A. La Rosa
PSU-Physics

PH 411/511
Fall-2009

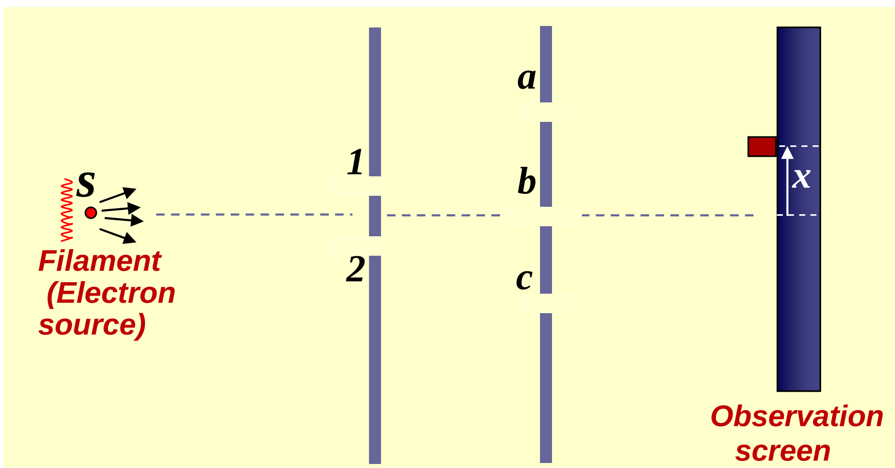
# INTRODUCTION TO QUANTUM MECHANICS

## Homework - 4

**Due: 11-05-2009**

---

**1.** The figure shows tow walls, one with two apertures (1 and 2) and another with three apertures (a, b, c). Behind the second wall there is a detector at the position $x$.




**1.A** Indicate the amplitude probability that an electron leaving from the source s, passes through the aperture 1 and the aperture b, and arrives at $x$.

**1.B** Indicate the probability that an electron leaving from the source *s* arrives at $x$, after passing through aperture 1 and through either apertures b or c.

**1.C** Indicate the probability that an electron leaving from the source *s* arrives at $x$, after passing through either aperture 1 or 2, and through aperture a.

**1.D** Calculate the amplitude probability $\langle x | s \rangle$

**2. Two-slit experiment**. During the lecture session we derive an expression to calculate the probability that an electron leaving from $s$ arrives at $x$ (while being watched by photons emitted from a source L and the scattered ones detected by either the photon counters $D_1$ or the photon counter $D_2$ ) is given by :

$$P(x;s)_{when\ a\ ligh\ source\ is\ present} =$$

$$= \left| \left\langle \begin{matrix} electron\ at\ x \\ photon\ in\ D_1 \end{matrix} \middle| \begin{matrix} electron\ from\ s \\ photon\ from\ L \end{matrix} \right\rangle \right|^2 + \left| \left\langle \begin{matrix} electron\ at\ x \\ photon\ in\ D_2 \end{matrix} \middle| \begin{matrix} electron\ from\ s \\ photon\ from\ L \end{matrix} \right\rangle \right|^2$$

$$= \left| \langle x|1\rangle a\langle 1|s\rangle + \langle x|2\rangle b\langle 2|s\rangle \right|^2 + \left| \langle x|1\rangle b\langle 1|s\rangle + \langle x|2\rangle a\langle 2|s\rangle \right|^2$$

Find the particular form this general expression will adopt if the following particular cases are considered:

**2.A** Case: The aperture-1 is closed

Find $P(x;s)$ and sketch a plot of the probability as a function of $x$.

(30% points)

**2.B** Evaluate $P(x;s)$ for the case $a=b$, and sketch a plot of the probability as a function of $x$.
Explain in detail, what physical situation does this choice may correspond to.

(30% points)

**2.C** If the photon detector-1 were removed, what would be the corresponding expression for $P(x;s)$ ?

(40% points)

**3.** Bohr postulated that the orbital angular momentum $L$ of an electron, moving in a circular orbit around the nucleus of charge $Ze$, can take only discrete values given by, $L = n\hbar = n\frac{h}{2\pi}$ with the quantum number $n = 1, 2, 3, ...$ and $h$ equal to the Planck's constant.

**3.A** Show that under the Coulomb potential, the possible values of the orbit-radius and the velocity are given, respectively, by

$$r = \frac{(4\pi\varepsilon_0)\hbar^2}{Zme^2} n^2 \quad \text{and} \quad \mathrm{v} = \frac{Ze^2}{(4\pi\varepsilon_0)\hbar} \frac{1}{n}$$

Hint: Equate the centripetal force with the Coulomb force

**3.B** Evaluate the potential energy and kinetic energy, and demonstrate that the corresponding total mechanical energy is given by,

$$E = -\frac{m}{2}\left[\frac{Ze^2}{(4\pi\varepsilon_0)\hbar}\right]^2 \frac{1}{n^2} = -\frac{mZ^2e^4}{(4\pi\varepsilon_0)^2 2\hbar^2}\frac{1}{n^2}$$

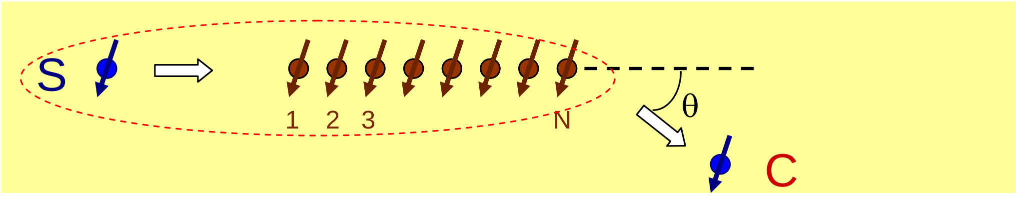
**4.** A crystal composed of nucleon that have spin ½ is bombarded by neutron. In each of the questions posted below, the initial state of the crystal is assumed to be known (as depicted in each corresponding figure.) The final states of the crystal are not shown.

Assume also:

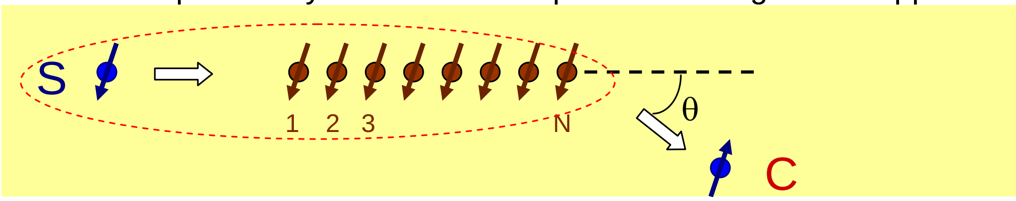
$a$ = amplitude probability that there is no interchange of spin due to the scattering

$b$ = amplitude probability that there is exchange of spin

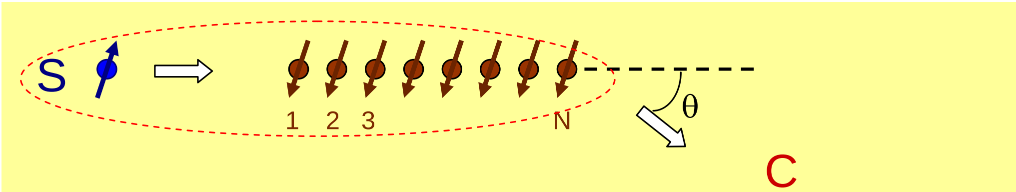
**4.1** For the case depicted in the figure, sketch the expected graph of number of neutron received at the detector as a function of θ.



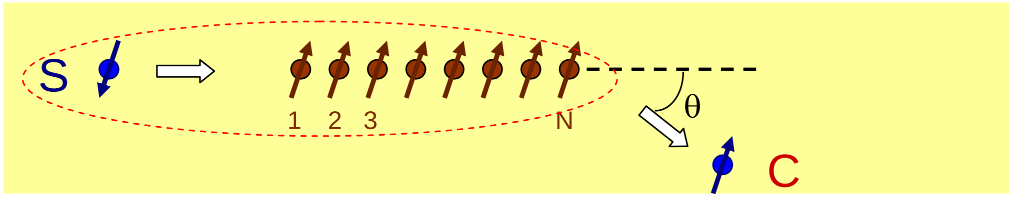

**4.2** Indicate the probability for the event depicted in the figure to happen.




**4.3** What is the probability that the incident neutron flip its spin via its interaction with the nucleon 4 (for a given fixed value of θ.)?




**4.4** What is the probability that this event occurs via the interaction of the incident neutron with the nucleon 4 or the nucleon 10.




**5.** A beam of silver atoms, for which $\mu_z = \pm \mu_B$, passes through an inhomogeneous magnetic field, as in the Stern-Gerlach apparatus. The filed gradient is $\frac{\partial B}{\partial z} \approx 10^3 T/m$, the length of the pole piece is L=0.1 m, the distance to the screen is d=1 m. The temperature of the oven is T=600K. Calculate the maximum separation of the two beam on the screen.

**Hint:** Assume that the velocity of the silver atoms is equal to the root-mean-square velocity $(3kT/M)^{1/2}$ (where $k$ is equal to the Boltzmann constant, and M is the mass of the silver atoms.)

**Hint:** The atoms will experience a vertical force due to the magnetic field gradient The motion of the atom can be considered then as the motion under constant vertical acceleration and constant horizontal velocity.