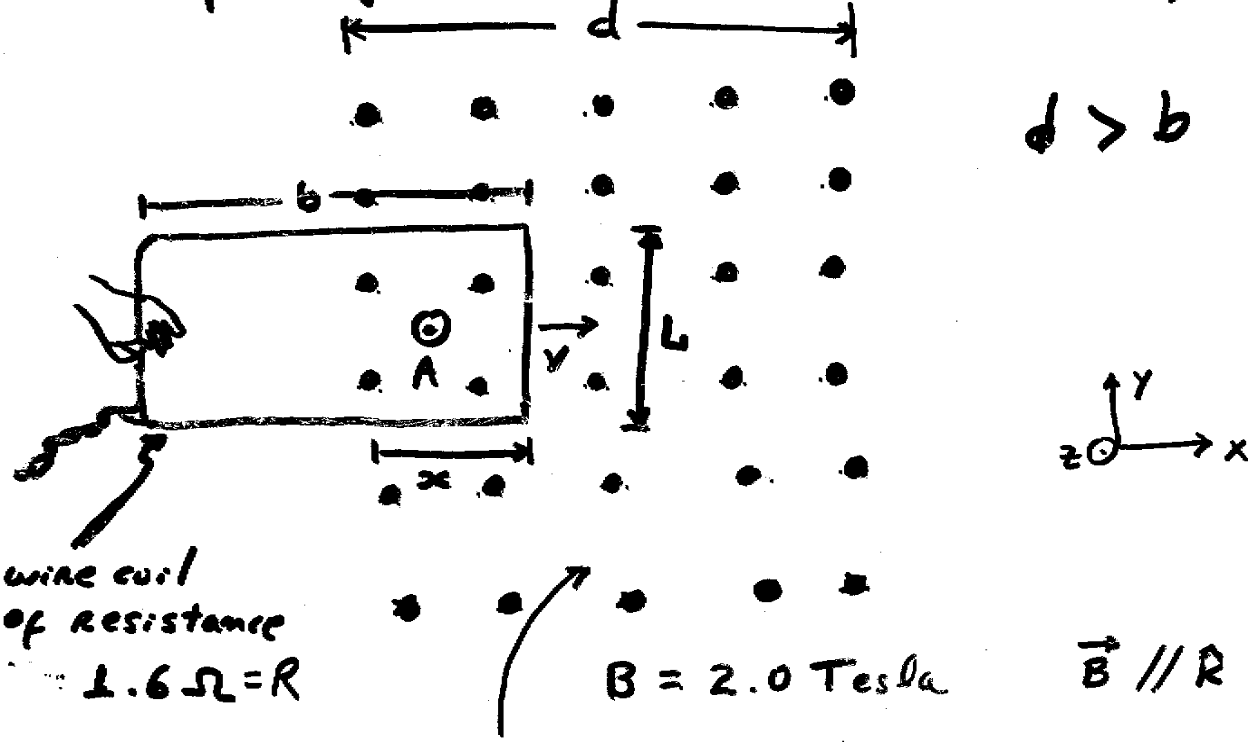
Notice also that the presence of an induced current I implies that:

there will be a magnetic force on the loop.

Where does that force points toward to?

HW * Do checkpoint 3, page 7..

Sample Problem 31-4, page 760 (5th Edition)
Loop being pushed at constant velocity v



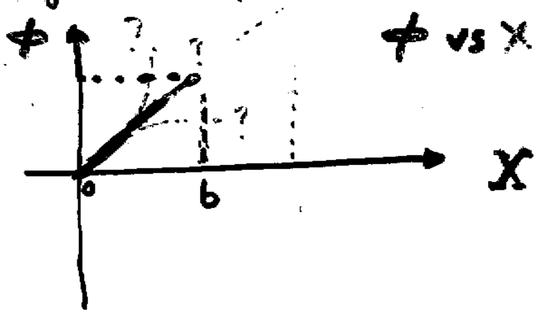
$d > b$

$B = 2.0$ Tesla $\quad \vec{B} \parallel \hat{k}$

For $x < b$

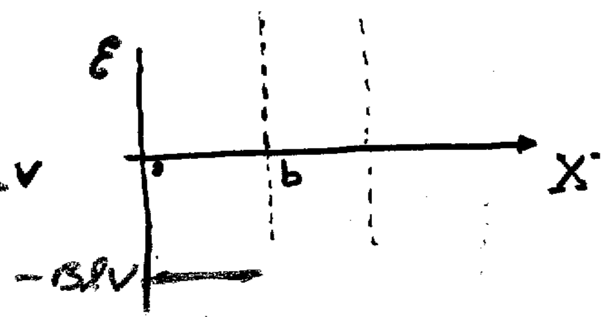
$\vec{A} = A\hat{k}$
$= Lx\,\hat{k}$

$\Phi = BLx$



$\mathcal{E} = -\frac{d}{dt}\Phi$

$= -BL\underbrace{\frac{dx}{dt}}_{\text{velocity}} = -BLv$



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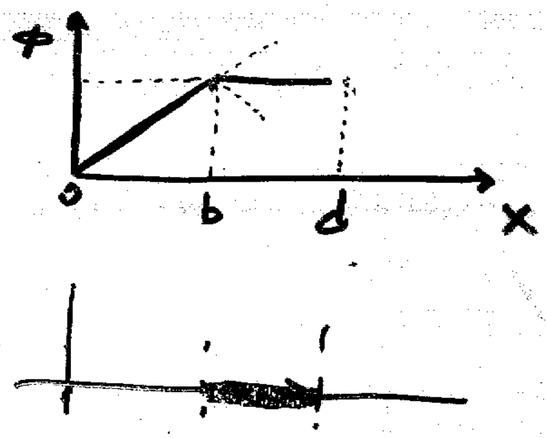
FOR $b < x < d$

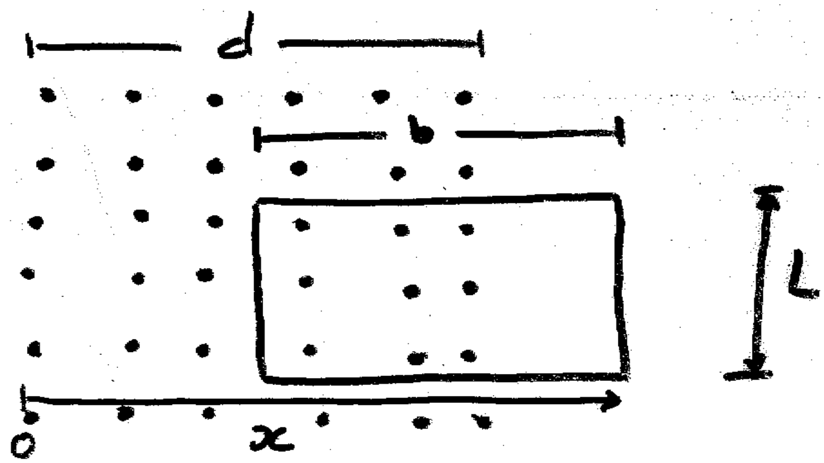
$\vec{A} = Lb$

$\Phi = BLb$

$\mathcal{E} = -\frac{d\Phi}{dt}$

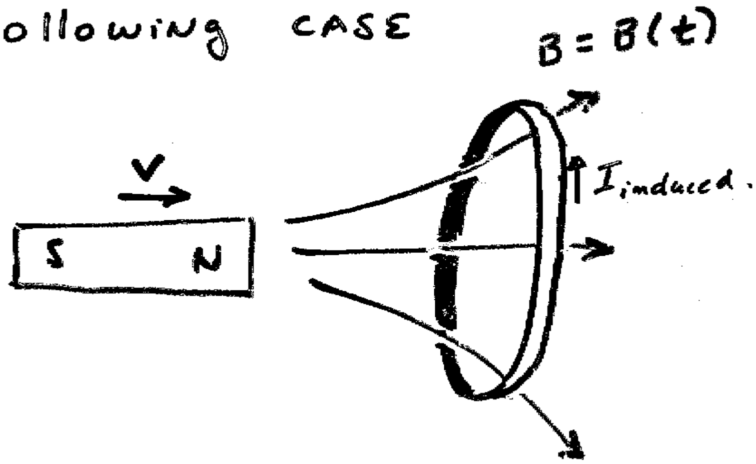


FOR $x > d$



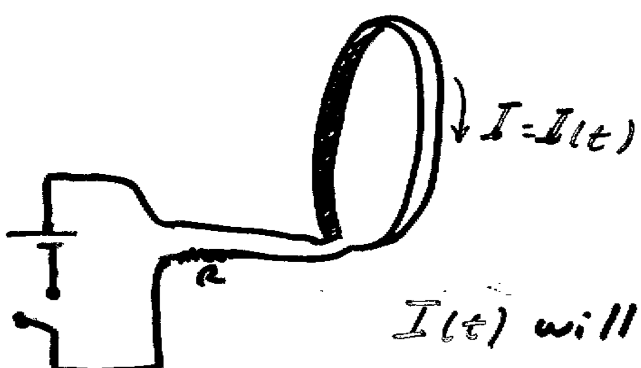
Summary

- So far we have considered the following case



the coil "sees" a magnetic field that changes with time

- What about if we establish on a coil a current I that is time dependent?



$I(t)$ will create a magnetic field $B = B(t)$
therefore; there will be a magnetic flux Φ
$\Phi = \Phi(t)$