A. La Rosa
PSU-Physics

PH 411/511
Fall-2009

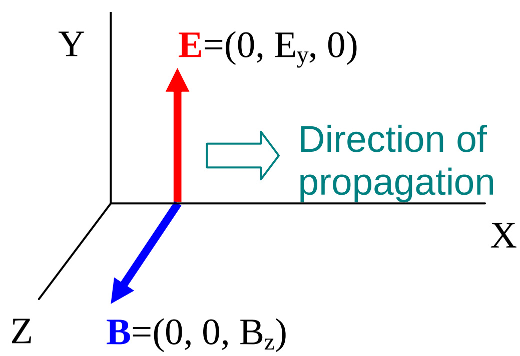
INTRODUCTION TO QUANTUM MECHANICS

Homework -1

Due: Th 10-08-2009

NOTE: Homework reports must be neat, clear, very explicit and correct in order to receive full credit.
Completion of all the questions is highly recommended. The grader will arbitrary and/or randomly select only five questions (out of the six) to grade.

1. The figure shows electromagnetic fields **E**=(0**,** E_y**,** 0) and **B**=(0, 0**,** B_z), where their corresponding components depend on the spatial coordinates as well as time; that is $E_y = E_y$ (*x, y, z, t*) and $B_z = B_z$ (*x, y, z, t*)



1A Find $\nabla \mathrm{x} \mathbf{E}$ and $\nabla \mathrm{x} \mathbf{B}$

Hint: Given the vector function

$$\mathbf{A} = A_x \hat{\mathbf{i}} + A_y \hat{\mathbf{j}} + A_z \hat{\mathbf{z}}) \quad \text{or simply} \quad \mathbf{A} = (A_x, A_y, A_z),$$

where each component depends on the variables x, y and z (i.e. $A_x = A_x(x, y, z)$, … etc), the rotational operator is defined as

$$\nabla \mathrm{x} \mathbf{A} \equiv \det \begin{vmatrix} \hat{\mathbf{i}} & \hat{\mathbf{j}} & \hat{\mathbf{k}} \\ \frac{\partial}{\partial_{\mathbf{x}}} & \frac{\partial}{\partial_{\mathbf{y}}} & \frac{\partial}{\partial_{\mathbf{z}}} \\ A_x & A_y & A_z \end{vmatrix} =$$

$$= \left(\frac{\partial}{\partial_{\mathbf{y}}} A_z - \frac{\partial}{\partial_{\mathbf{z}}} A_y \ , \ \frac{\partial}{\partial_{\mathbf{z}}} A_x - \frac{\partial}{\partial_{\mathbf{x}}} A_z \ , \ \frac{\partial}{\partial_{\mathbf{x}}} A_y - \frac{\partial}{\partial_{\mathbf{y}}} A_x \right)$$

1B Using, respectively, the third and fourth Maxwell's equations demonstrate that

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t} \quad \text{and} \quad \frac{\partial B_z}{\partial x} = -\mu_o \varepsilon_o \frac{\partial E_y}{\partial t}$$

1C From the results in 1B, demonstrate that the components of **E** and **B** satisfy the WAVE EQUATION

$$\frac{\partial^2 E_y}{\partial x^2} - \frac{1}{\mathrm{v}^2}\frac{\partial^2 E_Y}{\partial t^2} = 0 \quad \text{where} \quad \mathrm{v} \equiv 1/\sqrt{(\mu_o \varepsilon_o)}$$

Calculate the value of v

2. Reference **O'** moves with constant horizontal velocity u=0.6c with respect to observer (as shown in the figure below.) At t=0, the origins of **O** and **O'** coincided, and their clocks were synchronized.

2.A Assuming that the relationship between the coordinates is linear, that is,

$$x' = k(x - ut) \quad \text{and} \quad t' = a(t - bx)\,, \quad y' = y\,, \quad z' = z$$

where k, a, and b are parameters to be determined from the conditions

$$x^2 + y^2 + z^2 = c^2 t^2 \quad \text{and}$$

$$x'^2 + y'^2 + z'^2 = c^2 t'^2\,,$$

demonstrate that,

$$x' = \frac{x - u\ t}{\sqrt{1 - (u/c)^2}}\,, \qquad t' = \frac{t - ux/c^2}{\sqrt{1 - (u/c)^2}}$$

Hint: Replace $x' = k(x - ut)$ and $t' = a(t - bx)$, into the relationship $x'^2 + y'^2 + z'^2 = c^2 t'^2$. (In the middle of this resulting expansion, do not overlook to consider $x'^2 + y'^2 = (tc)^2 - x^2$

You should obtain an expression like this: $\mathrm{A}x^2 + \mathrm{B}xt + \mathrm{C}t^2 = 0$; where A, B and C being expressions depending on k, a, b.
(For example $\mathrm{A} = k^2 - a^2 b^2 c^2 - 1$; B= …)

One way to make this condition valid for any x and t is to require A=B=C=0. Hence, obtain three expression that relate k, a, b.

You will notice that one potential solution is to make k=a. From this selection, you will find the required expression for k and b in terms of u and c.

2.B At a given time t', observer **O'** locates an object **P** at the coordinates $x' = z' = 0$ and $y' = 2m$, having a velocity of components $V_{x'} = V_{z'} = 0$ $V_{y'} = 3.5 m/s$.

What is the corresponding y component coordinate of **P** relative to the observer **O**?

What is the corresponding V_y component of the velocity of **P** relative to the observer **O**?

3. 3.A Demonstrate that coordinates related by the Lorentz' transformation satisfy

$$x'^2 + y'^2 + z'^2 - c^2 t'^2 = x^2 + y^2 + z^2 - c^2 t^2$$

Hint: The idea is to set

A= $x'^2 + y'^2 + z'^2 - c^2 t'^2$ (1)

Then, according to the Lorentz transformation, replace the expressions x' = x'(x,y,z,t) ; t'=t'((x,y,z,t) ; y'=y, z'=z into expression (1) above

After some algebra manipulation, the expression for A should reduce to

A= $x^2 + y^2 + z^2 - c^2 t^2$

3.B Given $\mathbf{P} = m\,\mathbf{v} = \dfrac{m_o}{\sqrt{1-(\mathrm{v}/c)^2}}\mathbf{v}$ and $E = \dfrac{m_o}{\sqrt{1-(\mathrm{v}/c)^2}} c^2$

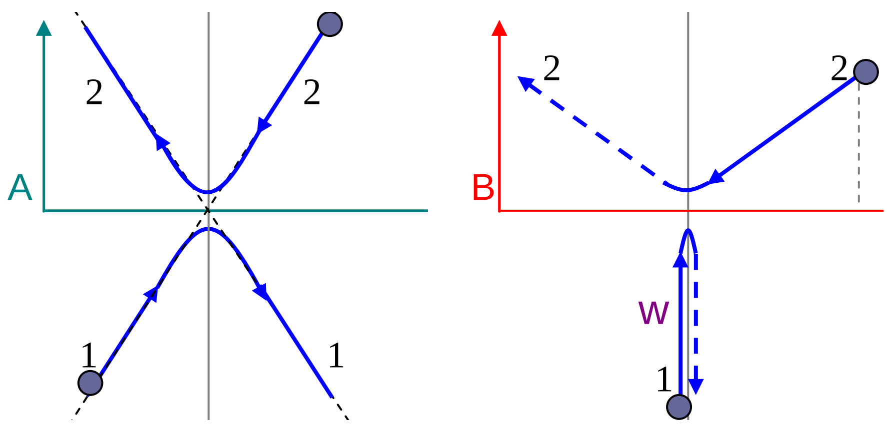
Demonstrate that

$$E^2 - P^2 c^2 = m_o{}^2 c^4 \quad \text{and} \quad Pc = E\mathrm{v}/c$$

4. **4A** Observer O' moves with speed $u_o = c/2$ with respect to observer O (here *c* is the speed of light). A particle moves with speed $V_{x'} = c/2$ relative to observe O'. What is the velocity of the particle respect to observer O.

4B Observer O' moves with speed $u_o = 0.95c$ with respect to observer O. A particle moves with speed $V_{x'} = 0.95c$ relative to observe O'. What is the velocity of the particle respect to observer O.

5. The figure below shows a collision between two identical particles observed from two different references A and B. (Reference A is given just to illustrate the symmetry of the collision, as the particles approach with the same speed. So the problem is simply about the conclusions made by Observer B.) Observer B sees particle-1 approaching with a speed w=0.4c; while the horizontal component of particle-2 is u=0.1c. What is the vertical component of particle-2 as seen by observer B?



6. During the lectures in class, the application of conservation of linear momentum in the case of the collision of two particles (see Fig. 2.9 in the Lecture notes, Chapter 2), led us to the following requirement,

$$\frac{m(\mathrm{v})}{m(\mathrm{w})} = \frac{1}{\sqrt{1-(u/c)^2}} \tag{1}$$

where the dependence of m on the particles' velocities is unknown.

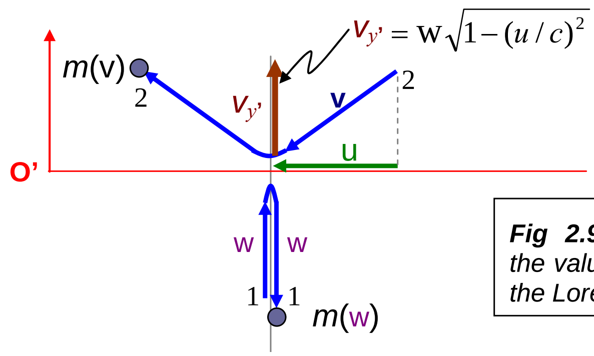


Fig 2.9 *Collision diagram showing the values of speeds compatible with the Lorentz transformation.*

The question now is to find the function m=m(velocity) that satisfies the relationship (1) above.

Let's propose the following solution:

$$m(\mathrm{v}) = \frac{m(0)}{\sqrt{1-(\mathrm{v}/c)^2}} \tag{2}$$

Demonstrate that the (2) indeed satisfies (1).

Hint:
Using (2) obtain an expression for m(v) and m(u), respectively, and replace them in (1). You should obtain

$$\frac{\sqrt{1-(\mathrm{w}/c)^2}}{\sqrt{1-(\mathrm{v}/c)^2}} = \frac{1}{\sqrt{1-(\mathrm{u}/c)^2}} \tag{3}$$

That is, the proposed solution (2 lead us to the (3). But is the last expression true?

The remaining task is then to verify whether or not expression (3) is true. Suggestion: expand the expression (3). You will arrive to an expression that is compatible with the conservation of linear momentum (i.e. compatible with the physical situation described in Fig. 2.9.)