A. La Rosa
PSU-Physics

PH 411/511
Fall-2009

INTRODUCTION TO QUANTUM MECHANICS

Homework -3

Due: 10-22-2009

---

1. **1.A** Demonstrate that the time average of $Cos^2(\omega\, t)$ over one time-period is ½.

Hint: Find the period of the function $Cos^2(\omega\, t)$ and integrate $Cos^2(\omega\, t)$ over that period of time

1.B Let $z_1 = a_1 + j\, b_1$ and $z_2 = a_2 + j\, b_2$ be two arbitrary complex numbers. Demonstrate that,

$(z_1 + z_2)^* = z_1^* + z_2^*$ and $(z_1 z_2)^* = z_1^* z_2^*$

where z* stands for the complex conjugate of z.

---

2. **2A.** Let $z_1 = r_1 Cos(\theta_1) + j\, r_1 Sin(\theta_1)$ and $z_2 = r_2 Cos(\theta_2) + j\, r_2 Sin(\theta_2)$ be two arbitrary complex numbers. Show explicitly, using the definition of two complex numbers) that

$z_1 z_2 = r_1 r_2 [\, Cos(\theta_1 + \theta_2) + j\, Sin(\theta_1 + \theta_2)\,]$

---

3. Spectral decomposition in complex variable.

3A Justify why

$$\int_0^{\infty} dk \left[\frac{1}{\pi}\int_{-\infty}^{\infty} \text{Cos}\,(k(x'-x)\;\psi(x')dx'\right] = \int_{-\infty}^{0} dk \left[\frac{1}{\pi}\int_{-\infty}^{\infty} \text{Cos}\,(k(x'-x)\;\psi(x')dx'\right]$$

Justify why

$$\int_0^{\infty} dk \left[\frac{1}{\pi}\int_{-\infty}^{\infty} \text{Sin}\,(k(x'-x)\;\psi(x')dx'\right] = - \int_{-\infty}^{0} dk \left[\frac{1}{\pi}\int_{-\infty}^{\infty} \text{Sin}\,(k(x'-x)\;\psi(x')dx'\right]$$

3B Demonstrate

$$\int_{0}^{\infty} dk \left[\frac{1}{\pi} \int_{-\infty}^{\infty} \text{Cos}\,(k(x'-x)\ \psi(x')dx'\right] = \frac{1}{2}\int_{-\infty}^{\infty} dk \left[\frac{1}{\pi} \int_{-\infty}^{\infty} \text{Cos}\,(k(x'-x)\ \psi(x')dx'\right]$$

Demonstrate

$$-j\frac{1}{2}\int_{-\infty}^{\infty} dk\ \left[\frac{1}{\pi} \int_{-\infty}^{\infty} \text{Sin}\,(k(x'-x)\ \psi(x')dx'\right] = 0$$

---

4. Given the set of harmonic functions $\{\ Cos_n\ ,\ Sin_n\ ;$ with $n = 0,1,2,3\ldots\}$, where λ is the period, and

$$Cos_0(x) \equiv \sqrt{\frac{1}{\lambda}}\ ,$$

$$Cos_n(x) \equiv \sqrt{\frac{2}{\lambda}}\ Cos\left(\frac{2\pi}{\lambda/n}x\right),$$ for n=1,2, … ; and

$$Sin_n(x) \equiv \sqrt{\frac{2}{\lambda}}\ Sin\left(\frac{2\pi}{\lambda/\text{n}}x\right);$$ for n =0,1,2, ...

Using the definition of scalar function between two periodic function, show explicitly that ,

$$< \text{Sin}_2 | \text{Cos}_2 > = 0$$

$$< \text{Sin}_2 | \text{Sin}_2 > = 1$$

$$< \text{Sin}_2 | \text{Cos}_1 > = 0$$

---

5. For the periodic function

$\Psi(x) = 1$ for $0 < x < \lambda/2$

$= 0$ for $\lambda/2 < x < \lambda$

$\Psi(x + \lambda) = \Psi(x)$

Find analytically the first 4 coefficients of its series Fourier expansion.
Using an excel file, plot the resulting 4-terms expansion (select your own value for λ.)