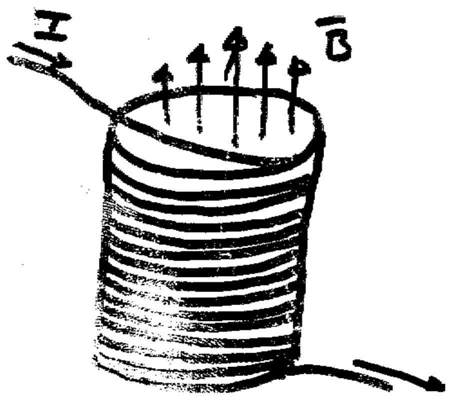
SELF INDUCTANCE (see also p. 725)



INDUCTOR

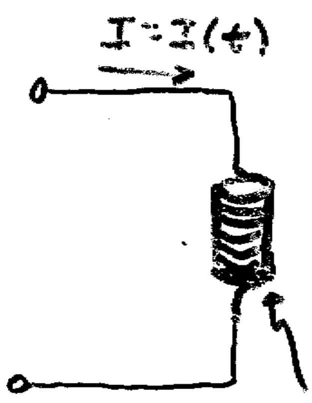
$d\Phi = \vec{B}\cdot d\vec{A}$

The current I produces a magnetic field $\bar{B}$ that varies from point to point, but $\vec{B}$ is propor-tional to I at every point.

Therefore
The magnetic flux Φ_m is also proportional to I

$$\Phi_m = L\,I$$

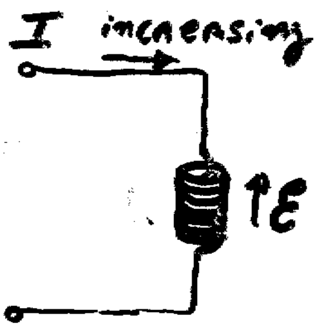
self inductance of the coil

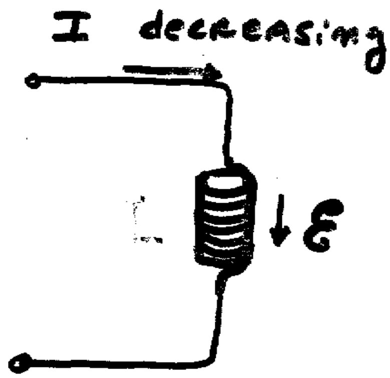


When a current $I(t)$ is changing with time, an electromotive force $\mathcal{E}$ will be induced across the thermi-nals

$$\mathcal{E} = -\frac{d}{dt}\Phi_m$$

$$\boxed{\mathcal{E} = -\,L\frac{dI}{dt}}$$





$$\mathcal{E} = -L \underbrace{\frac{dI}{dt}}_{\text{Amp/sec}}$$

$\mathcal{E}$: volts

L: Henry [H]

UNITS of the self inductance

L

or

$$\Phi_m = L\, I$$

L: Henry [H]

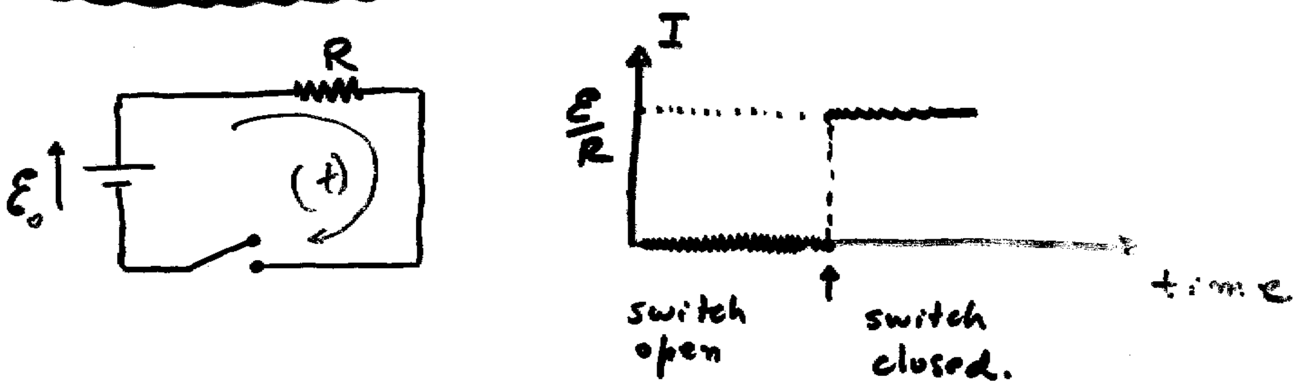
I: Ampere

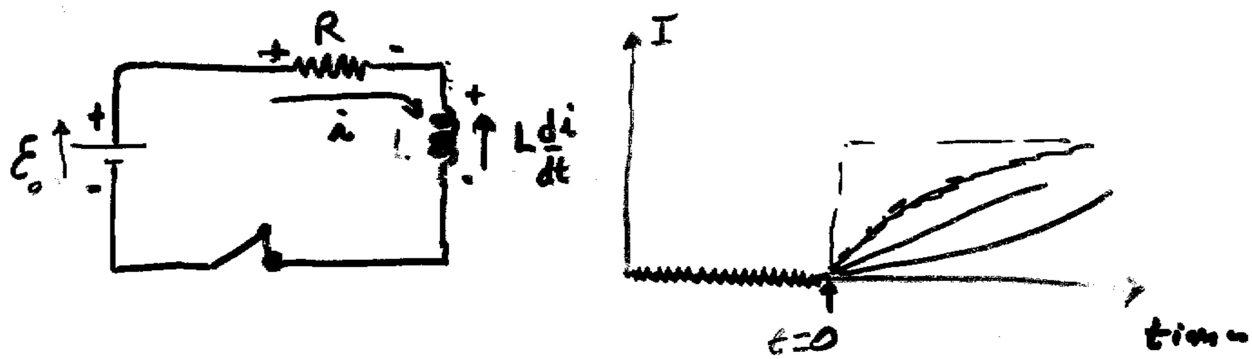
Since the wire that makes up the coil has some resistance, a REAL inductor (a solenoid is an inductor) is typically represented as

ideal inductor + resistor = real inductor

What is the effect of an inductor in a circuit ?

RL circuit





When the switch is closed, Kirchhoff's law indicates.

$$+\mathcal{E}_0 - iR - L\frac{di}{dt} = 0$$ ← ($\mathcal{E}$ is a const in this particular case)

differential equation.

$$L\frac{di}{dt} + iR = \mathcal{E}_0$$ ① $i = ?$

Let's try some solutions.

- Is $i_1 = \frac{\mathcal{E}_0}{R}$ a solution ? ✓

Well, we have to find a more general solution, one that involves . . .

we remember that: IF i_1 is a solution and i_2 is another solution of the same linear differential equation, THEN $i_1 + i_2$ is another solution.

So, let's try

$$i = \underbrace{\frac{\mathcal{E}}{R}}_{i_1} + \underbrace{A e^{-bt}}_{i_2} \qquad (2)$$

A, b are constants, so far unknown

In order to replace (2) in (1) we calculate first:

$$\frac{di}{dt} = -Abe^{-bt}$$

$$iR = \mathcal{E} + RAe^{-bt}$$

thus, replacing these values in (1) we obtain

$$\underbrace{-Abe^{-bt}} + \underbrace{\cancel{\mathcal{E}_0} + RAe^{-bt}} = \cancel{\mathcal{E}_0}$$

$$A b e^{-bt} = R A e^{-bt}$$

$$\Rightarrow \quad \boxed{b = \frac{R}{L}}$$

What about A ?

So far, our solution looks like:

$$i = \frac{\mathcal{E}}{R} + A e^{-(R/L)t} \qquad \text{solution}$$

"A" will be determined by the "initial conditions"

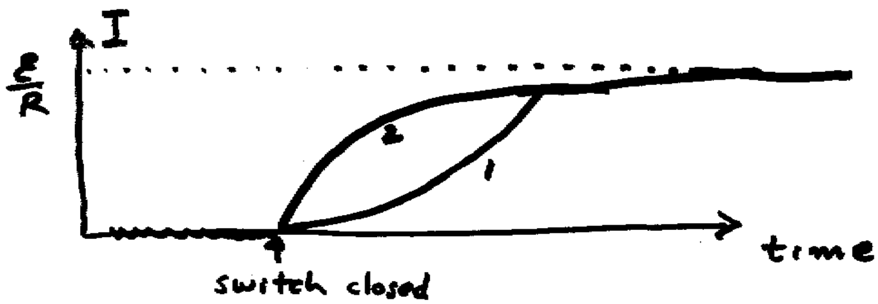
When $t=0$, $i=0$

$$0 = \frac{\mathcal{E}}{R} + A \underbrace{e^{-(R/L)\cdot 0}}_{=1} \Rightarrow \boxed{A = -\frac{\mathcal{E}}{R}}$$

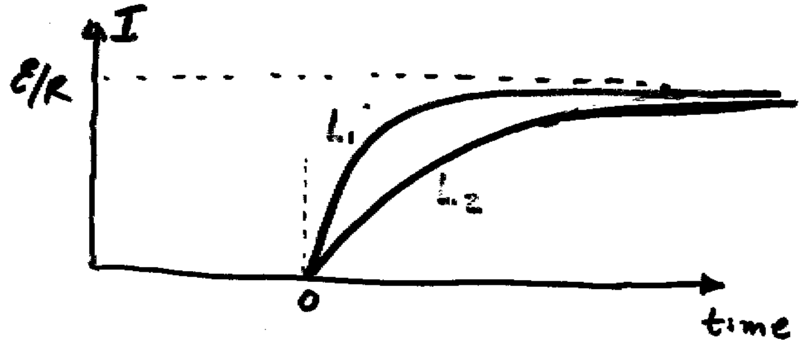
thus,

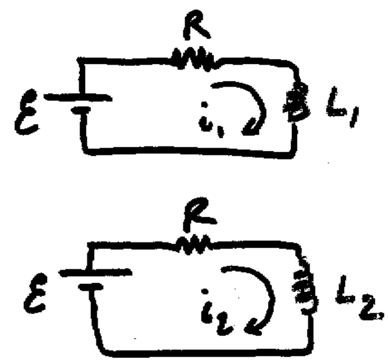
$$i = \frac{\mathcal{E}}{R}\left[1 - e^{-(R/L)t}\right]$$

where t starts to count just after the switch is closed



*Checkpoints

① 



Which self inductance is higher? L_1 or L_2?

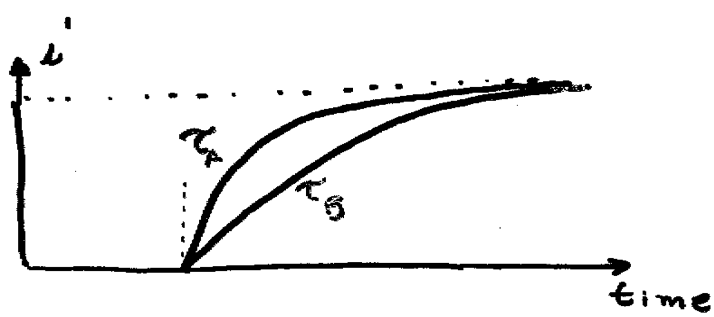
It is usual to define

$$\tau = \frac{L}{R} \longleftarrow \text{" inductive time constant}$$

The current on a LR circuit can, then, be expressed as

$$i = \frac{\mathcal{E}}{R}\left[1 - e^{-t/\tau}\right]$$

*Checkpoint : Show that the quantity $\frac{L}{R}$ has units of time

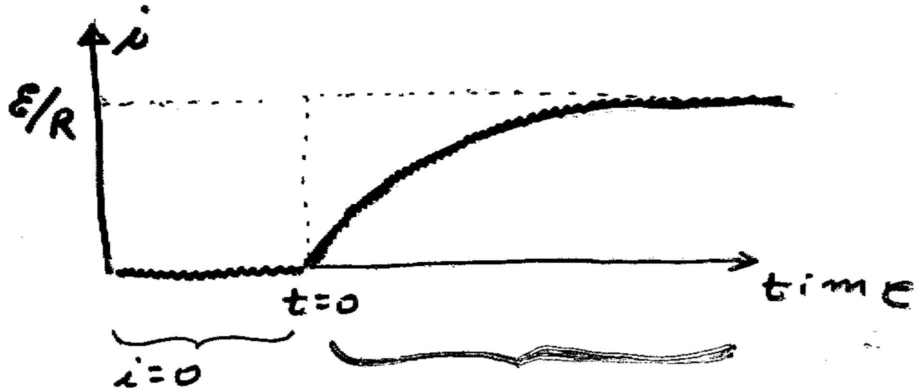
* Checkpoint 

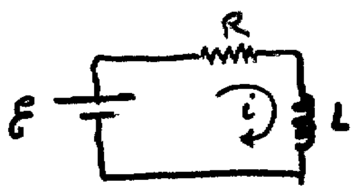
$\tau_A > \tau_B$

$\tau_A = \tau_B$

$\tau_A < \tau_B$ ✓

Interpretation of τ



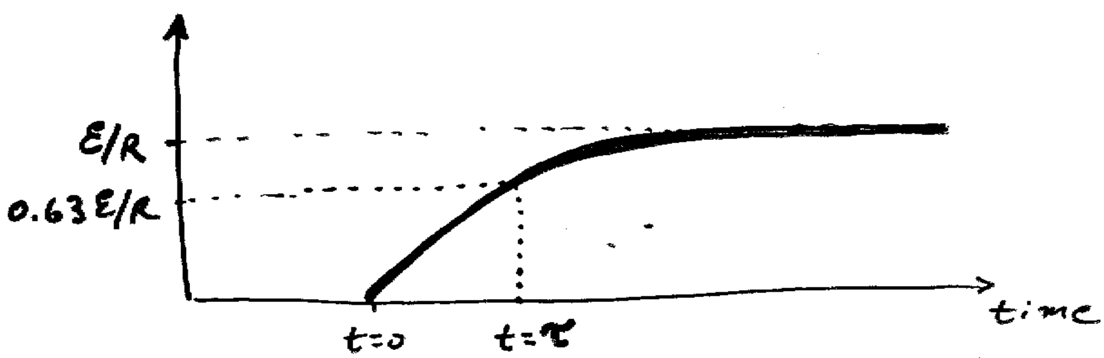


$$i(t) = \frac{\mathcal{E}}{R}\left[1 - e^{-t/\tau}\right]$$

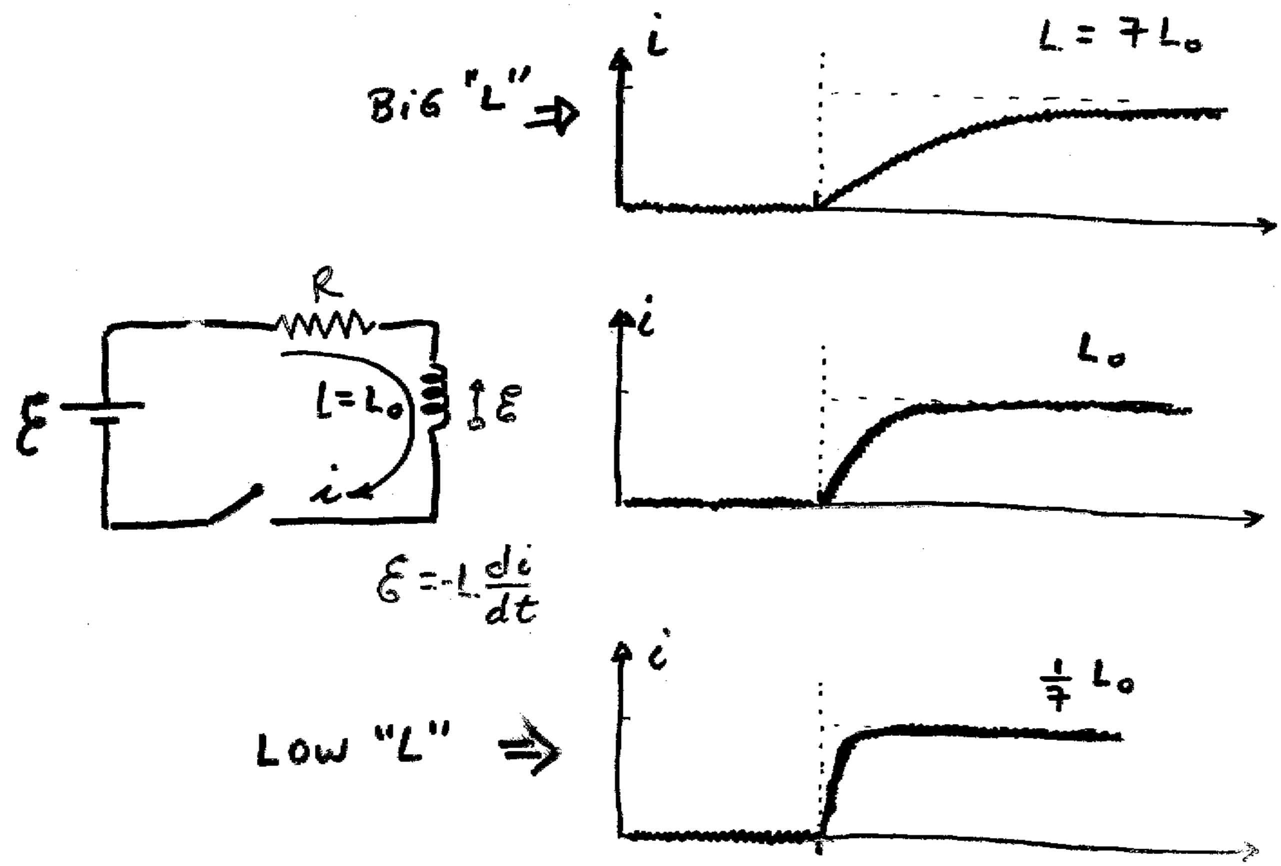
where $\tau = \frac{L}{R}$

when $t = \tau = \frac{L}{R}$

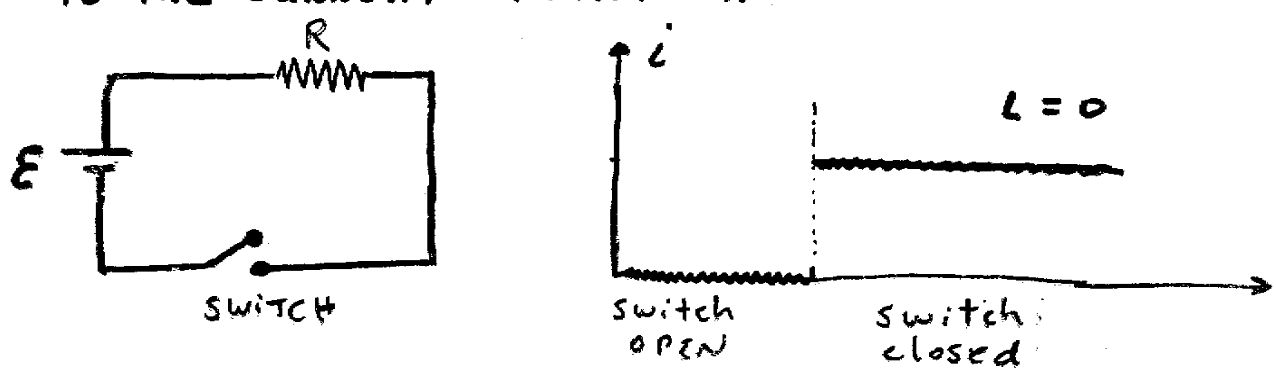
$$i(\tau) = \frac{\mathcal{E}}{R}\left[1 - e^{-\tau/\tau}\right] = 0.63\,\frac{\mathcal{E}}{R}$$



Physical interpretation



L_0 acts as a break that opposes any change in the current. The lower L, the less opposition to the current variation.

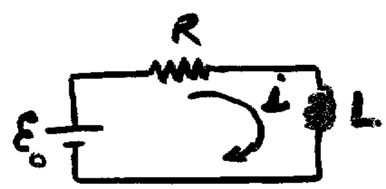


MAGNETIC ENERGY

STORED IN A INDUCTOR

We know

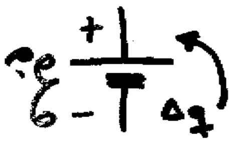
$$\mathcal{E}_0 = iR + \frac{di}{dt}$$



Let's multiply this equation by i:

$$i\mathcal{E}_0 = i^2R + i\frac{di}{dt}$$

$$i\mathcal{E}_0 = i^2R + \frac{1}{2}\frac{di^2}{dt}$$



work done on a charge Δq to take it across a potential difference $\mathcal{E}$ is:

$$(\Delta q)\mathcal{E}.$$

If it does it in a time Δt, THEN the rate at which the battery does work on the circuit is:

$$\frac{\Delta q}{\Delta t}\mathcal{E}$$

$$\boxed{i\mathcal{E}}$$

$$\boxed{i^2 R}$$

Rate at which energy is dissipated through the resistor R as Thermal Energy

$$\boxed{\frac{1}{2}L\frac{di^2}{dt}}$$

CONSERVATION OF ENERGY indicates that this term correspond to the rate at which the magnetic energy is being stored in the inductor.

This is to say:

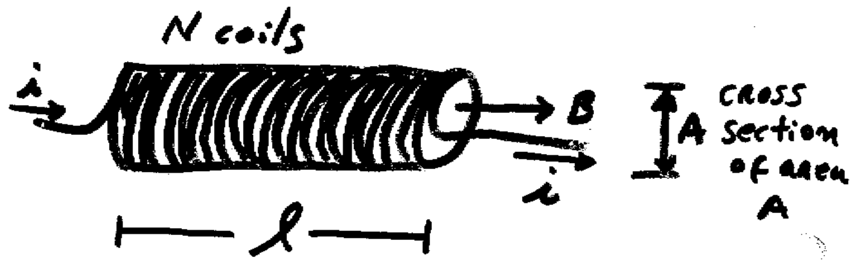
$$\frac{dU}{dt} = \frac{1}{2}L\frac{di^2}{dt}$$

$$U = \frac{1}{2} L i^2$$

MAGNETIC ENERGY stored in the inductor.

Question:

What is the selfinductance L of a selenoid ?



$n = \frac{N}{l}$ # of turns per unit length

We already know that

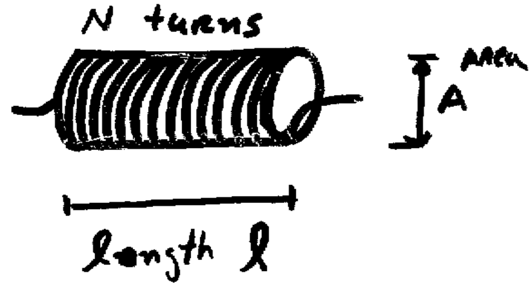
$$B = \mu_0 \frac{N}{l} i \quad ; \quad l \gg A$$

The total magnetic flux through the N coils is

$$\Phi = NBA$$

$$= \underbrace{\mu_0 \frac{N^2}{l} A}\, i$$

$: \mu_0 \frac{N^2}{\ell} A$ of a solenoide



QUESTION: What is the magnetic energy stored in a solenoid carrying a current i?

$$u = \frac{1}{2} \, i^2$$

$$= \frac{1}{2}\left(\mu_0 \frac{N^2}{\ell} A\right) \frac{B^2}{\left(\mu_0 \frac{N}{\ell}\right)^2}$$

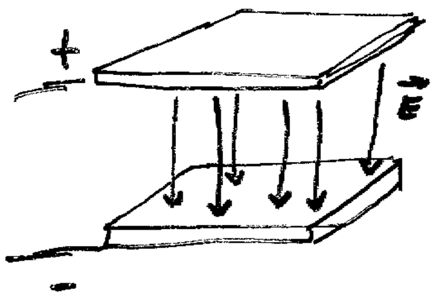
$$= \frac{1}{2} A\ell \frac{B^2}{\mu_0}$$

(Notice that $A\ell$ is the volume of the solenoid)

$$\boxed{u_B = \frac{B^2}{2\mu_0}}$$

MAGNETIC ENERGY density (Joules/m^3)

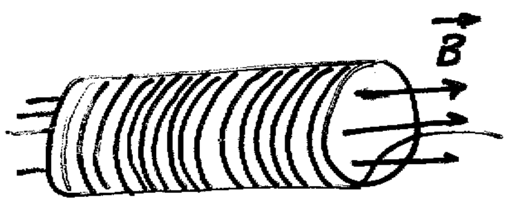
Remember, earlier we found that energy is stored in the electric field



$$u_E = \frac{\text{Energy}}{\text{Volume}}$$

$$= \frac{1}{2}\epsilon_0 E^2$$

Now, we just realized that



$$u_M = \frac{1}{2\mu_0} B^2$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$$

$$\frac{\mu_0}{4\pi} = 10^{-7} \frac{Tm}{Amp}$$

Wherever there exist an electro-magnetic field, there exist an electromagnetic energy density given by

$$\boxed{u = \frac{\text{electromagnetic energy}}{\text{volume}} = \frac{1}{2}\epsilon_0 E^2 + \frac{1}{2\mu_0} B^2}$$

u is in $\frac{\text{Joules}}{m^3}$

Exercise: Just for curiosity, find the value of

$$\frac{1}{\sqrt{\epsilon_0 \mu_0}}$$ in MKS

light

what is the speed of light? (in m/s)

* checkpoint #6, page 729.

* checkpoint

  At $t = t_1$ the switch is closed. Later, at $t = t_2$ the switch is oppened again.

  Make an sketch of i vs time.