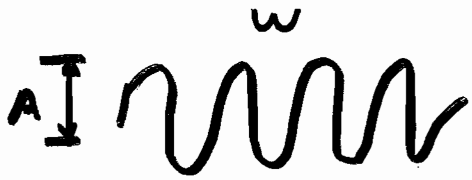
CLASSICAL VIEW



light viewed
as a wave

Since the value of the amplitude A can have any value (in a continuous range), the values of the energy imparted to the oscillator would be also in a continuous range.

QUANTUM PHYSICS VIEW

1900, Max Planck

He suggested, at a given ω, energy from the radiation would be available only in the form of $\hbar\omega$, $2\hbar\omega$, $3\hbar\omega$, ...

That is, rather than a wave, this hypothesis would make ligh to appear to be composed of "particles" of energy $\hbar\omega$

QUANTUM PHYSICS VIEW

In 1900, Max Plack introduced a revolucionary hypothesis:

In the expression

$$u(\omega) = \frac{1}{\pi^2 c^3}\omega^2 W$$

↳ He did not questioned the way used to count the modes

↳ Rather he suggested a new way to calculate W (the average energy of a mode of frequency ω):

↳ He discarted that the mode could have an arbitrary value of (continuos) energy.

INSTEAD

Planck's hypothesis

He proposed a given mode could take only discrete values of energy

$0, \epsilon, 2\epsilon, 3\epsilon, \dots$

The step ϵ would depend on the angular frequency ω of the mode.

The higher ω, the larger step ϵ

Following Planck's hypothesis, the average energy W would have to be calculated according to

$$W_{Planck} = \langle E \rangle = \frac{\sum_{n=0}^{\infty} (n\epsilon)\, P(n\epsilon)}{\sum_{n=0}^{\infty} P(n\epsilon)}$$

A descrete sum (rather than an integral as in the classic case)

The form of P still being of the same form:

$$P(n\epsilon) = \frac{1}{KT}\, e^{-\frac{n\epsilon}{KT}}$$

How could Planck's hypothesis lead to an agreement of $U(w)$ with the experimental results ?

<u>For low w</u>

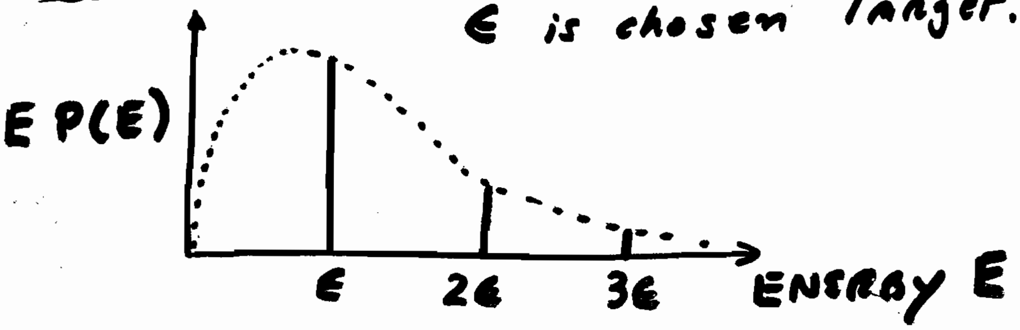
ϵ is chosen very small

When adding all those vertical segments we expect that the resulting W_{Planck} to be very close to the classical value W (Remember, the latter is close to the experimental value at low w)

So, Planck's hypothesis would do well at low ω.

For larger ω

ϵ is chosen larger.



When adding those vertical bars, the resulting W_{PLANCK} would be much smaller than the classical value W.

In fact,
the larger ω, the larger step ϵ to be chosen, the smaller W_{planck}

And, indeed, the experimental results indicate that W should decrease at large ω
Thus W_{planck} would reproduce better the experimental results. !

How to choose the step ϵ ?
ϵ should be chosen to increase with ω, but in what form ?

The simplest choice is to assume a linear relationship

$$\epsilon(\omega) = \hbar\omega$$

where $\hbar$ is a constant of proportionality to be conveniently chosen to fit the experimental results

Thus

$$W_{planck} = \frac{\sum_{n=0} (n\hbar\omega)\left[\frac{1}{KT} e^{-\frac{n\hbar\omega}{KT}}\right]}{\sum_{n=0}^{\infty} e^{-\frac{n\hbar\omega}{KT}}} = W_{planck}(\omega)$$

Average energy of the electro magnetic mode of freq. ω

- Let $\alpha \equiv \frac{\hbar}{KT}$, $A \equiv \sum_{n=0}^{\infty} e^{-n\alpha\omega}$, $\frac{\partial A}{\partial \alpha} = \sum_{n=0}^{\infty} (-n\omega)e^{-n\alpha\omega}$

$$W = \frac{1}{A}\sum_{n=0}^{\infty}(n\alpha\omega)e^{-n\alpha\omega}$$

$$= \frac{1}{A}(-\alpha)\frac{\partial A}{\partial \alpha} = -\alpha\frac{\partial}{\partial \alpha}\ln A$$

- Let $x = e^{-\alpha\omega}$; then
$A = 1 + x + x^2 + \ldots$ which is equal to $\frac{1}{1-x}$

$$A = \frac{1}{1-e^{-\alpha w}} \; ; \; \frac{\partial}{\partial x} \ln A = (1-e^{-\alpha w}) \frac{(-1)\, w\, e^{-\alpha w}}{(1-e^{-\alpha w})^2}$$

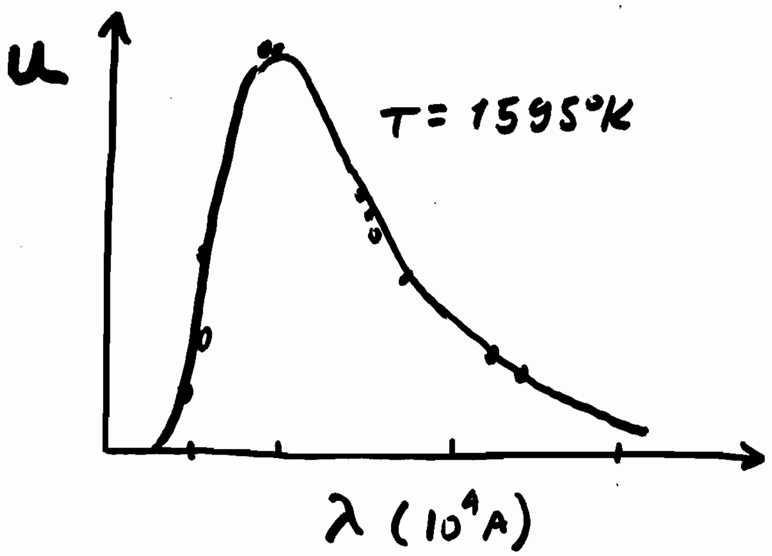
$$W = (-\alpha)(-1) \frac{w\, e^{-\alpha w}}{(1-e^{-\alpha w})} = \frac{\alpha w}{e^{\alpha w} - 1}$$

$$W_{Planck}(w) = \frac{1}{kT} \frac{\hbar w}{e^{\frac{1}{kT}\hbar w} - 1}$$

(This time, W depends on w)

and

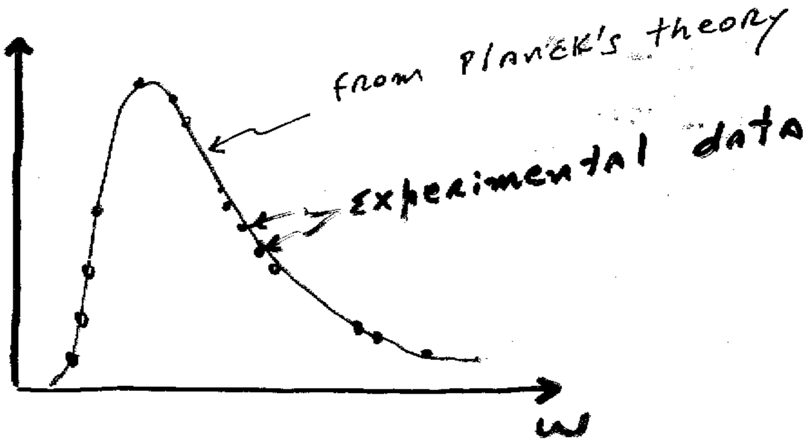
$$u(w) = \frac{1}{\pi^2 c^3} w^2 W = \frac{\hbar}{\pi^2 c^3 kT} \frac{w^3}{e^{-\frac{1}{kT}\hbar w} - 1}$$



$$I(\omega) = \# \, \omega^2 \underbrace{W}_{\text{average energy of the oscillator}}$$

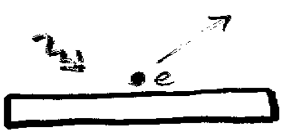
using Planck's hypothesis

$$W = \frac{\hbar\omega}{e^{\frac{\hbar\omega}{kT}} - 1}$$



WAVE THEORY OF LIGHT KEEP CRUMBLING

the photoelectric effect



ejection of electrons from a surface by the action of light

(1887) H. Hertz
Lenard

1905 Albert Einstein called into question the classical theory of light

Planck believed that light once radiated spreads out like a wave

Einstein proposed instead that

- radiant energy is quantized into concentrated bundles, later called photons
  $E = \hbar\omega$
- Experiments of interference and diffraction had been performed only in situations involving very large number of photons

  Interference = results from averages of individual photons behavior.

- Cited the photoelectric effect as an experiment to test his theory

1914 Millikan performs carefull experiment on the photoelectric effect and confirms the quantized theory.

So, LIGHT = WAVE or PARTICLE ?

$\Uparrow$ 19th century

$\Uparrow$ 20th CENTURY

QUANTUM MECHANICS

Statistical interpretation

λ

mass: m

linear momentum: p

1924 LUIS de BROGLIE

Particles are endowed with wave properties.

A particle of momentum p has an associated wavelength of

$$\lambda = \frac{h}{p}$$

h: Planck's constant

1927-1928 C. DAVISSON, L. LESTER observe the diffraction of electrons.