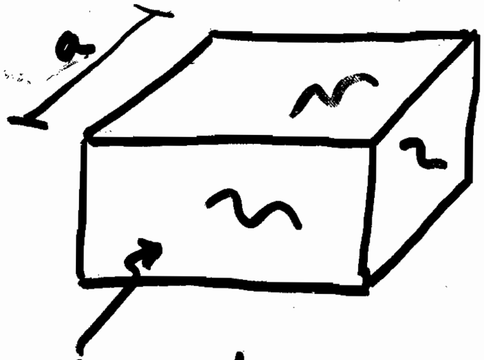
APPROACH #2 CALCULATION of the electromagnetic energy density u inside the cavity

First, let's count the number of electromagnetic modes inside a cavity



perfect reflecting walls

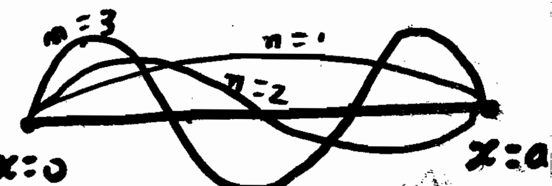
$$E(x) = E_0 \sin\left(\frac{2\pi}{\lambda} x\right) \sin(\omega t)$$

where $\lambda \dfrac{\omega}{2\pi} = c$

The value of ω specifies the electromagnetic mode.

Many modes can exist inside the cavity (a cube of side "a"). But, not All values of ω are allowed.

The requirement that $E=0$ at $x=a$ implies



$$\frac{2\pi}{\lambda} a = n_x \pi$$

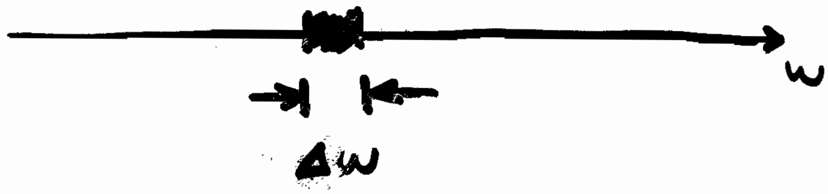
$$\Longleftrightarrow$$

$$\frac{1}{c}\frac{\omega}{2\pi} = \frac{n}{2a} \quad \text{or} \quad \boxed{\omega = 2\pi\, n \frac{c}{2a}}$$

$n = 1, 2, 3, \ldots$

modes allowed in the cavity

In a given interval $\Delta\omega$ how many modes there exists?



Answer: $\frac{1}{2\pi}\frac{2a}{c}\Delta\omega$ (one dimensional "cube")

In a three dimensional cube

$$\left[\frac{1}{2\pi}\times\frac{2a}{c}\right]^3\times\left[\frac{4\pi\omega^2}{8}\Delta\omega\right] =$$ Number of modes allowed (in a cube cavity of side "a") that have a frequency between ω and $\omega+\Delta\omega$

$$=\frac{V}{2\pi^2c^3}\omega^2\Delta\omega \quad , \text{ where } V\equiv a^3$$

Since for each mode, 2 polarizations are possible,

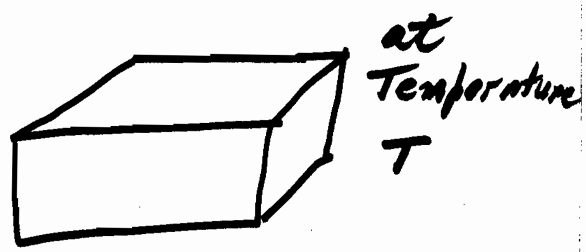
$$=\frac{V}{\pi^2c^3}\omega^2\Delta\omega$$

Thus, we find that

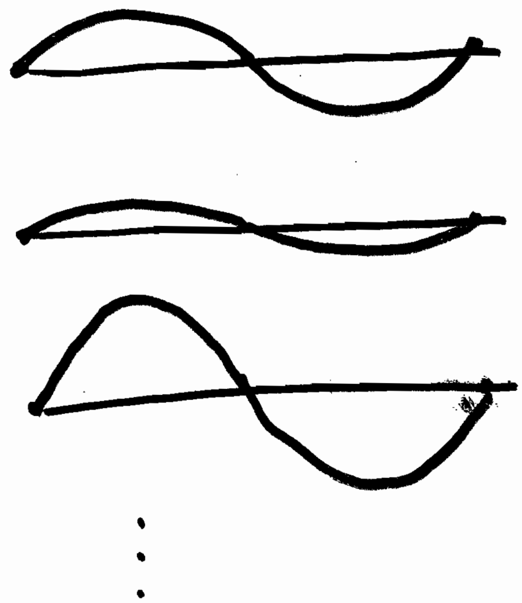
$$\frac{\omega^2}{\pi^2 c^3}\,\Delta\omega \equiv N(\omega)\,\Delta\omega$$

Number of modes per unit volume that have frequencies between ω and $\omega+\Delta\omega$

WHAT is the ENERGY STORED in EACH MODE?



Since the perfectly reflecting walls are kept at temperature T, we can picture a given mode interchanging energy with the walls. That is, sometimes the mode will have small amplitudes, other times large amplitudes, etc. But it will have an average amplitude, and, hence, an average energy. Let's call it W.



(W could depend on T and the frequency ω of the mode)

Accordingly, the energy density (J/m^3) in the cavity, contributed by the modes whose angular frequency lie between ω and $\omega + \Delta\omega$

$$N(\omega)\, W\, \Delta\omega =$$

$$= \frac{\omega^2}{\pi^2 c^3} \underset{\sim}{W}\, \Delta\omega \equiv u(\omega)\, \Delta\omega$$

$$u(\omega) = \frac{\omega^2}{\pi^2 c^3} \underset{\sim}{W} \qquad (4)$$

average energy of the mode of frequency ω

Notice the similarity between expressions (3) and (4)